Logic and phenomenology of incompleteness in illusory figures: New cases and hypotheses

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1. Introduction

Why is it relevant to analyze the role of incompleteness in illusory figure formation? Incompleteness probes the general problems of organization of the visual world and object segregation. The organization problem is one of the most important problems in visual neuroscience; namely: How and why are a very large number of unorganized elements of the retinal image combined, reduced, grouped and segregated to create visual objects? Within the problem of organization, illusory figures are often considered to be one of the best examples to understand how and why the visual system segregates objects with a particular shape, color, and depth stratification. Understanding the role played by incompleteness in inducing illusory figures can thus be useful for understanding the principles of organization (the How) of perceptual forms and the more general logic of perception (the Why). To this purpose, incompleteness is here studied by analyzing its underlying organization principles and its inner logic.

Incompleteness has been proposed to be one of the basic factors in inducing illusory figures. Indeed, some influential authors have hypothesized that illusory figures are perceived if and only if the inducing elements are incomplete. Gregory (1972, 1987) suggested that illusory figures are similar to perceptual hypotheses postulated to explain the unlikely gaps within stimulus patterns. This idea is derived

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from Helmholtz's Likelihood principle (Helmholtz, 1867). The well known Kanizsa's triangle (Kanizsa, 1955, 1979) would then be created by a top-down cognitive hypothesis to explain the gaps (missing sectors of the disks and missing parts of the outline triangle) within the stimulus. Similarly, Rock (1983, 1987) proposed that fragments similar to familiar figures elicit the cognitive hypothesis that a surface is occluding missing parts of inducing elements. Symmetry, incompleteness, interruptions, gaps, alignments among interruptions, familiarity, expectations and general knowledge are cues triggering the cognitive problem-solving process. Thus, in Kanizsa's triangle the alignment among gap terminations and the familiarity of the fragments would elicit a cognitive hypothesis of a triangle occluding three disks and an outline triangle. Coren (1972) considered the incompleteness of the stimulus as a depth cue that elicits the hypothesis of an occluding triangle.

Starting from a Gestalt background, Kanizsa (1955, 1979) suggested that the necessary factor for the formation of the illusory triangle is the presence of incompletenesses, or open figures, which activate amodal completion and closure processes that "create" complete perceptual elements behind a partially occluding illusory triangle.

The role of incompleteness is studied in the next three sections by: (1) defining the inner logic subtended by use of the term "incompleteness", (2) presenting new cases to clarify the phenomenology of incompleteness as a necessary and sufficient condition, and (3) suggesting an alternative hypothesis to explain illusory figures after analyzing problems with the incompleteness hypothesis.

2. *The logic and the paradox of "incompleteness"*

According to previous theoretical views, incompleteness was considered as a geometrical/structural factor that triggers illusory contour formation but, at the same time, as a perceptual property, beforehand, perceived as such and, afterwards, activating object hypotheses or completion processes. However, from a logical point of view if incompleteness is assumed as a geometrical factor:

(i) it cannot be defined as incompleteness, because the term "incompleteness" implies a perceptual phenomenon and not a geometrical property;

(ii) consequently, the use of the term "incompleteness" leads to the experience error (Köhler, 1947); that is, the implicit assumption that the structure of perceptual experience coincides with the optical array;

(iii) furthermore, as a geometrical property it does not require any solution, explanation or completion;

(iv) nevertheless, by assuming that incompleteness, open figures, or irregular elements can activate dynamical gestalt tendencies toward amodal completion and closure, as such, these tendencies should show two states and not just one as they do: the preceding uncompleted and open state, and the succeeding completed and closed one. This is even truer for the cognitive hypothesis. However, only one state is perceived. Thus, as a geometrical property, incompleteness cannot be a cause of itself as a perceptual phenomenon that in its turn determines something else, the illusory figure.

If incompleteness is assumed as a perceptual property, it implies that:

(i) as such it cannot be considered the cause of another perceptual property at the same epistemological level; that is, if incompleteness is perceived as incompleteness, there is no need to be discounted or completed to cause and elicit illusory figures, especially because incompleteness is in many cases a perceptual property independent from illusory figures;

(ii) it cannot be considered as such because as soon as it is perceived it is unperceived (reductio ad absurdum);

(iii) thus, the incompleteness of a perceptual object cannot be perceived because once it is perceived it is immediately completed and explained; that is, it is first perceived as incompleteness and then completed: to be completed it should first be perceived as incomplete. This is paradoxical: In fact, it is not perceived if and only if it is perceived, or it is perceived if and only if it is not perceived (the paradox of "incompleteness"). The same paradox can be also stated as follows: To perceive an incompleteness that has to be explained and completed, first, it should become a conscious phenomenal property and, then, it should become unconscious so that its perceptual result is annulled or completed. In other words, the incompleteness is not perceived (completed) only if it is perceived (not completed);

(iv) incompleteness considered as a depth cue implies that, as with all other depth cues, it should be perceived, but if it is perceived it is not perceived; thus, the argument is the same as that in point (iii);

(v) therefore, if it is a perceptual property, it is not a perceptual property. In other words, incompleteness can never be perceived because it should be always completed. Said yet another way, incompleteness should never be perceived even under conditions independent from illusory figures.

The previous arguments reject incompleteness as a factor for explaining illusory figure formation on the basis of merely logical reasoning. In the next section, the role of incompleteness is further weakened and redefined through new phenomenological conditions.

3. Phenomenology of incompleteness

The aim of this section is to argue phenomenologically that incompleteness is neither a necessary nor a sufficient factor in inducing illusory figures. The demonstration of this statement follows four steps: (i) incompleteness is not a sufficient condition; (ii) illusory figures do not necessarily complete incompletenesses (reductio ad absurdum of the role played by incompleteness); (iii) the shape of incompleteness does not predict the shape of illusory figures; (iv) incompleteness is not a necessary condition.

In this section, the phenomenology of incompleteness is accomplished by demonstrating that gestalt principles of grouping and figure-ground segregation of boundary contour discontinuities and brightness inhomogeneities underlie, acting as parallel factors, the formation of illusory figures without invoking incompleteness.

To demonstrate the phenomenological arguments, the spontaneous descriptions of subjects were recorded under the controlled conditions described in the next General Method section.

3.1. General Method

The stimuli were composed by the figures illustrated in the paper. The mean overall size of the stimuli was 10.8 x 8.6 deg of visual angle. The luminance of the white (background) paper under our test conditions was 80.1 cd/m². Black components had a luminance contrast of 0.97. Stimuli were presented on a computer screen under Osram Daylight fluorescent light (250 lux, 5600° K) and were observed binocularly from a distance of 50 cm with freely moving eyes.

Different groups of twenty observers, if not otherwise stated, described only one stimulus. This was done to avoid the possibility that the perception of one stimulus might influence the perception of another. New experiments, not reported here, demonstrated that the following phenomenal descriptions could change by presenting different sequences of the stimuli.

The subjects were naive as to the purpose of the experiments. All had normal or corrected-to-normal vision.

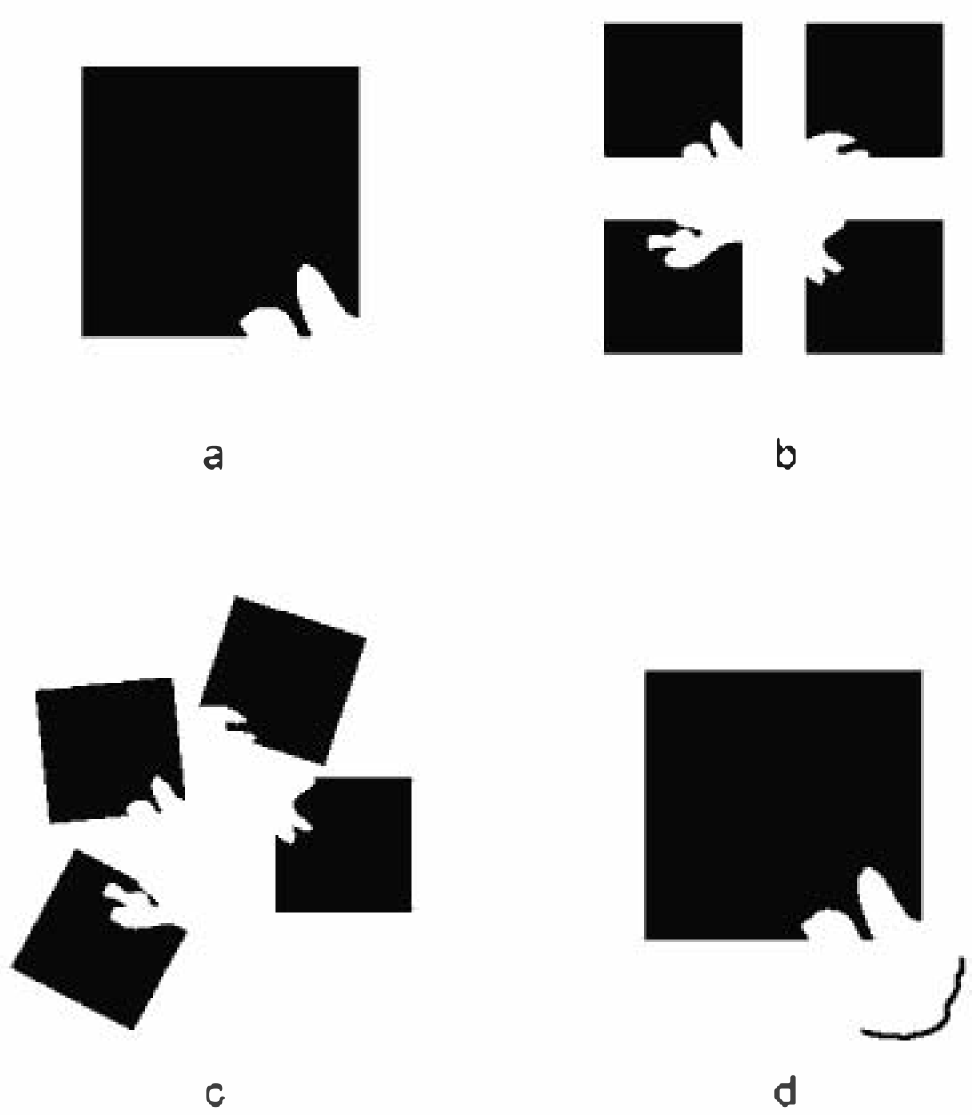
The subject's task was to report what they perceived by describing it. There was a training period preceding the experiment. During practice, subjects viewed some known figures from the illusory figure literature (*e.g.*, Kanizsa's triangle, etc.). Observation time was unlimited. All the observers quite promptly reported a description. Within the paper, if not otherwise stated, the reported descriptions contain the same words used by the subjects during the description of each stimulus and concern the phenomenal results obtained with a significant number of subjects (? 15). The subject descriptions are reported in italics and in quotation marks. In almost all studied conditions, the subjects were asked to sketch out what they perceived. Where necessary to clarify and specify the subjects' percepts, the results of the sketches are included.

3.2. Incompleteness is not a sufficient condition

The first phenomenological argument states that incompleteness is not a sufficient condition in inducing illusory figures. A condition *A (incompleteness)* is said to be *sufficient*
if (and only if) the truth (existence or occurrence) of *A* brings about the truth (existence or occurrence) of *B*. In other words, *A* cannot occur without *B*, or whenever *A* occurs, *B* occurs.

Even if the necessary logical condition cannot be easily disproved – namely, that all illusory figures contain incomplete inducing elements or that complete inducers cannot produce illusory figures (see Section 3.5), it can be easily demonstrated that not all occurrences of incomplete elements produce illusory figures (the sufficient condition). Within previous models, the sufficient condition is less interesting than the necessary one, but it is nonetheless important towards understanding the nature of incompletenesses and their organization.

Figure 1



Conditions showing that incompleteness is not a sufficient condition and the role of gestalt principle of grouping of concave discontinuities: (a) an incomplete square not inducing an illusory figure; (b) an irregular illusory amoeboid figure partially occluding four squares obtained by grouping incompletenesses; (c) by breaking the good continuation of (b) the illusory amoeboid largely disappears; (d) the articulation without rests factor segregates an illusory figure invading and partially occluding the black square.

Figure 1a is an example of a phenomenal incomplete square not producing an illusory figure. It is interesting to notice that Figure 1a is perceived simply as an "*incomplete square with two missing parts curved inward on the lower-right side and on the corner*". The term "incomplete" is merely phenomenal. More specifically, this kind of incompleteness appears like a gnaw or a nibble.

