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A NEOCLASSICAL GROWTH MODEL WITH PUBLIC SPENDING

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A Neoclassical Growth Model with Public Spending

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ABSTRACT**

This paper analyses the effect of public expenditures in the context of a modified Solow model of capital accumulation with optimising agents. The model identifies optimal government size and optimal composition of public expenditures which maximize the rate of growth in the dynamics to the steady state and maximize the long run level of per capita income. Different allocations of public resources lead to different growth rates in the transitional dynamics depending on their elasticity. However effects from fiscal policy are only temporary and disappear in the steady state. Finally we argue that neglecting the non-linear nature of the relationship between government spending and growth may lead empirical studies to biased results.

***Keywords**:* neoclassical and augmented growth models, fiscal policy, public spending composition.

***JEL classification**:* E62, O40, H50, E13, H20.

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1 INTRODUCTION

In their seminal contribution, Arrow and Kurz (1969) develop a neoclassical model of growth where aggregate production benefits from public capital services and government finances public capital by levying a proportional income tax, subtracting resources from private agents (see also Fisher and Turnovsky, 1998; Bajo-Rubio, 2000). This twofold influence implies a non-monotonic relationship between government size and growth. Given the properties of diminishing returns to private and public capital, the impact of government policy is limited to the transition path to the steady state to which the economy converges in the long run.

The property of convergence, implicit in neoclassical models of growth, is an appealing feature in the light of empirical analysis of growth. Convergence has, in fact, been shown to have considerable explanatory power (Temple, 1999).

Endogenous growth theories in the late Eighties caused a surge of interest in models of growth with fiscal policy. The first contribution in this area is the work of Barro (1990) who developed a model where government plays an active role in influencing long run growth (see also Futagami et al, 1993). All government spending is implicitly productive, it complements private inputs and it is included in the production function. The model determines the optimal level of public spending, using a non-monotonic relationship between government size and growth. Given the absence of diminishing returns to capital, endogenous models allow government to permanently influence economic growth (Romer, 1994).

Lee (1992), Devarajan et al (1996) expand on Barro's model, allowing different kinds of government expenditures to have different impacts on growth. By employing the traditional distinction between productive and non-productive spending (Glomm and Ravikumar, 1997; Kneller et al, 1999), they are able to determine the optimal composition of different kinds of expenditures, based on their relative elasticities. Following a similar line, Chen (2006) investigates the optimal composition of public spending and its relationship to economic growth. He established the optimal productive public service share of the total government budget and the optimal public consumption share, determined by policy and structural parameters.

In this paper we analyze the effect of fiscal policy, in the context of a modified Solow model of capital accumulation with optimising agents. Fiscal policy aims to create different kinds of public capital through accumulation financed through a proportional income tax. The model developed here determines the government size and the mix of government expenditures which maximize the rate of growth and the long run level of per capita income. We build the analysis within a neoclassical framework, which is still a central organizing framework for empirical research on economic growth (Barro and Sala-i-Martin, 2004).

The model focus on transitional dynamics to the steady state. There is, indeed, an empirical consensus on the fact that the process of convergence towards the steady state may take many years to play itself out (Temple, 1999). This works makes a contribution in this direction.

We find a non monotonic relationship between growth and government expenditures when government size and composition of public expenditures are both optimised. Such a relationship disappears during the process of convergence. This differentiates our model from those of endogenous growth, in which policies affect growth permanently such as those of Devarajan et al (1996) and, more recently, Ghosh and Gregoriou (2008). In these similar results for government size and public spending composition are obtained.

Our model differs from that of Bajo-Rubio (2000) and Carboni and Medda (2007), since we introduce representative agents utility maximization by a benevolent government (Ramsey rule) and we relax the assumption of exogeneity of private savings ratio. In addition, and unlike Bajo-Rubio (2000), our model considers the role of the composition of public spending in the accumulation process.

According to Fisher and Turnovsky (1998) we consider all government spending as a stock (public capital) but, differently from these authors, we divide public capital into two components which may have different productivities. Hence, the government can influence growth by deciding the extent of its intervention and/or by deciding on the allocation of its resources in the two different components of public capital.

An other important conclusion of this paper is that neglecting the characteristics of non-monotonicity of public spending and the different impact different types of government have on growth results in mis-specified models which bias traditional empirical analysis (Slemrod, 1995).

