



Università degli Studi di Sassari

Essays in Macroeconomics

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Abstract

A long-standing and fascinating puzzle in economics is the understanding of the determinants that make some countries poor while other rich. This question can be summarized by the following quote, taken from the Lucas' essay "On the Mechanics of Economic Development", which reads as follows " Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's?"

This thesis, from a macroeconomic perspective, has not the ambition to provide a full and clear answer to the above mentioned issues. Rather, it attempts to give a partial but sensible contribution dealing with two different but deeply intertwined topics.

The first chapter studies the interplay between infrastructure, informality and high skilled migration through a macroeconomic model which aims to depict a developing stylized country. In fact, infrastructure and human capital are two widely recognized key ingredients to achieve development. Large informal sector and substantial high skilled migration flows characterize developing countries eventually preventing both infrastructure and human capital accumulation. Two different frameworks are considered. In the first one high skilled migration flows are left exogenous to the model while in the second one the brain drain phenomena is fully accounted by the model. Theoretical results are derived along with dynamical properties of the model. Subsequently, the model is calibrated for 60 countries. Numerical experiments show that the informal sector slows down the infrastructure accumulation process while helping low skilled to escape from extreme poverty. A quantitative assessment of a benchmark restrictive fiscal policy suggests that, in the long run, an increase in the tax rate would be beneficial for high skilled individuals without worsening the welfare of low skilled individuals. The results are robust to the brain drain mechanism accounted.

In the second chapter sovereign debt crisis are the object of the study. Initially, a brief review of the literature on sovereign debt crisis is provided, in light of the OLG model developed in the second part. Inspired by Cole and Kehoe (2000), I assume that the government default choice leads to a permanent drop in productivity and as a result, the government's choice is driven by a cost-benefit analysis. It has been proved that when the economy starts with an initial level of capital lower than a threshold, the government finds it profitable to bankrupt. As a consequence, the model predicts that economies with

the same level of outstanding debt but different initial capital levels can have different default behaviors suggesting that developing countries may be credit constrained.

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Chapter 1

On the Effectiveness of Infrastructure Policies in Developing Countries

1.1 Introduction

This chapter provides a macroeconomic model which highlights the impact of infrastructure on development accounting for both informal sector and brain drain. Gramlich (1994) provides a valid definition of infrastructure: “natural monopolies such as highways, other transportation facilities, water and sewer lines and communication systems”¹. The former definition can be seen as good reference, anyhow I remind that education and sanitation facilities are sometimes embodied into the infrastructure concept as well. Moreover, it should be noticed that public capital and infrastructure are not synonyms. Furthermore, the latter does not represent a subset of the first, in fact public capital includes military expenditure and part of infrastructure can be privately owned as well.

Since the leading paper of Aschauer (1989) hundreds of papers attempted to estimate the quantitative impact of public infrastructure on private² output. He used the production function approach obtaining that a 1% increase in the

¹In this chapter by infrastructure is meant what in the literature is either called core or hard infrastructure. Core infrastructure is made up by transport facilities (highways, railroads, port), telecommunication network, electric generation capacity.

²Aschauer used as dependent variable private output, and most of literature followed his approach but some studies nevertheless employed total GDP as explained variable

public capital stock would increase private output by 0.39%. The subsequent literature has not converged to the same estimate, a useful summary of the results obtained can be found in Straub (2011) review. More recently, the discrepancy between the estimates has been documented in Bom and Ligthart (2013). The latter provides a useful meta-regression analysis of papers which estimated the private output elasticity of public capital. The authors considered 68 papers for the period 1983-2008 which provided them 578 estimates of the output elasticity of public capital. Estimated values of the elasticity of public capital markedly differ across studies, in fact they range between a lower bound equal to -1.7 to an amazingly value of 2.04, with a simple average equal to 0.188.

Exploiting their meta analysis approach Bom and Ligthart (2013) suggests an elasticity of core infrastructure equal to 0.17 in the long run. Using a different approach, Calderon et al. (2014)³ obtains a long run elasticity of output between 0.07 and 0.10. The former used as explanatory variable GDP of 88 countries with respect the time span 1960-2000. In order to assess the impact of infrastructure they built, using a principal component technique, a synthetic index of infrastructure using as variables physical measures of telecommunications, electric power and roads.

The focus of this chapter is on development countries. Estache et al. (2005) estimated an augmented Solow model with respect to sub-Saharan countries considering five different types of infrastructure: telecoms, electricity, roads sanitation and water. The estimated infrastructure elasticity coefficients show that infrastructure matters to explain growth in Africa⁴. Calderón and Servén (2010) report that thanks to infrastructure investment the annual growth rate among developing countries increased on average by 1.6 percent in 2001-2005 relative to 1991-1995. They found that the largest contribution of infrastructure to growth was achieved in South Asia, 2.7 % per annum. In sub-Saharan countries the contribution of infrastructure was much smaller, only 0.7% of growth was due by either enhanced infrastructure quality or enlarged stocks of it. Straub and Terada-Hagiwara (2011) focusing on developing Asian countries finds that the observed effect on growth was simply the results of higher than average infrastructure capital accumulation but they did not find clear-cut evidence of a positive effect on productivity.