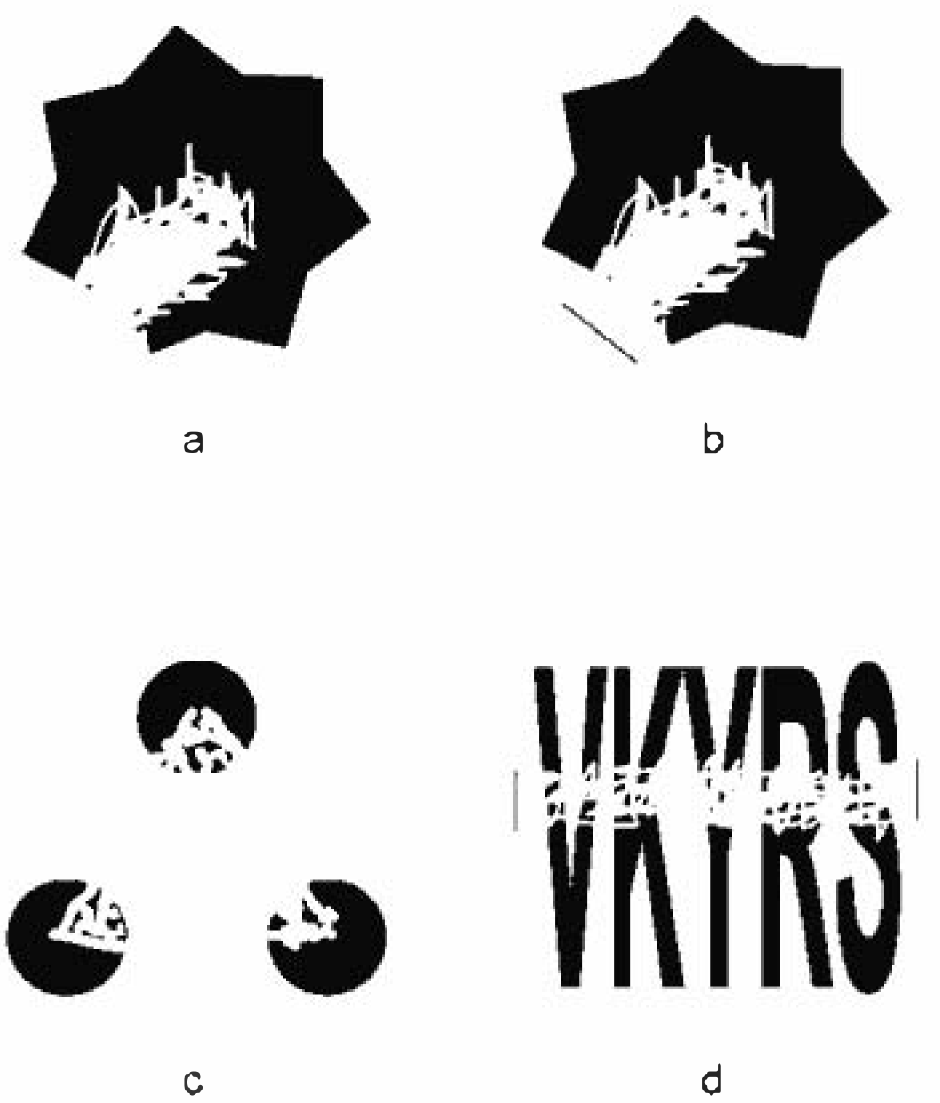
From an epistemological and logical point of view, the distinction between the term "incompleteness" and a mere geometrical term, like, for example, "concave discontinuities" of the contours of a square-like shape, is necessary: This geometrical property is not subjected to the previous logical arguments and paradoxes. At the same time, the expression "concave discontinuity" can also have a phenomenal meaning. The concave discontinuities are perceived, but they do not require any completion and, more generally, they do not tend to any other perceptual state. Thus, from an epistemological and phenomenological point of view, it can be stated that: "geometrical concave discontinuities, such as those illustrated in Figure 1a, induce the perception of an incomplete square".

By saying "concave discontinuities", our aim is not to define incompleteness. Instead, the term "concave discontinuity" highlights the role played by discontinuities along the boundaries that may be said to include incompletenesses, and that can be related to principles of grouping and figure-ground segregation. For example, if a concave discontinuity region segregates from other boundaries, because of similarity (or dissimilarity), they can group with similar concave discontinuous boundaries, as illustrated in the next figures. The replacement of incompletenesses with "concave discontinuities" is not considered either in geometrical or in phenomenal terms, but as a dynamic process of grouping and figure-ground segregation. It allows us to overcome the logical and phenomenal critiques that can be made against the term "incompleteness". Furthermore, it allows one to suggest hypotheses that are not restricted to illusory figures and that are basic for the segmentation and organization process of vision. This alternative way to consider incompleteness will be clarified below.

Concerning the result of Figure 1a, in defense of the cognitive and Kanizsa hypotheses, one can say that the reason why the incompleteness in Figure 1a does not induce an illusory figure is that the incompleteness shown does not need to be completed. Since this implies that incompleteness is not a sufficient factor, such examples elucidate the role played by incompleteness and, more particularly, the problem regarding under what conditions concave discontinuities are perceived as incompletenesses or as illusory figures. Thus, if incompleteness is not a sufficient factor, then the specific question is: Under which geometrical conditions does incompleteness get completed, as suggested by Gregory, Rock, Coren and Kanizsa? A possible answer to the question is illustrated in Figure 1b, where by arranging the incompletenesses of four squares similar to the one in Figure 1a in a grid, "*an irregular illusory amoeboid figure partially occluding four squares is perceived*".

By comparing these conditions with those in Figure 1a, the classical laws of organization in perceptual forms, as described by Wertheimer (1923), seem to play a basic role: "incompletenesses" or concave discontinuities group on the base of proximity, closure, similarity of contours and good continuation factors, creating an amoeboid shape. Therefore, by breaking their good continuation, as in Figure 1c, "*the illusory amoeboid largely disappears*". The roles of the closure factor and, in addition, of the articulation-without-rests factor are shown in Figure 1d. The curved line is the rest, included within the whole organization, assuming "*the perceptual role of the boundary of the illusory figure invading and partially occluding the black square*". A similar result is obtained in Figures 2a and 2b.

Figure 2



Different
different
complete (gnawed or nibbled) black polygon; (b) an illusory white paste, as painted with a tempera, in the outer and inner edge of the polygon; (c) and (d) non-homogeneous paintings across and along the inducing elements.

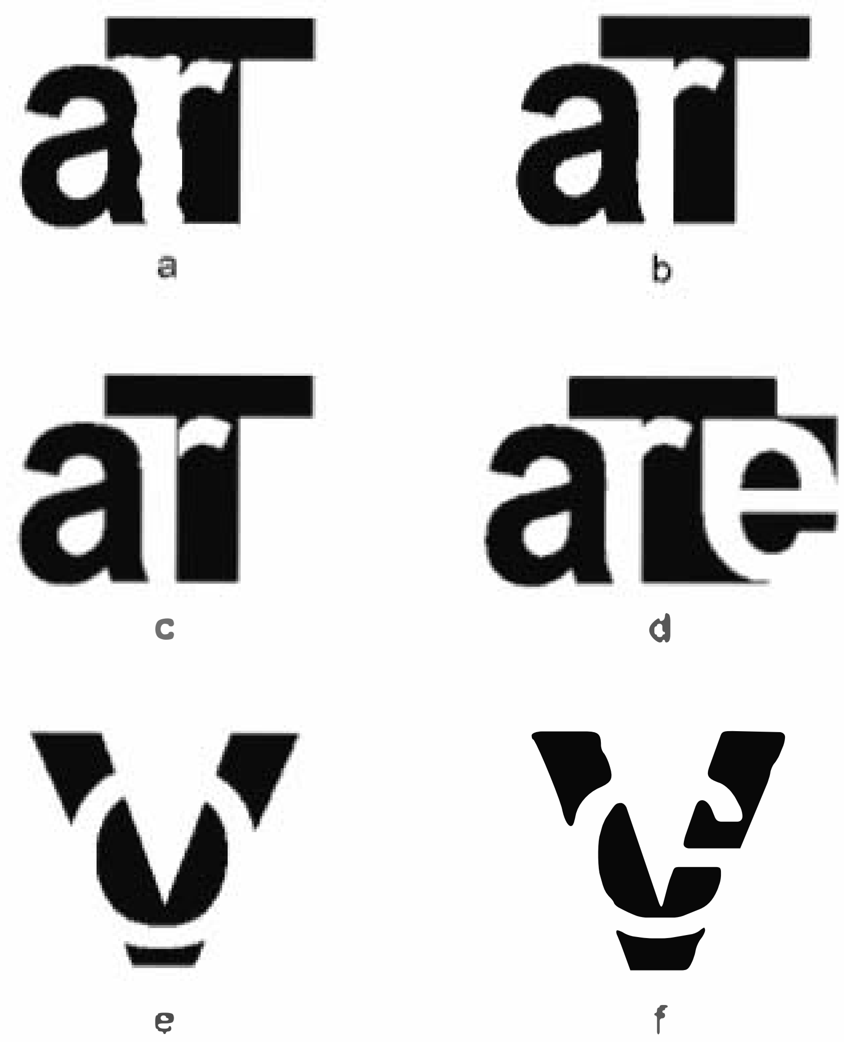
Figure 2a appears as "*an incomplete black polygon, whose incompleteness is perceived as an erased part of the polygon*", while Figure 2b appears as "*an illusory white paste, as painted with a tempera, in the outer and inner edge of the polygon*". In Figure 2a, differently from Figure 2b, "*very weak or no illusory contours are perceived outside the polygon*". It is worth highlighting that the incompleteness of Figure 2a assumes a specific form: "*Deletion or erasure*". As a consequence, from a phenomenological point of view it is not a mere incompleteness but "*something*" at a more specific level and different from the previous incompletenesses: "*A gnaw or a nibble*". Therefore, according to cognitive and gestalt models, the term incompleteness is an undetermined property that can assume a large number of forms. This uncertainty further weakens the phenomenological role played by incompleteness as considered by gestalt and cognitive models. As can be observed in Figures 2c and

2d, "*the illusory figures appear as non-homogeneous paintings across and along the inducing elements*".

Incompleteness can be reconsidered in the light of gestalt grouping principles even when it appears as a different and independent factor. In Figure 3a, "*the word ART is perceived*" (Pinna, 1990). The similarity/dissimilarity in the type and shape of the boundaries composing the R letter as well as the grouping of concave discontinuities, due to the good continuation principle, can be responsible for the perception of the word ART. In conjunction with similarity, the surroundedness principle of figure-ground segregation (Rubin, 1915, 1921) and past experience (Wertheimer, 1923) likely influence the pop out of the R letter. "*The illusory R appears in front of the A and the T, that each completes itself amodally*".

The role of past experience is more obvious in Figure 3b, where the similarity/dissimilarity principle is weakened and where the incompleteness within the T groups with the empty space between A and T, that by virtue of the surroundedness principle tends to appear as a figure. The surroundedness principle is working as well within the component R that can be considered as the incompleteness of T, which in its turn can be considered as a concave discontinuity that is a special case

Figure 3



Gestalt grouping principles as alternatives to incompleteness: (a) the word ART is perceived due to similarity/dissimilarity in the type and shape of the boundaries composing the R letter; (b) the illusory R emerges due to surroundedness and past experience principles; (c) the line separating the component of the R within the T weakens the emergence and segregation of the word ART and the R does not appear as an illusory figure; (d) The word ARTE is perceived and the E letter pops out due to past experience, surroundedness, good continuation, convex discontinuities along the T letter, closure and articulation without rests; (e) and (f) due to proximity factor mutilated or incomplete O and the G do appear in front of Vs or, alternatively, two holes having V shapes partially showing a black background and a white O and G partially perceived through them.

of the concave grouping principle (Wertheimer, 1923). It is interesting to note that "*the R appears on the same plane of the A, while partially occluding the T*".

In Figure 3c, "*the word ART emerges again but less strongly than in Figure 3b*". The presence of a line separating the component of the R within the T, and the empty space between the A and the T, together weaken the grouping of the R and favor the amodal completion of the former component behind the latter. "*The R does not appear as a clear illusory figure, even when it is clearly recognized*".

In Figure 3d, "*the word ARTE* (Italian for ART) *is now read*". The letter E is perceived despite the absence of incompletenesses or local concave discontinuities of the R. On the contrary, on its right side the T presents convexities or additions. Both R and E pop out from the background as figures, acquiring clear figural qualities. "*The letter E emerges from a black (upper-right and lower-left corners) and white (upper-left and lower-right corners) square that appears as a background*". "*The E appears with curved boundaries*". The perception of the E and its background demonstrates that the boundaries belong unilaterally to the illusory E. The E letter pops out as a figure likely because of the following principles: past experience, surroundedness, good continuation, convex discontinuities along the T letter, closure and articulation-without-rests. Despite the fact that perception of the four alphabetical letters seems to be ruled by different figural principles, they group together making the reading of the word ARTE easy.

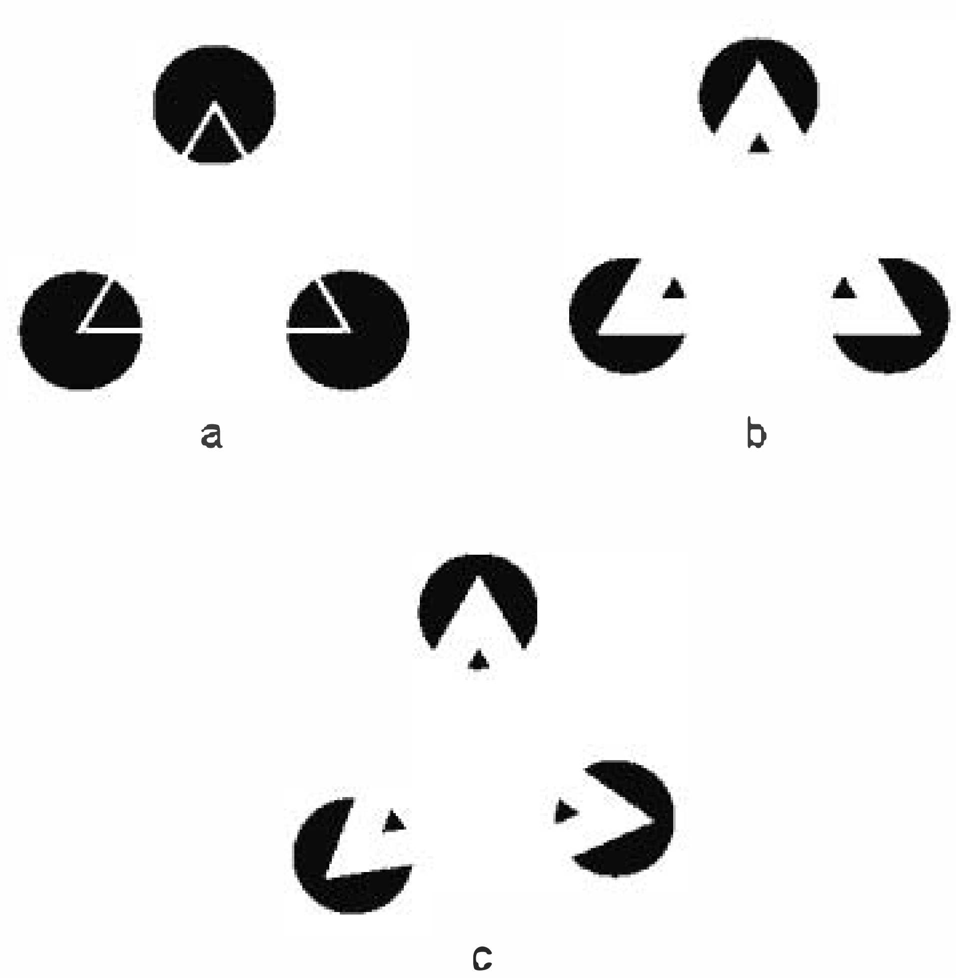
Because the convex discontinuities along the right side of the T are the opposite of the concave discontinuities on the left side that can be considered as the low-level description of the incompleteness, the illusory E letter is induced from something that is the opposite of incompleteness; i.e., convexities or additions.