The remainder of this paper is organized as follows. Section 2 presents the core model. Section 3 analyses the dynamics of the fiscal policies and the implications of the model. Conclusions are presented in section 4.

2 THE MODEL

In this section we model the effects of fiscal policy on growth as a part of the aggregate economy, using an augmented neoclassical framework. We explicitly include the public sector in the production function as a distinct input based on the rationale that government services are not a substitute for private factors, and resources cannot be easily transferred from one sector to another. Public capital provide flows of rival, non-excludable public services, which would not be provided by the market. Flows are proportional to the relative stocks and enter the production function together with private capital, labour, and exogenous labour-augmenting technological change.

We follow Arrow and Kurz (1969) by considering all government spending as an accumulation process designed to create productive public capital. It has been demonstrated (Rodrik et al, 2004) that even expenditures on maintenance of law and order, health, education, social security, distribution of wealth and public administration influence growth. This happens through the creation and improvement of institutions, which can be considered as a type of public capital (Glaeser et al., 2004). Government can also influence growth through investments in other types of public capital such as roads and highways, telecommunication systems, R&D capital stock, other infrastructures (Aschauer, 1989; Kneller et al, 1999). The different impact of each type of government capital on productivity makes it all the more necessary to disaggregate the public budget into its various components.

2.1 Aggregate production and public capital

Production of output Y is specified in a Cobb-Douglas form and represents a special case of that in Arrow and Kurtz (1969):

$$Y = K_P{}^{\alpha} \left(LE\right)^{1-\alpha-\gamma_1-\gamma_2} K_{G1}{}^{\gamma_1} K_{G2}{}^{\gamma_2} \tag{1}$$

where K_P is private capital stock, L is total employment, E is labour-augmenting Harrod-neutral technology and K_G is public-sector or government capital. Elasticities are bounded between 0 and 1. Positive but diminishing returns to single inputs and constant returns to scale are assumed. Unlike Arrow and Kurz (1969) and Bajo-Rubio (2000) we consider two categories of public capital (K_{G1}, K_{G2}) both of which are characterised by elasticities which depend to their productivity (γ_1, γ_2).

Following the main literature, we assume a permanent balanced government budget and rule out debt-financing of government spending (Fisher and Turnovsky, 1998). Public spending is financed by levying an average flat-rate tax on income τ ($0<\tau<1$):

$$\tau Y = G = G_1 + G_2 \tag{2}$$

$$G_1 = \phi G; \quad G_2 = \left(1-\phi\right)G \tag{3}$$

where G_1 are traditional core productive expenditures, G_2 are all others productive government expenditures and ϕ ($0\leq\phi\leq1$) is the share of G_1 on total spending.

Public capital accumulation depends on total government revenues. Assuming equal depreciation rates δ for different kinds of public capital, accumulation dynamics are defined by:

$$\dot{K}_{G1} = \phi G - \delta K_{G1}; \qquad \dot{K}_{G2} = (1-\phi)G - \delta K_{G2} \tag{4}$$

and from eq. (2) we get:

$$\dot{K}_{G1} + \dot{K}_{G2} + \delta(K_{G1} + K_{G2}) = \tau Y \tag{5}$$

where dots indicate time derivatives. If government sets ϕ=1, then only accumulation of public capital of type 1 will be financed. For ϕ=0, the government sets G_1=0 and net growth of public capital will involve only capital of type 2.

Equations 2-4 also show that, for a given ϕ, if government wants to raise investment in public capital it is necessary to augment the tax rate τ. The economy will benefit from increased public capital but it must support a greater fiscal burden, which subtracts resources from private firms. As long as public capital productivity is equal to private capital productivity, changes in fiscal policy will have neutral effects on overall production. By contrast, a trade-off between private and public capital productivity occurs and, given their different productivity, the effects of an expansion (reduction) in government spending will depend on the composition of expenditure.

2.2 Capital accumulation and dynamics

The accumulation of public capital builds on two conflicting aspects of government spending (G). One is a detrimental effect, taxes which reduces private resources, and the other is a positive one, investment in public capital (Aschauer, 1989).