Illusory figures are not completed in Figures 3e and 3f (Cocco, Pinna & Spillmann, 2000), where the best expected solutions to the incompletenesses on the basis of past experience are two black Vs, partially occluded, and an illusory O and G in front of them. However, "*illusory contours are not perceived in the outside edges of the Vs*" (20 out of 20 subjects for each stimulus). "*The O and the G do appear in front of Vs, but only partially, because they are seen as incomplete or mutilated*".

Other possible percepts are "*two holes having V shapes partially showing a black background and a white O and G partially perceived through them*" (12 out of 20 subjects for each stimulus).

These results suggest that grouping and figure-ground segregation principles play an important role in the explanation of illusory figures and, more specifically, in the understanding of their perceptual organization. However, by grouping "incompletenesses" through the synergistic combination of several principles, an illusory figure is not necessarily perceived (Figures 3e and 3f). How and whether it forms may depend on factors that are linked to grouping principles. In Figure 4a, the three corners of a virtual triangle are not grouped to produce an illusory triangular contour across the white space between the circles. Rather, the percept is typically of "*a white triangle partially perceived through three holes in a white surface on a black background*". "*The disks become circular holes and their boundaries belong to the white surface, while their black color completes itself amodally becoming a homogeneous black background*". "*The white space becomes a punched surface partially occluding a white triangle*". The effect is similar to the ones induced by Figures 3e and 3f, but it is interesting all the more so because each circular inducing element is perceived as an incomplete disk, not as a circular hole, when presented separated from others. It demonstrates the basic role of grouping principles that create emergent figures with new properties not reducible to the local components.

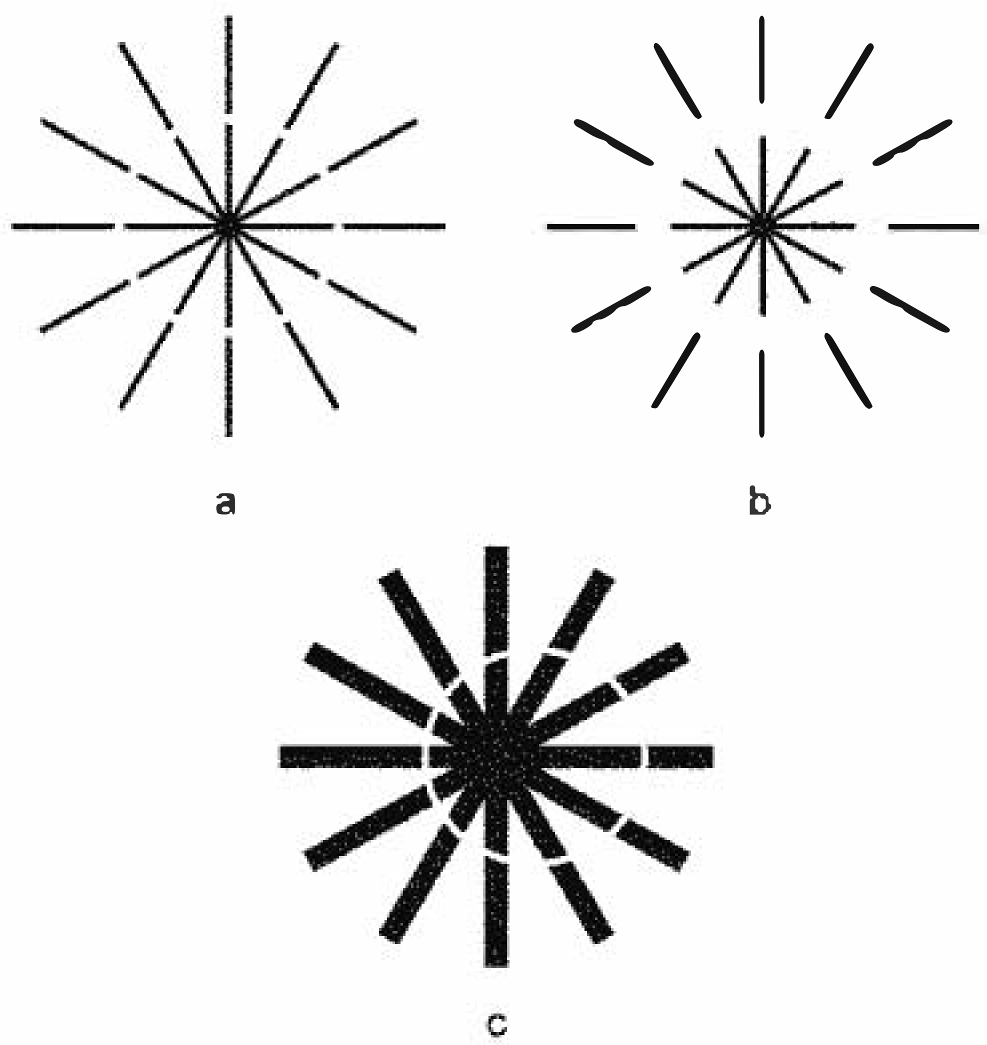
Figure 4



A special case of proximity factor induces: (a) a white triangle partially perceived through three circular holes in a white surface on a black background; (b) an illusory triangle; (c) an illusory triangle is perceived even when the good continuation factor is weakened.

Furthermore, these results demonstrate that, even if incompletenesses are aligned to favor good continuation, even if closure, proximity, and prägnanz factors work synergistically, even if the boundary contour of the triangle is thin enough to induce a strong figural effect (surroundedness and proximity principles of figure-ground segregation; Rubin, 1921), no illusory triangle may be created (Figure 4a). The opposite is perceived when viewing Figure 4b, where, by increasing the width of the boundary contours of the corners, "*an illusory triangle is clearly perceived*", even if the grouping factors are weakened or pitted against it (Figure 4c).

Figure 5



Variations in the ratio between the width of each interruption and the distance between the successive radial lines induces no illusory circumference in (a), a strong illusory annulus in (b) and not any illusory circumference in the right side of (c) but only disjoint bright interruptions or dashes, while the opposite is true in the left side of the (c).

The same argument is illustrated in Figure 5a, where bright spaces, interrupting each ray (incompleteness as interruption) in approximately its center, create "*a circumference that is virtual but not illusory*". In fact, "*the spaces may appear as disjoint bright spots on the radial lines*" (13 out of 20 subjects). However, "*the lines do not look like holes, as in Figure 4a, and the spots do not complete themselves amodally behind the white surface*". This result is interesting in the light of the following possible counterargument, using Figure 4a, to the critique of incompleteness. This counterargument states that, even if in Figure 4a an illusory triangle is not created, the reversed and complementary effect (disks as holes, illusory triangle as amodal triangle, etc.) is induced as an object hypothesis that can fill and explain the gaps within the stimulus pattern. Thus, the amodal triangle should represent the perceived solution to the incompleteness within the circular holes. Figure 5a provides a counterexample to this counterargument because "*it does not complete the spots or bright interruptions, as does happen by increasing the width of the circumference in Figure 5b*".

Without invoking the role of incompleteness, the factor that differentiates the percepts in Figures 5a and 5b is the ratio between the width of each interruption and the distance between the successive radial lines. By increasing the width of the amodal triangle of Figure 4a as in Figures 4b and 4c, or of the virtual circumference of Figure 5a as in Figure 5b, this ratio changes accordingly. All else being equal, by decreasing the distance (angle) between the radial lines, the completion of the bright interruptions becomes more likely and stronger, thereby creating an illusory circumference. As illustrated in Figure 5c, "*the right side of the figure does not show any illusory circumference but only disjoint bright interruptions or dashes, while the opposite is true in the figure's left side*".

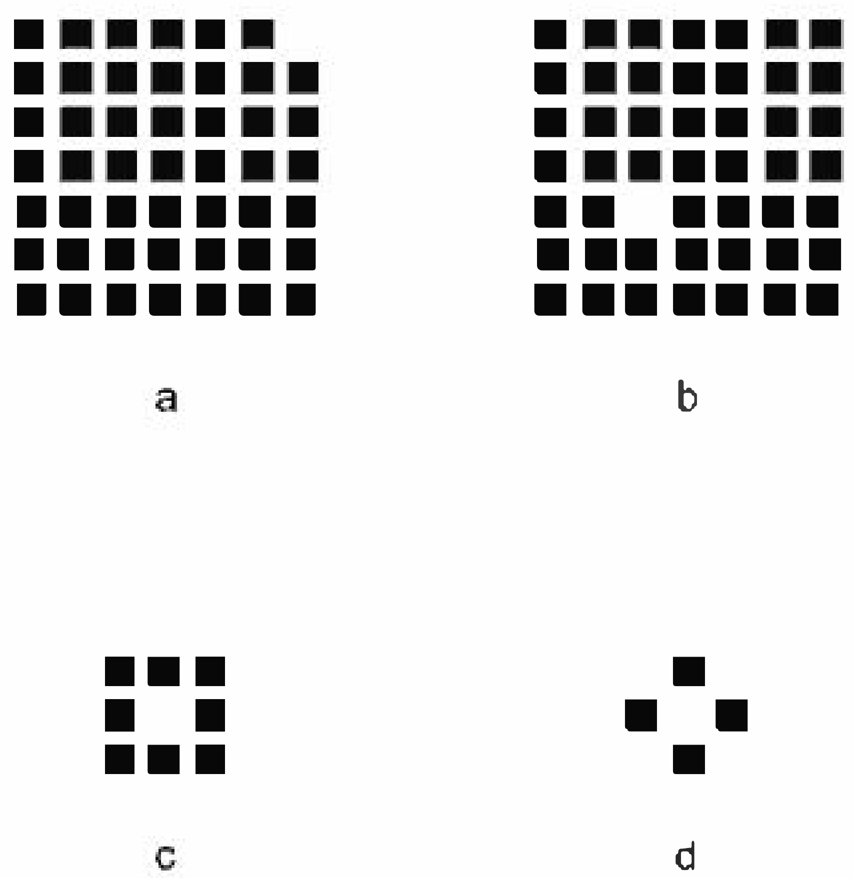
The perceived difference between the left and right sides of Figure 5c is strong enough to overcome any tendency of good continuation to weaken the difference between the two sides and enhance the perceptual grouping. This ratio property is a special type of the proximity factor, which pits horizontal versus vertical proximity against each other.

In summary, from a phenomenological point of view, one need not invoke completion processes of incomplete inducing elements to explain the previous results. They can be more easily explained in terms of perceptual grouping factors, suitably understood. In particular, it is not necessary that an illusory figure completes both local and global incompletenesses. Both these arguments are topics of the next section.

3.3. Illusory figures do not necessarily complete incompletenesses

In Figure 6a (Pinna, 1990), "*a square matrix, made up of small squares with a missing element in the left upper corner*", is perceived. This is perception of "incompleteness" without an illusory figure (not sufficient condition). In Figure 6b, "*the square matrix appears again incomplete but with an illusory bright square larger than the black ones and not occluding anything*". The occlusion of one small square is more an inference than a perceptual result. Furthermore, "*the four crossed black squares, all around the largest bright one, do not appear incomplete or partially occluded*" even though they are connected through T-junctions with the illusory square (as reported by the subject through a sketch) and are pairwise-colinear, which is a basic constraint that often leads to amodal completion. One may argue that the perceived incompleteness is global; i.e., of the square matrix and not of single black squares. Even if it were so, "*the square matrix does not appear completed*". To be completed, a small square should be perceived behind the illusory large square, that "*does not appear occluding anything even if it is tangent to the sides of four black squares all around, that in their turn do not appear incomplete or partially occluded*". To sum up, Figure 6b is a case of incompleteness that is not completed by an illusory square neither locally nor globally. This result represents a logical confutation of the role played by incompleteness if incompleteness is considered as the necessary condition.

Figure 6



Incompletenesses are not necessarily completed by illusory figures: (a) a square matrix with a missing element in the right upper corner; (b) an incomplete square matrix with an illusory bright square larger than the black squares and not occluding any element; (c) a square frame made up of black squares (control condition); (d) an illusory bright square occluding a black cross (control condition).

These perceptual results are particularly interesting even in the light of Figures 6c and 6d. By annulling the global effect of the matrix– that is, by leaving the just square frame around the large white square of Figure 6b, as illustrated in Figure 6c– neither the illusory figure nor the incompleteness is perceived, but "*only a square frame made up of black squares*". This result implies a global effect of the grid on the illusory figure formation in Figure 6b. By comparing this result with the one of Figure 6d, where the squares in the corners of Figure 6c are missing, another perceptual result emerges: "*an illusory bright square occluding a black cross*". Under these conditions, "*the black squares complete themselves amodally in a cross partially occluded by a large illusory white square*". This result is not perceived in Figure 6b and demonstrates that even complete squares can induce an illusory figure, contrary to theories that assert a necessary role for incompleteness.

However, a counterargument in favor of these theories states that, when the illusory figure is perceived, the inducing elements complete themselves amodally in a more global figure different from the geometrical inducing elements. Within this counterargument, incompleteness becomes an *a posteriori* result that can be defined and known only after the perception of the illusory figure. This kind of reasoning implies the same paradoxes defined in Section 2. Nevertheless, even accepting the logic of this counterargument, it is phenomenologically weakened by the perceptual result of Figure 6b, but it will be more definitively rejected in Section 3.5, when more probing counterexamples will be shown.

Figure 7 (Pinna, 1990) reports two conditions phenomenologically in agreement with the previous results: "*an illusory rectangular stripe behind the four black central squares*" (Figure 7a), and "*an illusory diagonal of five squares demarcating the separation between the two halves of an incomplete square matrix of squares*" (Figure 7b). Note that "*the illusory stripe* of Figure 7a *does not occlude any square, rather appears coplanar to the other squares on its sides above and below, yet appears behind the four black squares contained in its surface*". Figure 7c shows two alternative results: "*a large bright square similar to the one perceived in Figure 7d, with four black squares in its inner edges arranged along the arms of a virtual cross*"; and "*five bright squares at the same plane of the black small squares all around them*". These alternative percepts emphasize the role played by figure-ground principles in inducing illusory figures.

Figure 7

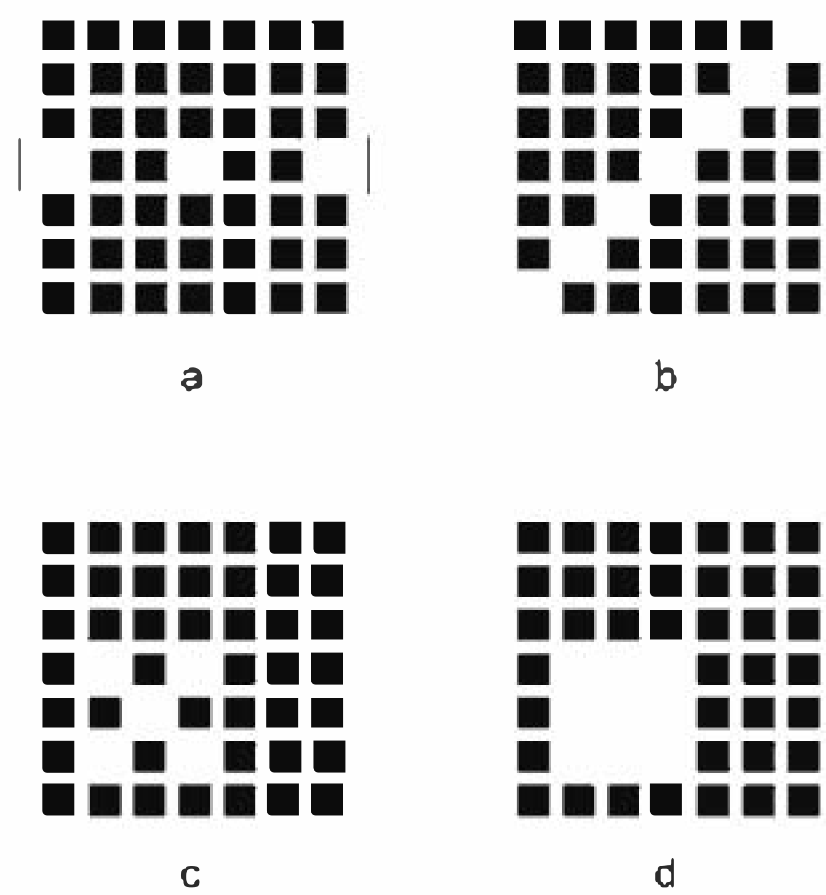


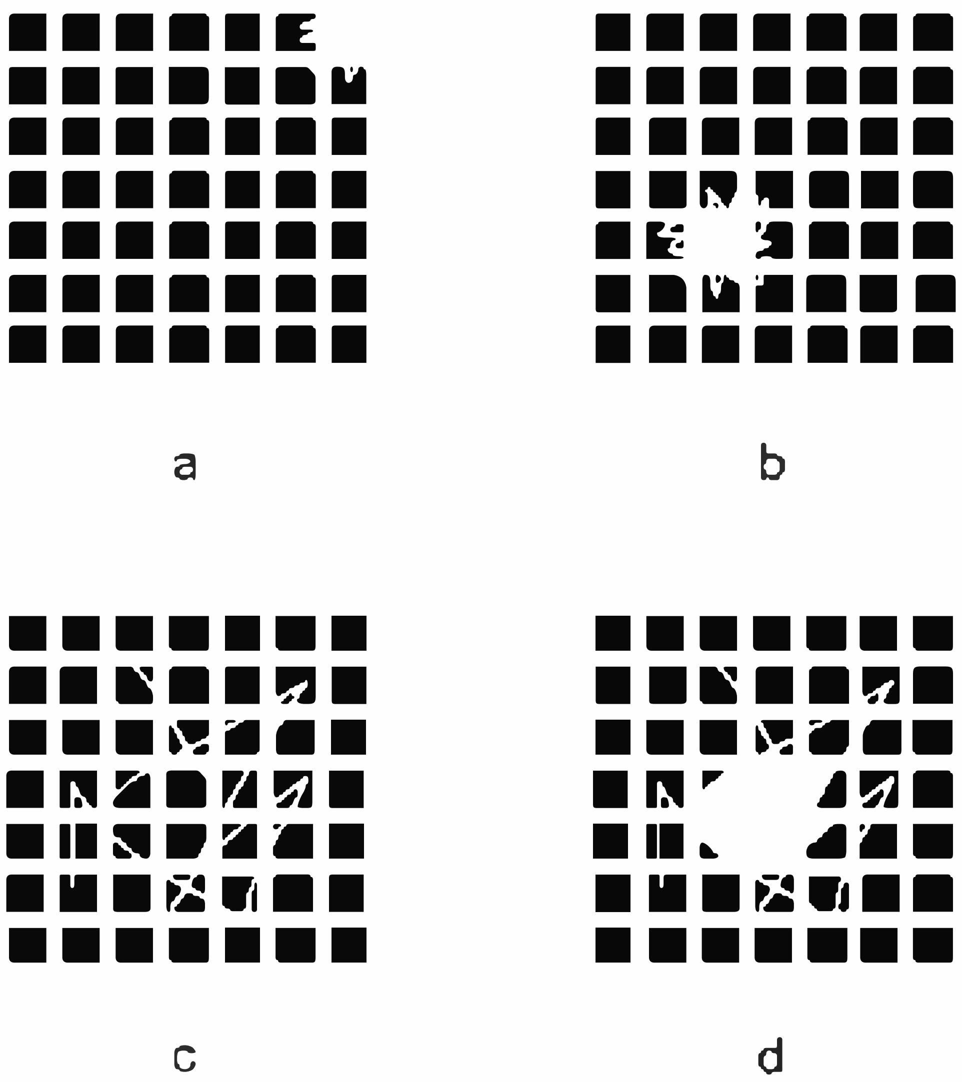
Figure-ground principles induce: (a) an illusory rectangular stripe behind the four black central squares; (b) an illusory diagonal of five squares demarcating the separation between the two halves of an incomplete square matrix of squares; (c) a large bright square with four black squares in its inner edges arranged along the arms of a virtual cross or, alternatively, five bright squares at the same plane of the black small squares all around them; (d) a large bright square.

The perceptual results of Figure 8 differ from those of Figure 6 in that "incompletenesses" are here accompanied by illusory figures similar to "*scribbles of white tint in front of one entirely occluded* (Figure 8a) *or some partially* (Figure 8b) *occluded squares*". In the experiment, 12 out of 20 subjects reported that, in Figure 8a, "*the scribble does not occlude the entire missing square at the corner, but only partially the three around it*".

The "*scribble*" perceived in Figure 8c, even if related to incomplete squares, "*appears to lie behind the crossed white bars of a window showing the dim interior of a room*". This result is similar to the one described in Figure 4a. In Figure 8d, the inside edge of the scribble of Figure 8a is totally white and the ratio between distances is changed, so that phenomenally "*the inner irregular white shape, similar to the body of an insect, appears in front of the partially occluded crossed bars while its legs appear intertwined in the bars*". Here, the bars are both occluded and occluding, depending upon the ratios.

On the basis of these results, the question is: Which are the inducing elements, the bars or the squares? If incompleteness is a necessary factor, then the question has a paradoxical answer: The effect is cause of its cause. In other words, the incompleteness of the bars and not of the squares can be decided only *a posteriori* and it becomes the cause of the perception of the incompleteness of the bars and not of the squares. This is paradoxical only if incompleteness is considered a necessary factor. However, this criticism has in

Figure 8



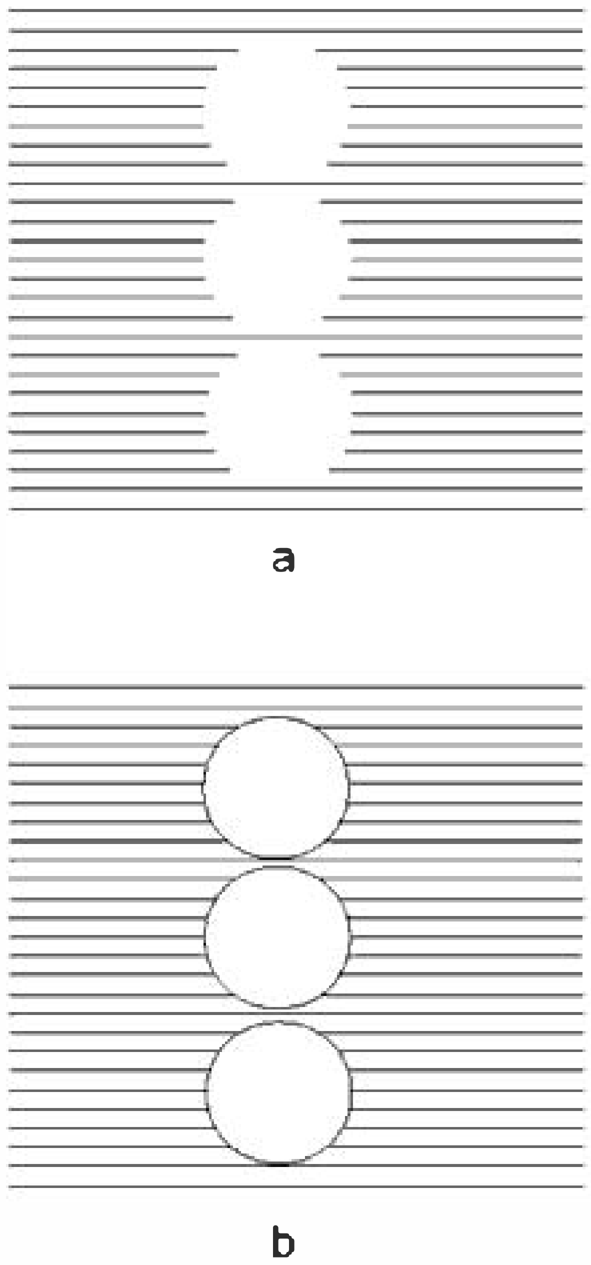
Different roles of incompletenesses in different form organizations: scribbles of white tint in front of one entirely occluded (a) or some partially (b) occluded squares; (c) the scribble lies behind the crossed white bars of a window showing the dim interior of a room; (d) the inner irregular white shape, similar to the body of an insect, appears in front of the partially occluded crossed bars while its legs appear intertwined in the bars.

the next Section 3.4 a further logical and phenomenological aspect from a different point of view. The new question is: Incompleteness of what?

3.4. The shape of incompleteness does not predict the shape of illusory figures

Once the geometrical shape of "incompleteness" has been defined, can the shape of the illusory figure be predicted? More particularly, does the shape of the illusory figure correspond to the shape of incompleteness? And does the shape of the illusory contours agree with the shape of real contours when these complete the shape of the same incompleteness? Many experimental data demonstrate the equivalence of illusory contours with real ones. Illusory contours are like real contours in: producing geometrical illusions (Farnè, 1968; Pastore, 1971; Gregory, 1972; Bradley and Dumais, 1975; Bradley and Petry, 1977; Meyer and Garges, 1979); in being enhanced by kinetic depth information (Bradley and Lee, 1982); in being subjected to apparent and stroboscopic motion (Sigman and Rock, 1974; Grünau von, 1979; Ramachandran, 1985); in producing figural after-effects (Smith and Over, 1976, 1979; Meyer and Phillips, 1980); in being used as targets or masks in visual masking experiments (Weissten et al., 1974; Reynolds, 1981); and in serving in information-processing tasks as a landmark aiding the localization of elements in visual space (Pomerantz et al 1981). Von der Heydt, Peterhans, and Baumgartner (1984), and Peterhans and von der Heydt (1987) found that neurons in cortical area V2 of macaques respond at locations where illusory contours are perceived. These neurons respond with a similar slightly delayed excitatory response to both illusory and real contours that crosses their receptive fields, and both real and illusory contours produced similar orientation tuning in these cells.