The rationale for a non-monotonic relationship is fairly simple: the growth rate increases with G up to a maximum level and then starts diminishing. One important target of public spending is to ameliorate growth performance by improving the marginal productivity of the private sector's physical capital and labour. This is generally attained by providing social and economic infrastructures, since these help private investment and promotes growth. Assuming private maximizing behaviour, the marginal product of capital receives beneficial effects from additional services. At the same time taxes have a detrimental effect, as they make individuals worse off. The optimal level of government infrastructure occurs when the marginal product of public infrastructure equals marginal social costs. Any public infrastructure beyond this level crowds out private investment, reduces the level of output and has a frictional effect on growth.

Private capital accumulation depends positively on the private saving (Y-C), and negatively on the average tax rate. For simplicity we assume a depreciation ratio δ equal to that of public capital[1]:

$$\dot{K}_P = (Y - C)(1 - \tau) - \delta K_P \tag{6}$$

We assume a representative consumer-producer agent who maximizes a constant inter-temporal elasticity of substitution utility function over an infinite planning horizon as given by:

$$U = \int_0^\infty \frac{c^{1-\theta} - 1}{1-\theta}\, e^{-\rho t}\, dt \tag{7}$$

where c represents per capita consumption, $\rho>0$ is the constant rate of time preference, and $\theta>0$ is the inverse of the inter-temporal elasticity of substitution. The inclusion of agents' utility optimization, and the relaxation of exogeneity of private saving ratio assumption, differentiate this work from that of Bajo-Rubio (2000). Here our goal is to find fiscal policies which maximizes a representative agent's lifetime utilities.

Expressing accumulation equations (4) and (6) and production function in terms of (technology-augmented) labour input we have:

$$\dot{k}_P = (y - c)(1 - \tau) - m k_P \tag{8}$$

$$\dot{k}_{G1} = \phi \tau y - m k_{G1} \tag{9}$$

$$\dot{k}_{G2} = (1 - \phi)\tau y - m k_{G2} \tag{10}$$

where $m = \delta + n + x$ and lower case letters indicate variables divided by (LE), n is the labour growth rate and x the labour-augmenting technological progress.

Output per unit of technology-augmented labour is:

$$y = k_P^{\alpha} k_{G1}^{\gamma_1} k_{G2}^{\gamma_2} \tag{11}$$

The maximization of utility is subject to costate variable equations (8)–(10). The optimization problem thus leads to the following Hamiltonian:

$$H = \frac{c^{1-\theta} - 1}{1-\theta} + \lambda_3 \dot{k}_P + \lambda_1 \dot{k}_{G1} + \lambda_2 \dot{k}_{G_2} \tag{12}$$

[1] Ai and Cassou (1995) develop a model with different δs. In their empirical investigation they estimate a lower depreciation rate for public capital.

where λs denote costate variables (Lagrange multipliers).

From the first order condition on the Hamiltonian and differentiating with respect to time we obtain the dynamic of consumption:

$$\frac{\partial H}{\partial c} = c^{-\theta} - \lambda_3(1-\tau) = 0 \tag{13}$$

Differentiating with respect to time:

$$\frac{\dot{c}}{c} = -\frac{1}{\theta}\frac{\dot{\lambda}_3}{\lambda_3} \tag{14}$$

2.3 *Steady state equilibrium*

Growth of public and private capital is bounded by the diminishing returns. We can then derive expressions for k_P, k_{G1} and k_{G2} in the steady state, as a result of the system of six differential equations given the production function (11):

$$\left\{\dot{k}_P = 0;\ \dot{k}_{G1} = 0;\ \ \dot{k}_{G2} = 0;\ \ \dot{c} = 0;\ \ \dot{\lambda}_1 = 0;\ \ \dot{\lambda}_2 = 0\right\} \tag{15}$$

given the dynamic of the costate variables:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial K_1} \tag{15.a}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial K_2} \tag{15.b}$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial H}{\partial K_3} \tag{15.c}$$