Figure 9

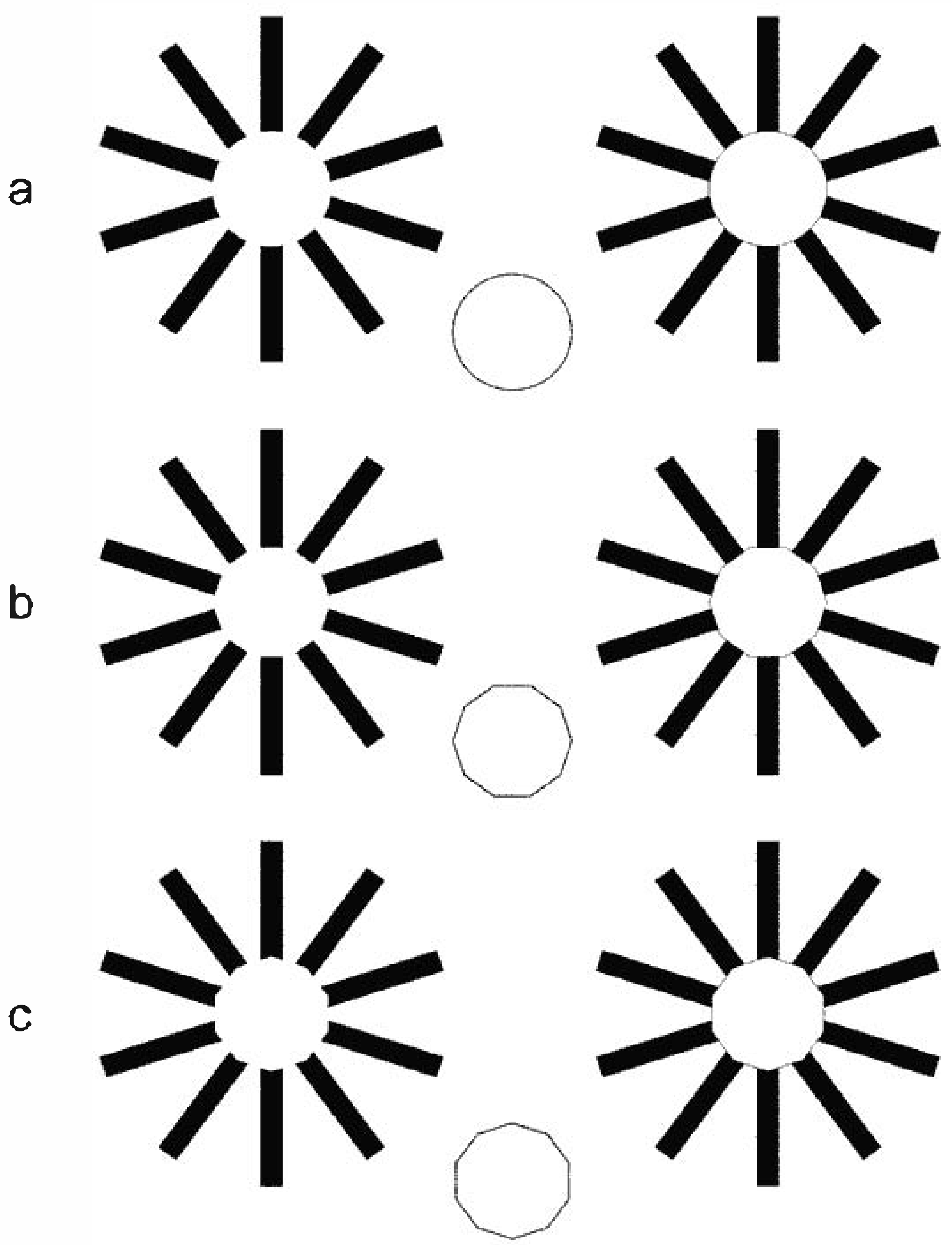


Illusory figure shape differs from the shape of the incompleteness. (a) An elongated or double lemon-like 8 amodally interweaved behind two single lines that appear curved is perceived despite the gaps correspond to three circles (b).

Despite these results, Figure 9 demonstrates differences between the two kinds of contours. In Figure 9a, a parallel distribution of horizontal lines encloses three circular gaps vertically arranged. However, they do not appear like three adjacent illusory circles but as "*a double 8 amodally interweaved behind two single lines that appear curved*". Furthermore, "*each circular shape of the 8 appears elongated or distorted with a shape similar to a lemon*". The "real" geometrical shapes of the incompletenesses, shown in Figure 9b to be "*three circles*", do not correspond to the shapes of these illusory figures.

The main point is: Why should three circular gaps be completed as described if the simplest figures that can solve the problem of incompleteness are three illusory circles? The complexity of the solution of the 8-like shapes, including their amodal completion, is far from being considered as the simplest solution. The general question raised by this figure is the following: If incompleteness is assumed to be a necessary factor, is it possible to predict the shape of the illusory figure that is induced to complete a gap in the simplest possible way? Figure 9a shows that the answer is No, and thus that real and illusory contours do not necessarily match.

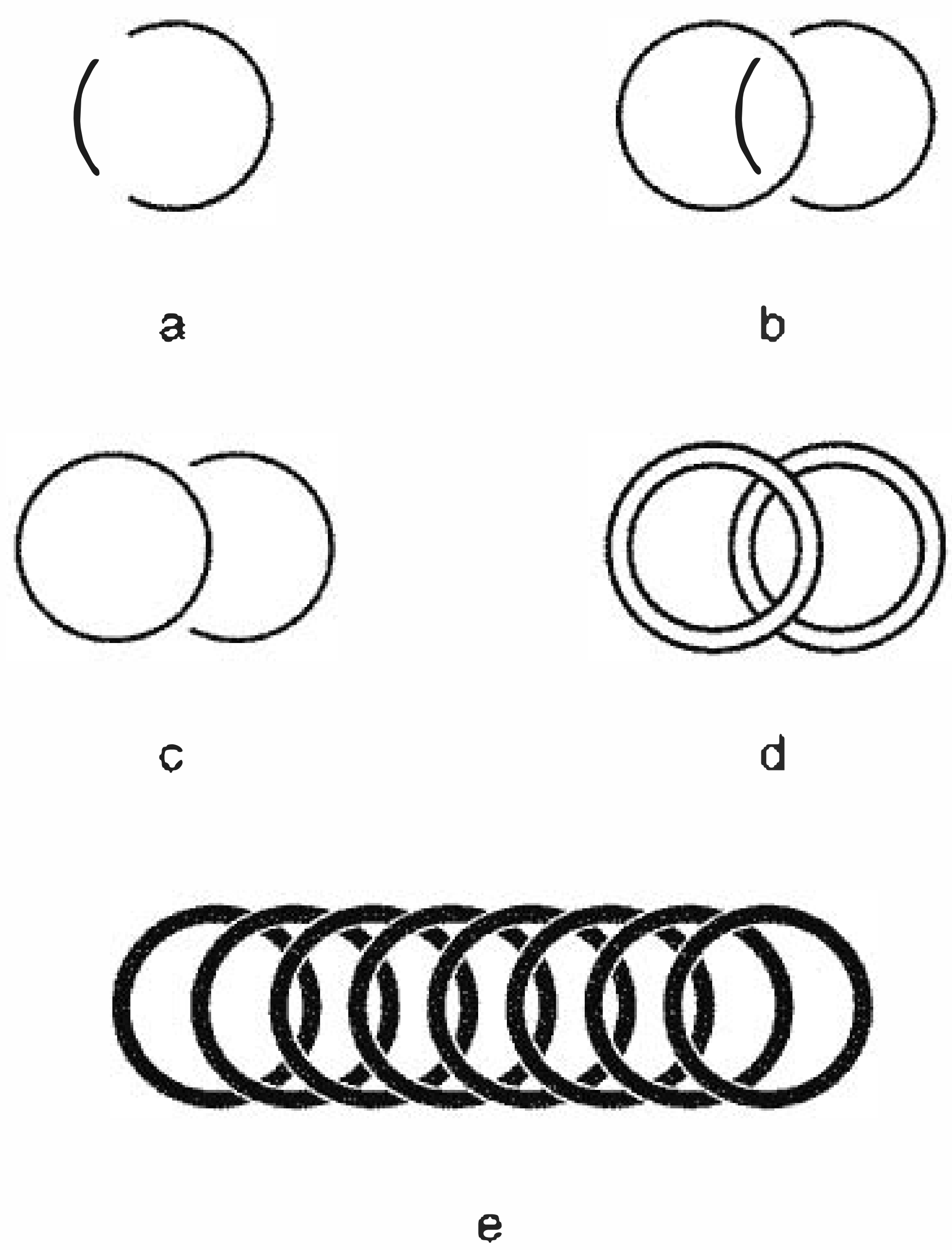
Figure 10



Illusion of angularity: the shape of the illusory figure concords with the shape of incompleteness, but real contours don't. (a left) An illusory disk – (a right) the circle appears to be polygonal with blunt angles directed towards the inside of the stripes. (b left) an illusory polygon – (b right) the sides of the polygon look convex with swellings inside the stripes and the vertices appear less pointed, blunter, and rounded off. (c left) an illusory polygon – (c right) the polygon appears more polygonal, pointed or sharper with the polygon sides appearing slightly concave and the vertices seeming to go even more inwards into the stripes.

Figure 10 sets the inverse problem: the shapes of illusory figures here are in agreement with the gaps, but, by completing the gaps with real contours, the perceived shapes do not match with the ones induced by the illusory figures. In Figure 10a left, "*an illusory disk*" is perceived filling the circular gap in the center of radial stripes. By replacing the illusory contours with real ones (Figure 10a middle), the "illusion of angularity" is perceived (Figure 10a right, Pinna, 1991). "*The circle appears to be polygonal with blunt angles directed towards the inside of the stripes*". When the gap is a polygon with each vertex lying between two contiguous stripes, "*an illusory polygon*", similar to the gap shape, appears (Figure 10b left). But when it is replaced by real contours (Figure 10b middle), "*the sides of the polygon look convex with swellings inside the stripes*". "*The vertices appear less pointed, blunter, and rounded off*" (Figure 10b right). By viewing globally, "*the polygon appears more circular than the real circle*" illustrated in Figure 10a right. If the gap is a polygon with vertices lying within the black stripes (Figure 10c left), "*an illusory polygon is perceived*". But, when it is replaced with real contours (Figure 10c middle), "*the polygon appears more polygonal, pointed or sharper with the polygon sides appearing slightly concave and the vertices seeming to go even more inwards into the stripes*" (Figure 10c right). On the basis of these results, the question is: If the illusory figure is the solution to the problem created by the gaps, then why by replacing illusory with real contours – that is, by adding a real solution to the gaps – does the phenomenal shape appears different?

Figure 11



Inadequacy of incompleteness to predict how gaps should be completed. (a) A circumference with two missing arcs; (b) the white gaps become "something" (neither an illusory figure nor a gap) that allows one circumference to pass behind the other; (c) "something" that induce the incomplete circumference to be amodally completed behind the complete circle; (d) the amodal completion is less strong than in (b) and (c); (e) the gaps become the white illusory boundaries of the circumferences that appear as surfaces.

A demonstration from another point of view of the scientific inadequacy of incompleteness in predicting how gaps should be completed is shown by the illusory contours illustrated in Figure 11. A circumference with two missing arcs, as illustrated in Figure 11a, is not completed, but rather appears "*broken even if within the gaps some weak brightness induction is* *perceived*". By inserting an identical circumference intersecting the gaps (Figure 11b), the white gaps are not perceived anymore as such but they become "*something that allows one circumference to pass behind the other*", that is "*the incomplete circumference is* amodally *completed behind the complete circle*". A similar effect is perceived in Figure 11c. What is this "*something*" is hard to define phenomenally, maybe an "illusory amodal completion". It is not an illusory figure in the known sense; "*it is not an illusory contour; it belongs to neither one nor the other circle; it does not appear as a gap or as incompleteness*". This illusory amodal completion "*is comparable or even stronger than the one illustrated in Figure 11d*". By increasing the amplitude of the gap, the strength of the amodal completion and the depth stratification effects increase accordingly.

By increasing the width of the woven circumferences and by decreasing the amplitude of the gaps (see Figure 11e), the illusory amodal completion persists but it assumes a different appearance: "*The gaps become the white boundaries of the circumferences that appear as surfaces*". Note that "*the bright illusory boundaries are perceived only along the crossover points and not everywhere along the circumferences*". They are "illusory" in the sense that they are detached from the real black region that they bound. These figures show that the real contours define a preferred orientation for attaching parallel illusory contours to their figure, even though these illusory contours, perceptually speaking, are real contours in the figure that bound, and thus complete, the black figures from which they are detached. In other words, these illusory contours define T-junctions with respect to the real contours from which they are detached. Again, these perceptual results and these phenomenal variations cannot be predicted from incompleteness as a sufficient condition for explaining illusory contour formation.

3.5. Incompleteness is not a necessary condition

The most important logical condition to be studied is incompleteness as a necessary condition; that is, A is *necessary* for B if B *cannot* be true unless A is true. Consequently:

(i) if the inducing elements are complete, then no illusory figure should be induced or, in other words, a case of complete elements inducing an illusory figure can never happen.

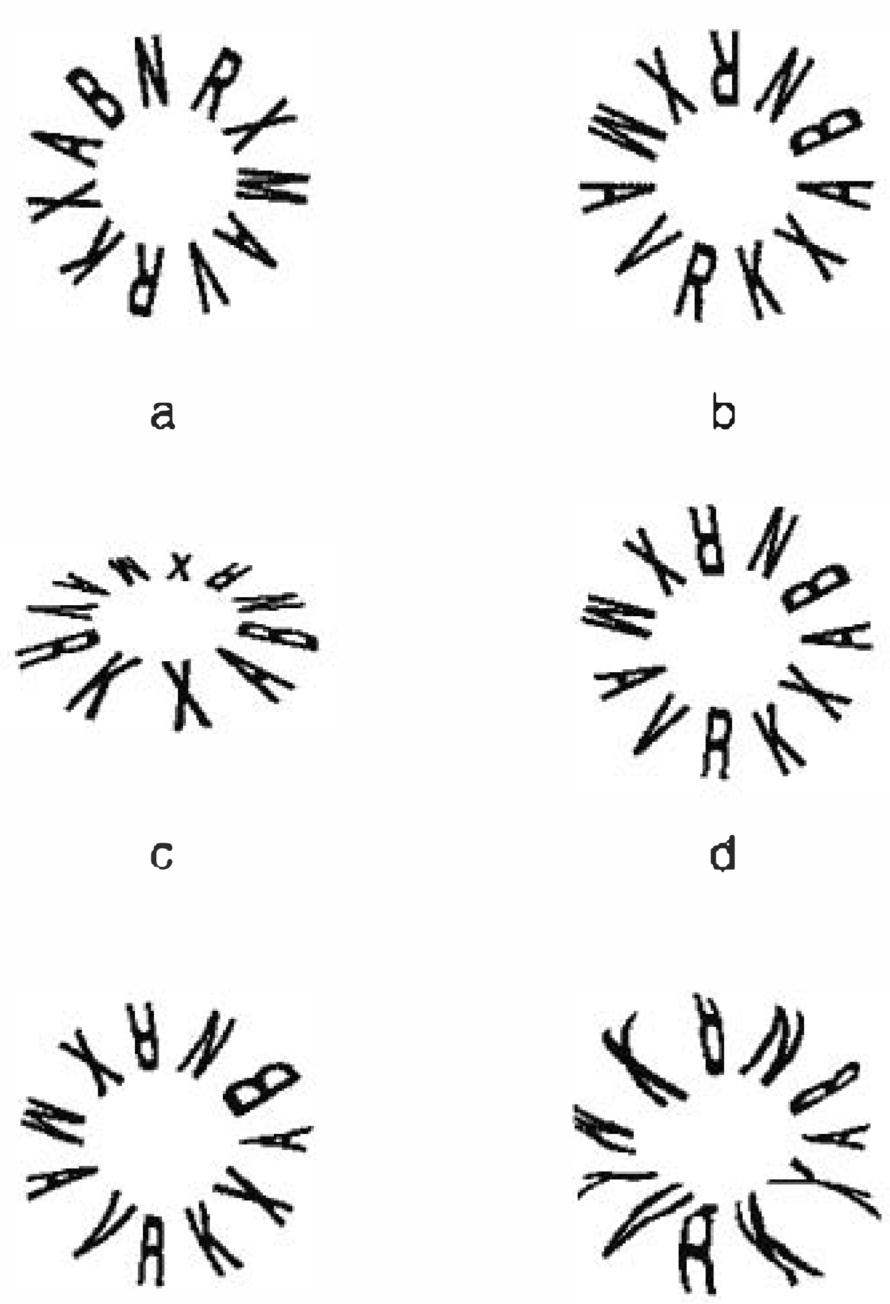
Within the logical rationale of cognitive and gestalt models, incompleteness is completed through depth segregation and amodal completion. Some consequences follow:

(ii) Whenever an illusory figure is perceived, necessarily the *depth segregation* of the illusory figure relative to the inducing elements, and their *partial occlusion* should be also perceived.

(iii) There should *never* be the case of an illusory figure *on the same depth plane* (coplanar) to its inducing elements.

At least three attempts have been made to disconfirm the necessary condition. One of them is the "Sun effect" by Kennedy (1976, 1978). By using triangles pointing towards a central open area, Kennedy demonstrated that illusory brightness can occur without amodal completion of the inducing elements. This is an interesting counter-example but does not completely disprove the role of incompleteness and completion. In fact, in the Sun effect, the illusory contours are perceived not as sharp boundaries but fuzzy ones, and the resulting brightness does not have the surface qualities of the Ehrenstein illusion, but rather appears diaphanous like a bright fog without defined depth segregation. Thus, this percept might be considered as a case of acute terminators inducing fuzzy illusory contours.

Figure 12



Purghé (1990) suggested another interesting case to refute the necessary condition. He arranged four black octagons in a way that the central illusory octagon appears that is tessellated to the other four all around it. This limiting case of amodal completion uses implicit Y-junctions between the illusory figure and the inducing elements. The strongest refutation of the necessary condition would contain T-junctions between the inducing elements and the illusory figure.

By modifying the Ehrenstein figure, Ehrenstein (1941), Pinna (1996), and Pinna et al. (2004) investigated whether illusory figures are affected by cognitive cues that do not allow amodal completion. This was done by replacing Ehrenstein inducing lines by alphabetical letters (either normal or modified) with similar line terminators (Figure 12a and 12b). The perception of the letter of the alphabet requires a cognitive process defining the completeness of it. "*Each letter is perceived complete, but their radial arrangement induces a strong illusory circle*", thereby contradicting rule (i) whereby incompleteness as a necessary factor. Rules (ii) and (iii) are also contradicted, since "*the illusory circle is perceived coplanar and tangent to the letter terminators*". "*An illusory disk with sharp contours is perceived*" even if curved terminators (R and B) and line-end terminators are mixed.

The three rules are refuted even by rotating Figure 12b in 3D space as illustrated in Figure 12c. "*The strength and sharpness of illusory contours and the coplanarity of the illusory disk are here even stronger than previously*". The illusory figure persists even when there are misalignments and deformations both in the inducing letter and in the illusory contours (Figure 12d, 12e and 12f), or even if the letters of the alphabet appear incomplete due to their bottoms being cut in the shape of a disk.

These various results indicate that incompleteness is neither a necessary nor a sufficient condition for inducing illusory figures, and indicate the need for a different explanation. However, incompleteness carries the idea that illusory figures make the understanding of the general problem of figural organization easier. This is an important point to be considered and included within such an alternative explanation. These are topics of the next section.

4. The hypothesis of figural organization of local inductions of contours and brightness

The questions to answer in this section are: How can the previous results be interpreted? Can the incompleteness hypothesis be replaced by a new hypothesis that overcomes the previous logical and phenomenological problems of the incompleteness theories? Section 3.2 showed the role of grouping and figure-ground segregation principles in inducing illusory figures. It is proposed below that these principles involve both *boundary contour discontinuities* like the wiggly concavities of Figure 1, as well as *brightness inhomogeneities* that are induced along the same wiggly concavities or at line terminators (Figures 5, 9, 11, 12). The line terminators are a special case of boundary contour discontinuities, because of the abrupt change in contour direction. These two factors clarify the more general problem of figural organization, where illusory figures still play a leading role. The next sections propose how to overcome inadequacies of the incompleteness hypothesis in explaining figural organization in terms of how percepts are completed in response to boundary contour discontinuities and brightness inhomogeneities.

4.1. Figural organization of boundary contour discontinuities

The problem of figural organization has his roots in opposite qualities of figure and background (Rubin, 1915, 1921) that depend on a phenomenal scission between figure and background and, more generally, between object and reference frame (Koffka, 1935; Köhler, 1920; Metger, 1963); namely: (i) a border is unidirectional, and belongs to the figure (border ownership), not to the background; (ii) a figure possesses a bright, compact, and opaque surface color, whereas a background shows a diaphanous color and appears empty; and (iii) a figure appears to lie above the background, whereas the background is unlimited and continues underneath the figure.

These points are illustrated in Figure 13. Consider the "*electric plug-like shape*" of Figure 13a, showing two additions, and the "*half-circular shape*" of Figure 13b. When they are put together, as shown in Figure 13c, "*a clear illusory E is perceived and the whole pattern is read as D-ED*". "*The two Ds appear on a white background, while the E is perceived on a black background*". "*No incompletenesses are present in the component elements*". They can be defined only after the illusory figure is perceived, but this implies a paradox as has already suggested. Instead, convexities and additions, clearly perceived in Figure 13a, are not anymore perceived as such in Figure 13c, because their boundaries there belong to the E and not to the plug-like shape, in Figure 13a, and thus they become background.

Similar examples useful for a comparison of the strength of the illusory letters are illustrated in Figure 13d and 13e, where convexities and additions are complementary to concave discontinuities and gaps. "*The illusory E is clearly perceived in Figures 13d and the illusory 5 in Figure 13e*". While "*the concave discontinuities* (for example the three white arms of the E in Figure 13d) *tend to appear as a figure*", "*the convex additions* (for example the two black additions to the D in Figure 13d) *are grouped and perceived as a black background*". Because the same border can be perceived both concave and convex depending on the direction of the perception – that is, from the left to the right or from the right to the left – the border of the two boundary conditions belongs to complementary regions. It is worthwhile noting that the two black additions to the D in Figure 13d, perceived as a black background, are not perceived as additions anymore and, as a consequence, they split from the D, thus inducing "*two illusory vertical boundary contours that separate the two regions and that belong to the D*". These illusory contours disappear when the electric plug-like shape is perceived.

Figure 13

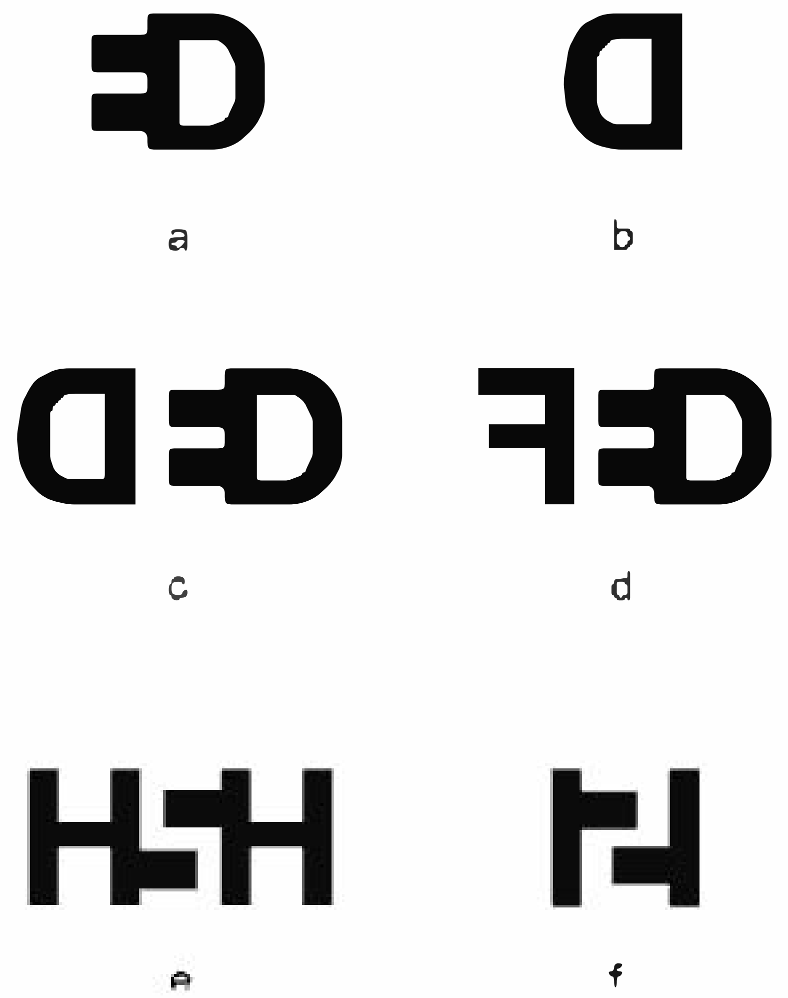


Figure-ground organization of boundary contour discontinuities. (a) Electric plug-like shape; (b) half-circular shape; (c) by putting together (a) and (b) a clear illusory E is perceived and the whole pattern is read as DED; (d) an illusory E is perceived; (e) an illusory 5 digit emerges; (f) an illusory number 2 on a black background or; alternatively, an illusory 2 on a black background with two adjacent I letters on both sides of the 2 digit.

These examples illustrate that the problem of illusory figures can be considered as part of the more general problem of figure-ground organization, notably of how boundaries are grouped and ungrouped in a way that is sensitive to figure-ground constraints. Figure-ground organization plays a role in illusory figure formation even if the effects of past experience are weakened. In particular, grouping and ungrouping can influence border ownership of the ungrouped boundaries. For example, in

Figure 13f, the perceptual results were described as (i) "*an illusory number 2*", and "*an illusory reflected S both on a black background*" or (ii) alternatively "*an illusory number 2 on a black background with two adjacent I letters on both sides of the 2 digit (I2I)*". The inducing elements, observed separately, can be phenomenally described as "*two small horizontal rectangles, each connected to two larger vertical rectangles*". The percept of a 2 on a uniform black background depends on the grouping of the small horizontal rectangles with the two larger vertical ones. The percept of I2I is facilitated by the ungrouping of the small rectangles from the larger ones, due to the discontinuity in the directions of their boundaries, thus eliciting the illusory contour formation in between them, as described in Figure 13d. This ungrouping makes the perception of the illusory 2 easier, enabling the small rectangles to lose their boundaries to the illusory 2 digit and enabling the large rectangles to appear as such or as I letters, in agreement with past experience through cognitive priming of the 2.