Imposing eq.(9) and (10) equal to zero we obtain:

$$k_p^{\alpha}\ \dot{k}_{G1}^{\gamma_1 - 1}\ \dot{k}_{G2}^{\gamma_2} = \frac{m}{\phi\tau} \tag{15.d}$$

$$k_p^{\alpha}\ \dot{k}_{G1}^{\gamma_1}\ \dot{k}_{G2}^{\gamma_2 - 1} = \frac{m}{(1-\phi)\tau} \tag{15.e}$$

substituting (15.d) into (15.a) and imposing $\dot{\lambda}_1 = 0$:

$$\dot{\lambda}_1 = \left(\rho + m\left(1-\gamma_1\right)\right)\lambda_1 - \frac{1-\phi}{\phi}\ m\,\gamma_1\lambda_2 - \frac{1-\tau}{\phi\tau}\ m\,\gamma_1\,\lambda_3 = 0 \qquad (15.f)$$

substituting (15.e) into (15.b) and imposing $\dot{\lambda}_2 = 0$:

$$\dot{\lambda}_2 = \left(\rho + m\left(1-\gamma_2\right)\right)\lambda_2 - \frac{\phi}{1-\phi}\ m\lambda_1 - \frac{(1-\tau)}{(1-\phi)\tau}\ m\,\gamma_2\,\lambda_3 = 0 \qquad (15.g)$$

Obtaining y by imposing eq.(8)=0 and substituting it into $\dot{\lambda}_3 = 0$:

$$\dot{\lambda}_3 = \left(\begin{array}{l} \left(\rho + m\left(1-\alpha\right)\left(1-\tau\right)\frac{c}{k_p}\alpha\right)\lambda_3 - \phi\tau\alpha\left(\frac{m}{1-\tau} + \frac{c}{k_p}\right)\lambda_1 - \\ +\left(1-\phi\right)\tau\alpha\left(\frac{m}{1-\tau} + \frac{c}{k_p}\right)\lambda_2 \end{array}\right) = 0 \qquad (15.h)$$

Dividing (15.g) and (15.h) by λ_3 and solving the dynamic optimization problem yields the following expression for steady state values:

$$k_P^* = \left[\frac{\frac{\phi\,\tau}{m}\Delta^{1-\gamma_1-\gamma_2}}{\left(\frac{\phi}{1-\phi}\right)^{\gamma_2}}\right]^{\frac{1}{1-\alpha-\gamma_1-\gamma_2}} \qquad (16)$$

$$k_{G1}^* = \left[\frac{\frac{\phi\,\tau}{m}\Delta^{\alpha}}{\left(\frac{\phi}{1-\phi}\right)^{\gamma_2}}\right]^{\frac{1}{1-\alpha-\gamma_1-\gamma_2}} \qquad (17)$$

$$k_{G2}^* = \left[\frac{\frac{\phi\,\tau}{m}\Delta^{\alpha}}{\left(\frac{\phi}{1-\phi}\right)^{1-\alpha-\gamma_1}}\right]^{\frac{1}{1-\alpha-\gamma_1-\gamma_2}} \qquad (18)$$

where stars denote steady state values and

$$\Delta = \frac{\alpha\, m\left(1 + P + Q\right)}{\phi\,(\rho + m)}\,\frac{\left(1-\tau\right)}{\tau} \qquad (19)$$

P and Q are parameters deriving from algebraic transformation:[2]

$$P = \frac{m(\rho + m)\gamma_1}{-2m^2\gamma_1 + \rho^2 + 2\rho m - \rho m \gamma_1 + m^2 - m\rho\gamma_2 - m^2\gamma_2 + m^2\gamma_1\gamma_2} \tag{20}$$

$$Q = \frac{m\left[m(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)\right] + \gamma_2\rho}{-2m^2\gamma_1 + \rho^2 + 2\rho m - \rho m \gamma_1 + m^2 - m\rho\gamma_2 - m^2\gamma_2 + m^2\gamma_1\gamma_2} \tag{21}$$

Substituting (16)-(18) into (11) gives the long-run steady state output per unit of technology-augmented labour:

$$y^* = \left[\left(\frac{\phi}{m} \tau \right)^{\gamma_1 + \gamma_2 + \alpha} \Delta^{\alpha} \left(\frac{\phi}{1-\phi} \right)^{-\gamma_2} \right]^{\frac{1}{1-\alpha-\gamma_1-\gamma_2}} \tag{22}$$

Rearranging terms and substituting Δ:

$$y^* = \left[\left(\frac{1}{m} \right)^{\gamma_1 + \gamma_2 + \alpha} \left(\frac{\alpha\, m(1 + P + Q)}{\rho + m} \right)^{\alpha} \tau^{\gamma_1 + \gamma_2} (1-\tau)^{\alpha} \phi^{\gamma_1} (1-\phi)^{\gamma_2} \right]^{\frac{1}{1-\alpha-\gamma_1-\gamma_2}} \tag{23}$$

The steady state level of output is related to exogenous and endogenous factors, as well as to the elasticities in the production function. Exogenous factors are the rate of depreciation of capital inputs, the rate of population growth and technological progress which are implicit in m (all negatively related).