These phenomenal results show that the role of incompleteness can be similarly explained in terms of boundary contour grouping processes and figure-ground organization, as they apply to the perception of illusory figures. Such an analysis avoids the paradoxes raised by "incompleteness" as an explanatory principle, yet also provides explanations of the phenomena that motivated this hypothesis. In particular, boundary contour grouping principles:

(i) are not restricted to specific figural conditions like incompleteness but cover multiple boundary contingencies, including additions, line terminators, and parallel contours (see in particular Sections 3.5 and 4.1);

(ii) incorporate both local and global boundary conditions, as illustrated in Section 3.3;

(iii) do not necessarily require amodal completion of inducing elements (Sections 3.3 and 3.5), but can induce illusory figures without amodal completion;

(iv) can predict opposite and complementary figure-ground organizations, as shown in Section 3.2;

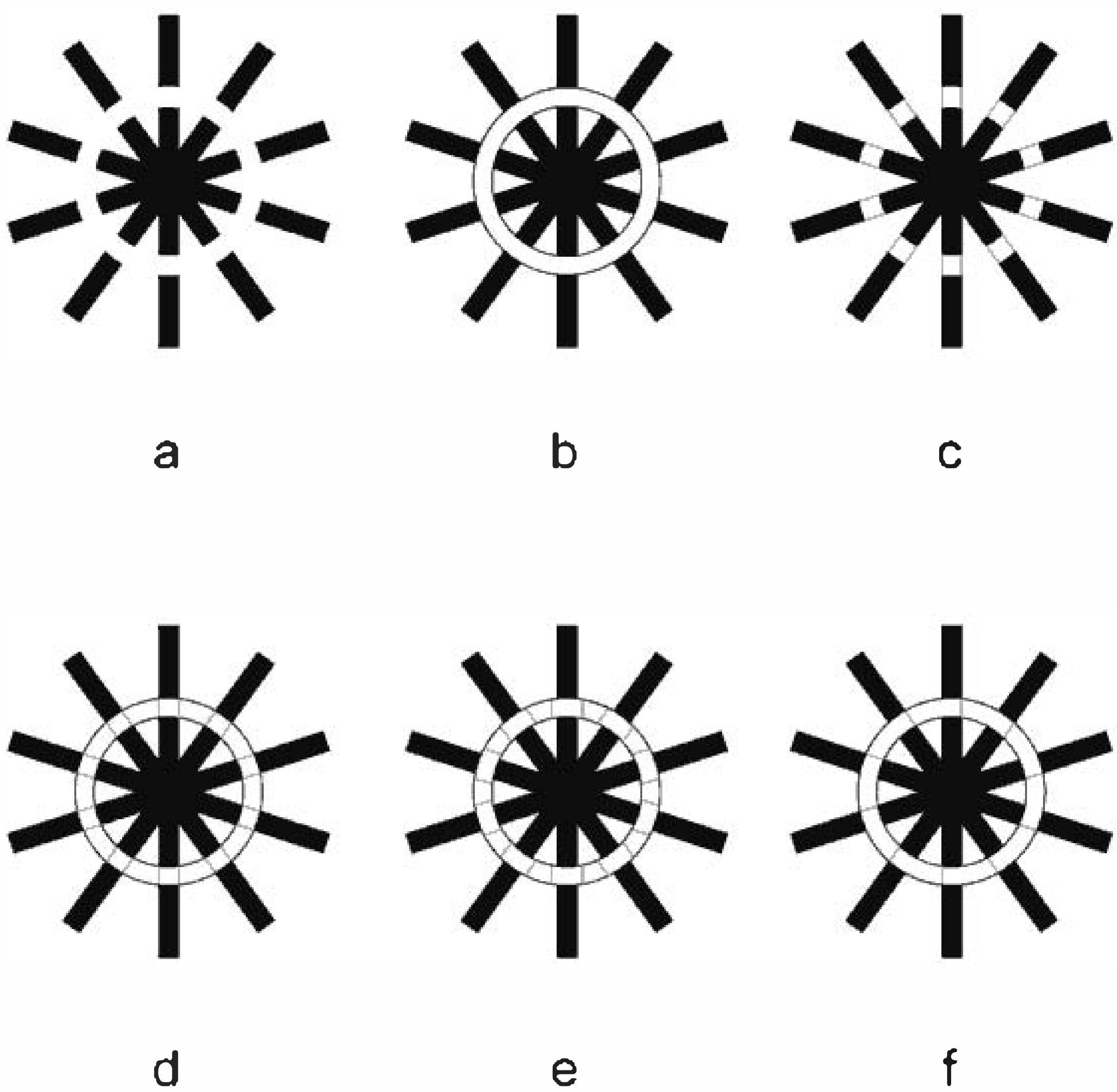
(v) predict the shape of illusory figures on the basis of the grouping of local discontinuities and their connections (Sections 3.2 and 3.4). Thus the shape of illusory contours is not necessarily equal to the shape of real contours and to the shape of incompleteness.

The figural properties of the illusory figures are accompanied by brightness enhancements along with the illusory figures that cannot be accounted for solely by the organization of brightness discontinuities. Both boundary grouping and surface filling-in processes work in parallel, indeed are represented by parallel cortical interblob and blob streams from cortical areas V1 through V4, to synergistically create the strong figural properties of real and illusory figures. The modeling articles of Grossberg (1994, 1997), Grossberg and Mingolla (1985a, 1985b), Grossberg and Swaminathan (2004), Kelly and Grossberg (2000), and of Raizada and Grossberg (2003) propose and simulate cortical mechanisms of perceptual grouping and figure-ground perception whereby these properties may be realized; see Section 5.

4.2. Figural organization of brightness inhomogeneities

In Figure 14a, "*the radial stripes appear partially occluded by an illusory bright annulus*". By replacing the illusory contours with real ones (see Figure 14b), "*the annulus loses its large brightness enhancement, although weak brightness enhancement persists within the interspace between the stripe terminators*". Furthermore, as in Figure 10, "*the inner and outer circles of the annulus appear as pointed arcs wedged in the black strips*", illustrating the illusion of angularity (Pinna, 1991). "*In Figure 14c, the brightness enhancement is stronger than in Figure 14b*", due to the pairs of lines that continue the boundaries of the stripes within the interspaces. In Figure 14d, by bordering the annulus with real contours, "*the induced brightness next to the stripe terminators pops out relative to the dark connecting regions within the annulus*". "*The brightness and darkness inductions are contained in separated sectors of the annulus*". Due to this separation, "*they are enhanced compared to Figure 14c*". Functioning as both filling-in generators and filling-in barriers (Grossberg, 1994, 1997), the separation lines stop and contain both the brightness filling-in (see Figure 14a) and the darkness filling-in (see Figure 14b) along the annulus. By widening the brightness area, as illustrated in Figures 14e and 14f, "*the brightness and darkness spreading effects are diluted, though not homogenously*", as in the Kanizsa and Minguzzi (1986) anomalous brightness differentiation and impossible staircase (Escher, 1961; Penrose and Penrose, 1958) brightness illusions, respectively, both of which are simulated in Grossberg and Todorovic (1988) by using boundary and surface interactions.

Figure 14



Figural organization of brightness inhomogeneities. (a) The radial stripes appear partially occluded by an illusory bright annulus; (b) the annulus loses its large brightness enhancement, although weak brightness enhancement persists within the interspace between the stripe terminators; (c) the brightness enhancement is stronger than in (b); (d) the induced brightness next to the stripe terminators pops out relative to the dark connecting regions within the annulus; (e) and (f) the brightness and darkness spreading effects are diluted, though not homogenously.

The conclusions suggested by these phenomenal results are the following:

(i) Brightness induction peculiar to illusory figures is elicited whether or not there are figural incompletenesses (Figures 14b-14f);

(ii) brightness enhancement is induced next to each stripe terminator;

(iii) brightness inhomogeneities are induced independently from the presence of illusory or real contours;

(iv) brightness inhomogeneities spread and fill long distances (Figures 14a and 14b);

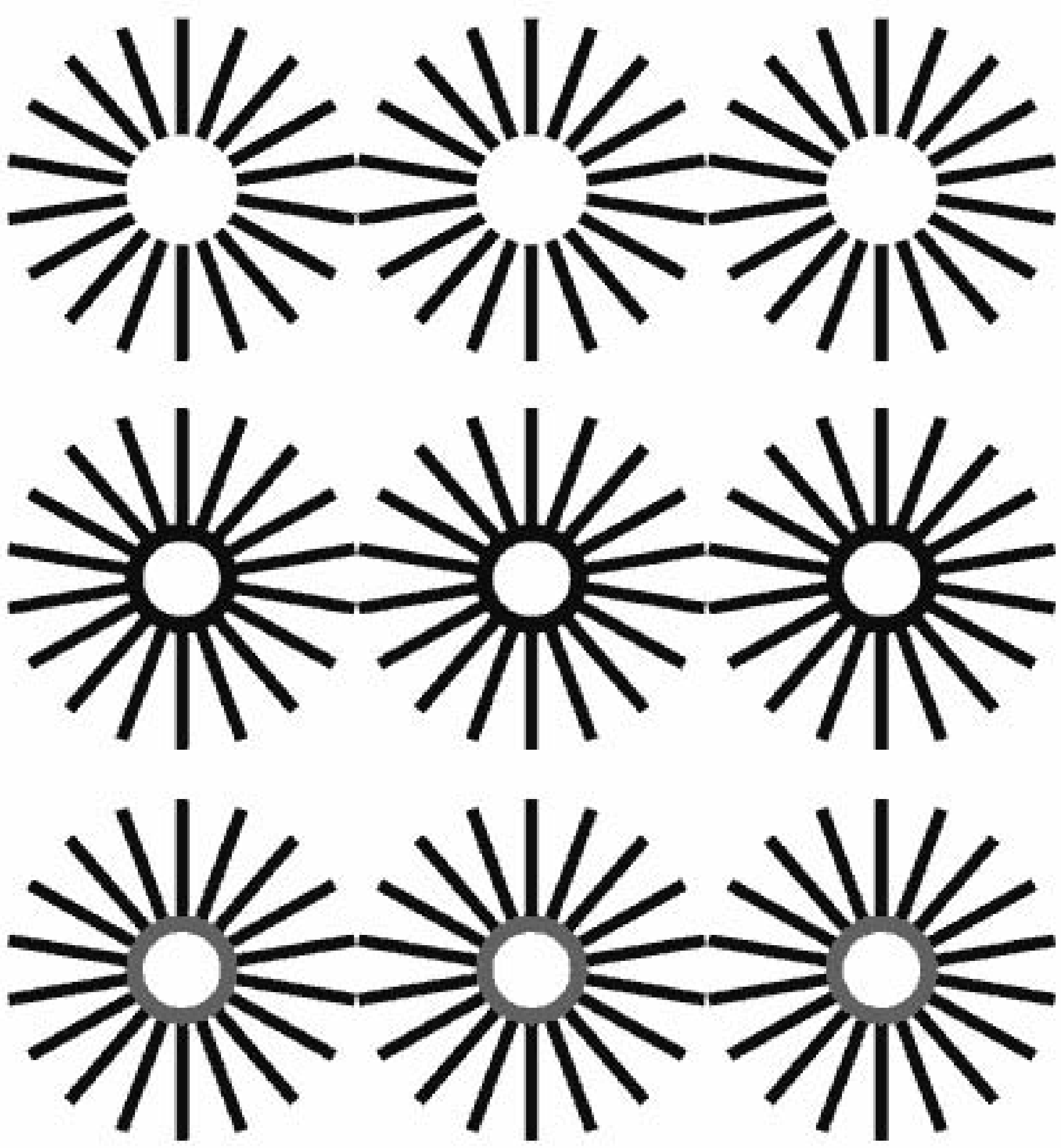
(v) brightness spreading can be contained by both illusory and real boundaries (Figures 14a-14e) acting as barriers;

(vi) among brightness inhomogeneities, darkness enhancement may be perceived, especially when a real boundary separates the respective regions (Figure 14d);

(vii) bright and dark inhomogeneities mix while they spread (Figures 14a, 14b, 14f) if they are not separated by boundaries.

The amount of brightness and darkness induction changes by replacing illusory with real contours but in a manner that does not depend upon incompletenesses. "*Going from Figure 14a to Figure 14b, the decrease of brightness*" may not depend on the completion of the gap, but rather on darkness assimilation due to the parallel black lines among the radial stripes. Figure 14d supports this hypothesis. Here, the assimilation effect is enhanced due to the presence of the pairs of perpendicular parallel lines enclosing the darkness assimilation regions and separating them from the brightness induction areas. The hypothesis of darkness induction due to assimilation processes may be only part of the explanation of the perceptual result of Figure 14d. In fact, it appears similar to the dark spots perceived at the crossroads of the Hermann grid and within the circular arrangement of triangles or zigzag elements of Kennedy's Sun Effect. This hypothesis deserves further experimental attention.

Figure 15



Chromatic assimilation effect. Surface processing occurs parallel to boundary formation, and its properties do not necessarily derive from figural incompleteness: (top) brightness enhancement conspicuously fills the central gaps between the radial lines (Ehrenstein illusion); (middle) when black rings are superimposed onto the illusory disks, the brightness enhancement largely disappears; (bottom) however, when colored annuli replace the black rings, an effect much brighter than in the Ehrenstein illusion occurs with a dense appearance comparable to a paste of bright and quasi-luminous white color added to the surface of the paper.

These examples of brightness induction, among others (Brigner & Gallagher, 1974; Frisby & Clatworthy, 1975; Day & Jory, 1978; Jory & Day, 1979; Grossberg, 1983, 1984, 1987a; Grossberg & Mingolla, 1985a, 1985b, 1987), illustrate that surface processing is a distinct process that occurs parallel to boundary formation, and that its properties do not necessarily derive from figural incompleteness properties. Figure 15 illustrates this conclusion in a vivid way. In the classical Ehrenstein illusion (Figure 15, top), "*brightness enhancement conspicuously fills the central gaps between the radial lines*". Ehrenstein (1941) showed that, when black rings are superimposed onto the illusory disks, "*the brightness enhancement largely disappears*" (Figure 15, middle). This property is in agreement with incompleteness theories. However, when wide colored annuli (e.g. purple, as in Figure 15, bottom) are used that replace the black rings, brightness cancellation does not occur. Instead, it produces "*an effect much brighter than in the Ehrenstein illusion. The white disks also show a dense appearance comparable to a paste of bright and quasi-luminous white color added to the surface of the paper*" (Pinna et al., 2003).

The cases illustrated in this section demonstrate that boundary and surface formation processes are parallel, distinct, but mutually interactive processes that together give rise to figure-ground segregation properties. Indeed, brightness inhomogeneities are especially induced next to line terminators (e.g., Figures 9a, 10a 12), as well as concave (e.g., Figures 1b and 2) and convex (e.g., Figure 12a and 15) boundary discontinuities.

5. Conclusions

According to cognitive and gestalt theories, incompleteness was considered as a necessary and sufficient factor for illusory figure formation. The role of incompleteness has herein been studied in terms of its inner logic and through a phenomenological analysis of new cases. It is shown that incompleteness is neither sufficient nor necessary to induce illusory figures. These problems are eliminated when the incompleteness hypothesis is replaced by concepts concerning interacting boundary grouping and surface filling-in processes during figure-ground segregation. These interactive boundary grouping and surface filling-in dynamics give rise to the three basic figure-ground properties (phenomenal scission between figure and background, see Koffka, 1935; Köhler, 1947, Metzger, 1963): the unidirectional belongingness of boundaries to the illusory figure, not to the background; the enhanced brightness of surface color under certain conditions; and the illusory figure lying above a background that continues underneath the illusory figure.

Compared to the logical and phenomenological problems coming from the incompleteness hypothesis, advantages of this new dynamic hypothesis are:

(i) Incompleteness becomes just one property to be explained, similarly to illusory figures, in terms of boundary and surface dynamics;

(ii) boundary and surface dynamics are not restricted to properties of incompleteness, and are thus not subjected to incompleteness paradoxes;

(iii) boundary formation processes can handle other kinds of discontinuities than incompleteness;

(iv) surface and boundary processes can explain both local and global levels of object representation, including both bottom-up and top-down levels of object representation.