Endogenous factors are the public policy instruments: *1)* the size of the government, expressed as the ratio of total government spending over total output, τ, and, *2)* the allocation of the public budget to the accumulation of K_{G1} and K_{G2} expressed by ϕ and $(1-\phi)$ respectively.

Public policy instruments have ambiguous effects on the steady state level of output per worker. The term τ in equation (23) represents a positive impact of the share of government size on steady state output, since a fraction τ of output is devoted to the creation of productive public capital. This latter positively influences total output at elasticity equal to $\gamma_1+\gamma_2$. By contrast the term $(1-\tau)$ represents a detrimental aspect of government spending, since only a fraction $1-\tau$ of total output (i.e. the private agent's disposable income) remains to influence production at elasticity α.

Equation (23) supplies another interesting piece of information. Given the size of government, the composition of public spending plays a significant role in determining the level of output per worker. The level of output per

[2] The complete algebra is not reported here for reasons of space. It is however available upon request.

unit of labour and the share of government spending used for investment in public capital of type 1 (or type 2), captured by the parameter ϕ (or 1-ϕ) are linked by a non-monotonic relationship. As long as $\gamma_1 \neq \gamma_2$, an allocation of resources in favour of public capital with higher elasticity will raise the steady state level of output per worker. However this process of shifting public resources cannot be continued indefinitely due to the diminishing returns on public capital. It is worth highlighting that Devarajan et al (1996) obtained a similar result but within an endogenous framework.

3 FISCAL POLICY AND TRANSITIONAL DYNAMICS

The aim of the government in a decentralised economy is to administer the public sector in the nation's interest, taking the private sector's preferences as given. Concerning our framework the government's problem is to choose τ and ϕ in order to maximize the representative agent's utility given the budget constraint (2).

In this section we examine the relationship between τ, ϕ and the level of income per capita in a dynamic framework. Equation (23) represents the level of income per unit of technology augmented labour in the steady state where the growth rate of y, k_P, k_{G1}, k_{G2} is zero. If the economy experiences a shock, transitional dynamics designed to reach a new equilibrium will be stimulated. Equilibrium will be reached after a transition period characterized by positive but declining growth rates. When this process ends the economy is in a new the steady state, the capital stock and output has reached levels at which the new rate of net investment is only sufficient to maintain a constant capital/labour ratio.[3]

Log-linearising equations (16)-(18), and given the production function (11), we can write the expression for the growth rate of output per unit of labour (Barro and Sala-I-Martin, 2004):

$$\frac{\dot{y}}{y} = \frac{dy}{dt} \Big/ y = \frac{d\left(\ln y(t)\right)}{dt} \cong \beta \cdot \left(\ln y^* - \ln y(t)\right) \qquad (24)$$

where β represents the convergence rate, depending on parameters from production and utility function[4] and y^* is the steady state output per unit of labour determined by equation (23).

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[3] The presence of three state variables severely limited our ability to formally investigate the transitional dynamics of the system, which is likely to be characterized by saddle-point behaviour. Employing models with two state variables Futagami et al. (1993); Glomm and Ravikumar (1994) and Turnovsky (1997), analyse the linear approximation around the steady state. However in our model when the economy departs far from its steady state, the linear approximations may become both quantitatively and qualitatively erroneous.

[4] See Barro and Sala-i-Martin (2004) for details.

Equation (24) shows that the rate of growth of output per unit of labour depends, negatively, on the level of y at time t (the convergence effect β), and, positively, on the level of y in the steady state.

The growth rate during the transition is related to the policy instruments τ and ϕ in the same way in which we described above, where we illustrated the influence of policy on the steady state level of output per worker. In detail, government can influence the growth rate of y by determining the size of its intervention and the relative shares of the two kinds of expenditure, G_1 and G_2, which are committed to the accumulation of public capital K_{G1} and K_{G2}. However, since the relationships between the rate of growth and τ and ϕ are non-monotonic, the influence of the effects of government policy is ambiguous, depending upon the current levels of τ and ϕ.

Taking logs of (23) and rearranging equation (24), it gives an expression for the average growth rate of y between the initial period 0 and time T.

$$\frac{1}{T}\cdot\frac{\dot{y}}{y}=\psi\left\{\frac{1}{1-\alpha-\gamma_1-\gamma_2}\begin{bmatrix}(\gamma_1+\gamma_2+\alpha)\ln\left(\frac{1}{m}\right)+\\+\alpha\ln\left(\frac{\alpha m(1+P+Q)}{\rho+m}\right)+\\+(\gamma_1+\gamma_2)\ln\tau+\alpha\ln(1-\tau)+\\+\gamma_1\ln\phi+\gamma_2\ln(1-\phi)\end{bmatrix}-\ln y(0)\right\} \qquad (25)$$

where $\psi=\frac{1-e^{-\beta T}}{T}$

From equation (25) one can see that the government size τ and the allocation parameter ϕ have two effects on the growth rate. There is a positive effect, due to the productive role of public capital $(\ln(\tau)$ and $\ln(\phi))$, and a negative effect, due to collecting resources from the private sector $(\ln(1-\tau))$ and the (mis)allocation of government expenditures with different levels of productivity $(\ln(1-\phi))$. From equation (25) it is straightforward to see that as τ goes to, either 0 or 1, then the growth rate goes to -∞. This implies that a too low or too high level of taxation can lead to negative growth rates.

Taking derivatives with respect to τ and ϕ separately and setting them to zero, we obtain the levels of τ and ϕ which maximize the growth rate (τ_{opt}, ϕ_{opt}):

$$\frac{\partial\left(\dot{y}/y\right)}{d\tau}=0 \Rightarrow \tau_{opt}=\frac{\gamma_1+\gamma_2}{\alpha+\gamma_1+\gamma_2} \tag{26}$$

$$\frac{\partial(\dot{y}/y)}{d\phi}=0 \Rightarrow \phi_{opt}=\frac{\gamma_1}{\gamma_1+\gamma_2} \tag{27}$$

For a given ϕ eq. (26) tells us that the maximizing value of τ is determined by the relative magnitudes of private and public capital elasticities as an increasing function of the ratio $(\gamma_1+\gamma_2)/\alpha$ (eq (26)).

The growth maximising level of government spending occurs when the marginal product of public capital equals marginal costs. Any public spending beyond this level crowds out private investment and reduces growth and the steady state level of output per worker. In other words, up to a certain point the distortional effects of tax are more than compensated for by the productive effects of public investment. As government grows, the detrimental effects of a high level of taxation prevail over productive effects. Further increases in τ will make the situation worse.

When government size is below τ_{opt} the marginal product of public capital is above the marginal product of private capital. In this case the economy is not making full use of all public capital potentialities and so it will reach a relatively low level of steady state income per unit of labour after a transition period characterized by a low rate of growth compared to the maximum. The opposite occurs for any $\tau>\tau_{opt}$. Clearly, the shape of this relationship depends on both private and public capital elasticities. The higher the relative share of the contribution of public capital to overall production the higher should be government investment in order to maximize growth. The same result is also achieved in Bajo-Rubio (2000) employing a neoclassical setting, where however, consumer maximization is not considered.

Successively we analyse the growth effects assuming that the government has different kinds of expenditures. From equation (25), given government size, the share of different kinds of expenditure in the public budget influences the growth rate of the economy during transition to the steady state and also the long run level of output per worker (eq. (23)). Again from equation (25), as ϕ goes to either 0 or 1, then the growth rate goes to $-\infty$. The direction of the composition effect depends on two aspects: 1) relative elasticities of different kind of public capital γ_1 and γ_2, and 2) the actual share ϕ and $1-\phi$ of government spending devoted to the accumulation of two different kinds of public capital.

Growth-maximizing values of ϕ can differ substantially across economies. When $\gamma_1=\gamma_2$ the best composition of the public budget assigns equal resources to G_1 and G_2, occurring when ϕ=0.5. The relationship between ϕ and the growth rate becomes asymmetrical when $\gamma_1 \neq \gamma_2$. In detail, when $\gamma_1 < \gamma_2$ the maximizing level of ϕ is less than 0.5, which corresponds to a relative higher share of resources attributed to G_2. The opposite occurs for $\gamma_1 > \gamma_2$.