The suggested hypothesis is consistent with neurophysiological experiments. Von der Heydt et al. (1984) and Peterhans and von der Heydt (1987) demonstrated that neurons in area V2 of the monkey cortex are respond to both real and illusory contours. Sasaki and Watanabe (2004) reported distinct fMRI signatures in the human visual cortex for illusory contours and for color spreading processes, including color spreading in V1. Zhou et al. (2000) found a substantial fraction of cells that are sensitive to border-ownership in area V2; cf., Section 4.1. Baylis and Driver (2001) found that neurons in monkey IT (inferotemporal) cortex, which are supposed to be involved in object recognition, respond differentially to figure or ground, and as a consequence to border ownership. Kleinschmidt et al. (1998) in fMRI studies on figure-ground reversals found activation over a number of areas in occipital, temporal, parietal and even frontal cortex. fMRI studies of Kanizsa squares by Hirsch et al. (1995) revealed that there was activation of the occipital cortex lateral to V1 where signals related to segmentation were present. Mendola et al. (1999) found that signals related to illusory contours were observed in cortical area V3 and also in LO, the lateral occipital area (Malach et al., 1995).

The FACADE neural model of boundary and surface formation during figure-ground segregation (Cao and Grossberg, 2004; Grossberg, 1994, 1997; Grossberg and Howe, 2003; Grossberg and McLoughlin, 1997; Grossberg and Swaminathan, 2004; Grossberg and Yazdanbakhsh, 2005; Kelly and Grossberg, 2000; Raizada and Grossberg, 2003) provides a conceptual foundation with which to replace hypotheses about incompleteness with new perceptual and neural organizational principles and mechanisms that are not subject to the problems of the incompleteness hypotheses and which have begun to provide explanations of the types of data that have been summarized herein. Various of the percepts induced by Figures 1 – 15 can be explained by using FACADE model mechanisms that were described and simulated in Grossberg (1994) and Kelly and Grossberg (2000). Several such explanations are sketched here to illustrate this claim.

Some of the percepts illustrate how Petter's principle (Petter, 1956) is realized by FACADE grouping and figure-ground mechanisms. For example, the percepts in Figures 4, 5, and 8c can be explained by the fact that bipole grouping cells (Grossberg and Mingolla, 1985a, 1985b; Raizada and Grossberg, 2003) form real and illusory boundaries via long-range excitatory connections and shorter-range inhibitory connections. Some of the shorter-range inhibitory connections ensure that the groupings form inwardly between pairs or greater numbers of (almost) collinear contours. Other inhibitory connections suppress weaker boundary groupings at nearby positions and at different orientations and depths.

In response to Figure 4a, boundary completion can occur between the exterior contours of the black disk that are interrupted by the thin white triangular shape, since the excitatory connections that group over the thin white shape have a high *support ratio* (Shipley and Kellman, 1992). In contrast, completing a collinear boundary is much more difficult between the aligned contours of two thin white triangular shapes that belong to different black disks. Here the support ratio is much smaller for the corresponding bipole cells. When the shorter-range inhibitory connections of the two classes of bipole cells compete, the boundaries within the circular disk win. This event is enough to trigger the figure-ground percept of a triangle that is partly seen through three circular holes in a white surface. The explanation is the same as the one given and simulated for Kanizsa stratification in Kelly and Grossberg (2000).

A key step in this explanation is based on a prediction made in Grossberg (1987b, 1994, 1997) that the stronger boundary can break the weaker boundary where it abuts the stronger boundary, thereby creating a gap in the weaker boundary. This step triggers depthful figure-ground percepts through a feedback interaction between boundary and surface representations. This hypothesis has been useful in explaining many figure-ground data, most recently data about how the visual cortex generates 3D transparency and neon color spreading percepts (Grossberg and Yazdanbakhsh, 2005). The predicted boundary breaks may change perceived brightnesses due to the spreading of contrast, through the boundary gap, between contiguous surface regions. Percepts of bistable transparency provide a particularly useful way to test this hypothesis, since focusing attention on the boundary of one region can cause its surface to be seen in front of the other region, and should cause a perceived change in brightness. Shifting attention to the boundary of the other figure can cause its surface to be seen in front, again with a perceived change in brightness. Tse (2005) has recently reported just such a perceived brightness change during percepts of bistable transparency.

Figure 4b greatly improves the support ratio of the bipole cells that group collinearly across a pair of black disks, since the pac man inducers are considerably larger. In addition, thickening the white triangular regions greatly weakens the support ratio of the bipole cells that might attempt to complete a boundary around each black disk. In particular, the small triangular black regions that lies between the thicker white bars provide very little support for completing a boundary across each bar. Thickening the white regions also makes the orientations of successive contours in the black region differ more. All these factors work together to enable the collinear grouping into a triangle to occur. This event triggers the percept of a triangle in front of partially occluded disks, again with the same explanation as Kanizsa stratification. Figure 4c has a similar explanation. Figures 5a and 5b have an explanation that is similar to that for Figures 4a and 4b. Figure 5c combines several of these factors by altering the relative support ratios in the left and right sides of the figure.

Several of the figures probe the properties of the bipole grouping kernel, which was designed in Grossberg and Mingolla (1985b) to already include properties that were later called *relatability* by Kellman and Shipley (1991) and *association field* by Field, Hayes, and Hess (1993). For example, in Figure 1b, the irregular bounding contours in each pair of contiguous black figures are relatable, and are capable of causing boundary completion across the intervening white regions. These boundaries can then pull the figure thereby formed forward and the black regions backwards, by the same mechanisms as in Kanizsa stratification, thereby leading to a percept of an irregular figure in front of four partially occluded black squares. Figure 1c supports this interpretation by making the irregular contours unrelatable. Figure 1d, the case of "articulation-without-rests", again enables illusory contours to form with respect to the two exterior irregular boundaries and the two ends of the rests. This case of boundary completion depends upon the fact that line ends generate a band of inducing orientations that are almost perpendicular to the line orientation (Grossberg and Mingolla, 1985b), a fact that is crucial in generating the illusory circular boundary in the Ehrenstein illusion of Figure 15c. Once these illusory contours are formed, the same figure-ground mechanisms operate again. Various cases in Figure 2, notably Figures 2b, 2c, and 2d, have similar explanations. So does Figure 7a.

The explanation of the percept derived from Figure 1b also raises the issue: Why do we not *see* the illusory bounding contour of the central figure? This property can be explained by the fact that "all boundaries are invisible" within FACADE theory (Grossberg and Mingolla, 1985b; Grossberg, 1994). That is, boundaries pool over opposite contrast polarities at each position, hence cannot discriminate between dark and light. They do this in order to create complete boundaries of objects that are seen in front of textured backgrounds whose relative contrasts with respect to the object reverse as the object boundary is traversed. This pooling of opposite contrast polarities occurs at complex cells of cortical area V1. Boundaries become visible when they separate surfaces that fill-in different surface lightnesses, brightnesses, or colors. In Figure 1b, any such difference in filled-in surface lightness or brightness is small between the irregular figure and the background white regions.

The main differences between Figure 9a and 9b can also be explained by how boundary relatability and figure-ground properties interact. In

Figure 9a, the horizontal lines that separate the circular open regions generate boundaries which are stronger than the illusory boundaries that bound the open regions. FACADE mechanisms explain how these horizontal boundaries can be seen in front. In particular, the horizontal boundary that is seen in front inhibits possible redundant copies of itself at the same positions but further depths. This across-depth inhibition is part of the surface-to-boundary feedback that is used in all FACADE figure-ground explanations. In this case, the surface is the black filled-in representation of the line itself. Such surface-to-boundary feedback ensures that perceived boundary and surface representations are *consistent*, even though the laws that they obey are *complementary* (Grossberg, 1994). Figure-ground separation is predicted to be a consequence of this consistency operation. Because the redundant copies of the horizontal boundary are inhibited at further depths, the illusory contours of the circular regions can complete behind it via relatability constraints, thereby completing the double 8 percept that is amodally perceived behind the horizontal lines.

The percepts in Figure 14 can be explained using some of the same ideas that Grossberg and Todorovic (1988) used to explain and simulate Kanizsa-Minghuzzi (1986) anomalous brightness differentiation and the impossible staircase (Escher, 1961; Penrose, 1958). One idea is that all brightness and darkness inductions combine when filling-in occurs within a bounded region. In Figure 14a, these are brightness inductions that are generated within the anular region by the black radial stripes. These brightness inductions are trapped inside the illusory annular boundaries, and can fill-in freely throughout the annulus.

In Figure 14b, the situation is more complicated due to the existence of the black circular contours that bound the annulus. Here, the black radial stripes can again cause brightness induction. However, the boundaries that are formed at the stripe-annulus *edges* are stronger than the boundaries that are formed by the thin black annular bounding *lines*. Whenever a stronger boundary collinearly abuts a weaker boundary, a boundary weakening or even a gap can form at the end of the weaker boundary, just as in the case of figure-ground separation that was discussed above. One can see the effects of such boundary gaps in percepts such as neon color spreading, where part of the explanation concerns how they are formed by sudden changes in boundary strength (Grossberg and Mingolla, 1985a). In the present case, some darkness spreading occurs from the black lines into the annular region. The brightness and darkness inductions spread during filling-in within the annulus, leading to a darker percept in response to Figure 14b than Figure 14a.

The fact that the brightness enhancement is greater in Figure 14c than Figure 14b has a similar explanation, but here the weaker line boundaries are collinear with the stripe edge boundaries. Figure 14d prevents the brightness inductions from spreading through the annulus, as in Figure 14b, but combines darkness inductions from weaker line boundaries that are collinear with the ends and the sides of the stripe edge boundaries. Figures 14e and 14f support the hypothesis that the brightness and darkness inductions can spread within their bounding contours. In particular, Figure 14f is a version of an impossible staircase.

These explanations illustrate how FACADE concepts about 3D boundary completion and surface filling-in, particularly as they lead to 3D figure-ground percepts, can be used to replace concepts about incompleteness without leading to any logical paradoxes. Percepts such as those derived from Figures 3 and 13 can be explained using similar mechanisms, with the addition that top-down attention to familiar letters from inferotemporal recognition categories can also influence the pre-attentive grouping process. How such pre-attentive and attentive factors may combine to enhance attended groupings is reviewed elsewhere; e.g., Grossberg (1999) and Raizada and Grossberg (2003).

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Abstract

Cognitive and gestalt vision theories consider incompleteness to be a necessary and sufficient factor for inducing illusory figures. The role of incompleteness is studied herein by defining the inner logic subtended by use of the term "incompleteness", presenting new cases to clarify the phenomenology of incompleteness as a necessary and sufficient condition, and suggesting an alternative hypothesis to explain illusory figures after analyzing problems with the incompleteness hypothesis. It is demonstrated that incompleteness is not a sufficient condition, illusory figures do not necessarily complete incompletenesses, the shape of incompleteness does not predict the shape of illusory figures, and incompleteness is not a necessary condition. Finally, it is noted that the incompleteness hypothesis can be replaced by concepts concerning interacting boundary grouping and surface filling-in processes during figure-ground segregation. The suggested hypothesis is consistent with neurophysiological experiments and is described in terms of the FACADE neural model of boundary and surface formation during figure-ground segregation.

Résumé

Les théories cognitives et de la vision de la Gestalt considèrent l'incomplétude comme une condition nécessaire et suffisante pour induire des figures illusoires. Le rôle de l'incomplétude est donc étudié en définissant la logique interne que l'emploi du terme «incomplétude» sous-tend, en présentant de nouveaux cas pour clarifier la phénoménologie de l'incomplétude en tant que condition nécessaire et suffisante, et en suggérant une hypothèse alternative pour expliquer les figures illusoires après avoir analysé les problèmes avec le modèle de l'incomplétude. Nous démontrerons, au contraire, que l'incomplétude n'est pas une condition suffisante, que les figures illusoires ne saturent pas nécessairement l'incomplétude, que la forme de l'incomplétude ne prédit pas la forme des figures illusoires, et que l'incomplétude n'est pas une condi-

tion nécessaire. Pour finir, nous observerons que l'hypothèse de l'incomplétude peut être remplacée par l'emploi de concepts qui concernent le groupement de contours en interaction et les procès de remplissage de surface pendant la ségrégation figure-fond. L'hypothèse que nous suggérons est cohérente avec les expérimentations neurophysiologiques et elle s'inscrit dans le modèle neural des contours FACADE.

Riassunto

Sia le teorie cognitive che quelle gestaltiste considerano l'incompletezza come un fattore necessario e sufficiente per l'induzione dei contorni illusori. Scopo di questo lavoro è (1) studiare il ruolo dell'incompletezza attraverso l'analisi della logica implicita all'uso del termine "incompletezza", (2) presentare nuove condizioni percettive utili per chiarire fenomenologicamente se l'incompletezza costituisca di fatto ragione necessaria e sufficiente e, infine, (3) proporre, successivamente all'analisi del ruolo svolto dall'incompletezza, un'ipotesi alternativa in grado di spiegare le figure illusorie. L'analisi logica e fenomenologica dell'incompletezza ha dimostrato che quest'ultima non è una ragione né necessaria né sufficiente per il costituirsi delle figure illusorie, infatti queste ultime non completano le incompletezze così come la forma delle incompletezze non può predire la forma delle figure illusorie. L'ipotesi dell'incompletezza può essere sostituita con una più efficace spiegazione basata sul raggruppamento dei contorni e sui processi di filling-in che avvengono durante la segregazione figura-sfondo. Questo tipo di spiegazione è coerente con i dati neurofisiologici ed è, in questa sede, proposta nei termini del modello neurale FACADE che descrive la formazione dei contorni e delle superfici durante la segregazione figura-sfondo.

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