For low levels of ϕ (specifically when $\phi < \frac{\gamma_1}{\gamma_1 + \gamma_2}$), an increment in G_1, that is, major accumulation of K_{G1}, results in a higher rate of growth for each economy even when $\gamma_1 < \gamma_2$. This is because at low levels K_{G1} exhibits high marginal returns relative to K_{G2}. However beyond a certain limit, determined by eq. (27), there are decreasing returns to K_{G1}. This reduces the advantages of investing in this kind of capital.

It is worth noting that eq. (26) and eq. (27) also represent maximising values for the steady state level of output per worker (y*) given by eq. (23) which in turn strongly depends on fiscal policy. However, and here we come to the essential point, given the properties of diminishing returns on public capital implicit in the model, any effect of policy on growth tends to disappear in the long run. In this state the transitional dynamics leave the economy with a growth rate determined by the rate of exogenous technological progress.

This is in fact what makes this work different from similar results obtained within endogenous frameworks (Devarajan et al, 1996; Ghosh and Gregoriou, 2008)[5], where the absence of diminishing returns do not arise when government variables grow along with private capital. So in these cases and unlike in this paper, endogenous models allow government to permanently influence economic growth.

Improvements decrease as the distance from optimality becomes narrower. Given $\boldsymbol{\Delta}\phi$, the gain in terms of output growth is larger when public capital elasticities are different. Once again, it becomes clear that the effect of a certain variation in the composition of government spending on growth depends on its proportion of the optimal value of ϕ. The higher the ratio $\boldsymbol{\Delta}\phi/\phi_{opt}$ the higher the final effect.

Finally, the speed at which a dynamic system approaches the steady-state equilibrium is clearly an important aspect of public intervention. For example, should the speed of convergence be rapid, then it would be possible to evaluate public policies with respect to their long-run effect on growth. Conversely, if the speed were relatively slow, then a consistent part

[5] The expression for the optimal share of the first public good is derived by Ghosh and Gregoriou (2008) as: $\phi^* = \left(\frac{\beta^{1/(\xi+1)}}{\beta^{1/(\xi+1)} + \gamma^{1/(\xi+1)}} \right)$ which, for the Cobb-Douglas case (ξ = 0), turns out to be the expression obtained in eq. (27).

of the dynamic adjustment would take place far away from the equilibrium. This highlights the need to analyse the transitional aspects of public policies (Atolia et al 2009).

4. CONCLUDING REMARKS

This paper shows that it is possible to maximise growth during the transition path to the steady state by controlling the size of the government and, also, the composition of government expenditure within the traditional neoclassical framework.

The model considers two different categories of government spending. It allows public capital productivity to differ and assumes that all government investment positively affects the productivity of private factors. For a given composition of public expenditure the aim is to find the spending level which maximise growth, reallocating resources between private and public capital according to their relative elasticity which maximise the agent's utility.

In the same way, for a given level of public spending (which can be easily considered fixed in the short-medium term) the aim of the model is to find the growth maximizing composition of public spending given private agent's choice. Changes in the spending structure lead to different growth rates, depending on their relative elasticity and share. This should induce governments to redistribute budgets between less and more productive public capital to achieve the optimum balance, thereby yielding stronger positive transitional growth effects than would otherwise be obtained.

The economy in the long term is in the steady state where growth only depends on exogenous factors. Fiscal policy has considerable influence on the levels of capital and output. Given the properties of diminishing returns on public capital, any effect of policy on growth tends to disappear in the long run. This differentiates this work from endogenous models where diminishing returns do not arise and the government can permanently influence economic growth.

Nevertheless, the transitional period of increased growth resulting from an optimal public spending can be rather long. As Barro and Sala–i–Martin (2004) suggest, at least five years are necessary to reach half of the transition, and if a broad concept of capital is used, this becomes 27 years. This highlights the need for short-medium term analysis such as that in this work.

Finally, the model has an important empirical implication which comes from the hypothesis of non-monotonicity between public spending and growth and from the effects of the composition of government expenditures. Research on optimal tax rates should take into account all the effects that public capital has on the economy. To be more precise, an increase in public capital at the expense of private capital is likely to accelerate or brake the economic growth rate. The latter effect typically depends on the marginal product of public and private capital respectively. Studies on fiscal policy which postulate a monotonic relationship (either positive or negative) and merely add an *ad hoc* government variable may well suffer from mis-specification and (log)linear regression analysis will be misleading.

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