³Bom and Ligthart (2013) did not consider studies that used as a proxy of infrastructure physical indicators

⁴For instance electricity related facilities have an estimated elasticity equal to 0.50. Only the sanitation variable does not affect the GDP

Studies exploiting quasi-natural experiments help to explain how infrastructure investments interplay with development. Duflo and Pande (2007) studied the impact of dams construction in India for 1971-1999. They find that dams construction makes better off downstream populations while not improving the welfare of people living in the vicinity of the dam. The former study highlights the point that the construction of large infrastructure facilities may deeply affect the income distribution between districts and eventually policies that equally share costs and benefits may be required. More recently, Wang and Wu (2012) studied the effect on local development of the Quinzang railway construction. They found that GDP per capita in the counties on which the railway pass through increased on average by 33% after the completion of the project. It is worth to remind that the railway construction was funded almost entirely by the central government.

It is then possible to claim that there is a large consensus on the idea that lack of infrastructure is often a key bottleneck to growth⁵. The former statement could be reinforced giving some numbers. Lin and Doemeland (2012) reports that roughly 1.4 billion of individuals have no access to electricity. Moreover, they mention Foster et al. (2010) which records that in sub-Saharan Africa (excluding South-Africa) the per capita consumption of electricity is not enough even to power one light bulb per person for six hours a day. The lack of infrastructure clearly decreases the well being of individual but it makes also more difficult to do business, depressing firm productivity by 40% percent as reported by Escribano et al. (2009) .

Moreover, Fay et al. (2005) finds that infrastructure is a key ingredient to achieve millennium goals through positive effects on both sanitation and education. It is worth to mention that nowadays the idea to rise growth through an infrastructure push is taking place even with respect developed countries, see IMF (2014) and again Lin and Doemeland (2012)

Given the abundant evidence on the positive effects of an infrastructure push here it comes the question: Why aren't enough resources invested in infrastructure?

An attempt to answer to the former question comes in two steps. First it should be noticed that infrastructure investments in developing countries are primarily funded through domestic resources. In fact, Foster et al. (2010) re-

⁵World Bank President Kim Yong Kim recently declared that "There's never been a country that has developed with intermittent power" see <http://mobile.bloomberg.com/news/2014-08-05/world-bank-may-support-african-coal-power-kim-says.html>

ports, with respect to Africa, that as much as two-thirds of the overall spending are financed by the African taxpayer. Secondly, the amount of the resources collected is not enough to achieve substantial results given the small size of the tax base in developing countries. Moreover, as reported by Jutting et al. (2009) and Schneider (2012) half of the workforce could be considered informal as a consequence of escaping taxation and eventually not contributing to public funded infrastructure. Furthermore, the huge size of the informal sector in developing countries is not explained by over the average tax rates. In fact, as Docquier et al. (2014)⁶ discusses, the nature of informality differs among developing and developed countries. They distinguish between tax-based informality and poverty-based informality. With the latter is meant another sector of the economy which through a low-skilled intensive technology give a way of living to many low-skilled individuals that have not many opportunities to be hired by formal firms. Here I account for both types of informality.

Another phenomena which shrinks the tax base in developing countries is migration⁷. Through migration individuals escape taxation, at least in their home country. Docquier and Rapoport (2012) reviews the literature which studied the link between development and high skilled migration. There is clear evidence that in developing countries low skilled migration rate are always lower than high skilled ones and brain drain decreases with development.⁸Moreover, human capital is widely recognized as a key ingredient to achieve development and since the Bhagwati and Hamada (1974)⁹ the negative impact of brain drain on development has been highlighted.

In this chapter a simple theoretical framework which sheds light on the interaction between infrastructure, informality and brain drain is provided. To the best of my knowledge this is the first study that makes such attempt¹⁰. I start

⁶Here the interplay between informality and education is highlighted leaving aside taxation. The authors show that while the informal sector help low-skilled to escape from extreme poverty may prevent convergence in the long-run

⁷The interplay between taxation and migration has not been extensively studied in the literature. Claus and Claus (2011) reports, with respect ASEAN and OPEC economies, that taxation raises migration rates and that educated individuals are more responsive to restrictive fiscal policies than uneducated ones

⁸For instance in Africa the high skilled migration rate was equal to 10.3% in year 2000 whereas the average migration rate was only equal to 1.3. The same patten characterizes South and Center America and Asia Docquier and Rapoport (2012)

⁹It has been showed that some high skilled migration may be beneficial to the source country due to the positive feedback on education decisions (Mountford (1997); Stark et al. (1997); Beine et al. (2001))

¹⁰Close to this study there is Grossmann and Stadelmann (2011) which defines a overlapping generation framework that analyzes the interaction of high skilled labor migration and the accumulation of a public good. The authors, theoretically, find that optimal public investments

defining a two sector economy. The formal sector represents the official economy with total factor productivity affected both by infrastructure and human capital. Infrastructure is provided by the government which raises resources by taxing formal sector workers, differently, the informal economy produces according to a low skilled-intensive technology. The interaction between the accumulation of a public good and the informal sector has already been analyzed in Loayza (1996) through a simple endogenous growth model. It is derived that a larger informal sector has a detrimental effect on growth driven by a lower available amount of public capital.

Two variants are going to be addressed. In the first one the brain drain will be kept exogenous while in the second the high skilled location choice will be embodied into the model¹¹. It results that the model allows to highlight the interrelations between high skilled migration, informal sector and infrastructure accumulation. Theoretical implications and dynamic properties are derived. Finally, numerical experiments are made to assess the effectiveness of infrastructure policy reforms eventually allowing to estimate the impact of high skilled migration on the results obtained.

decreases in response to higher migration

¹¹It is quite common to focus only on high skilled migration leaving exogenous the low skilled individuals migration flow. Examples are De la Croix and Docquier (2012) on which is shown that that the bi-directional link between poverty and high skilled migration may give rise to coordination failures, the already mentioned Grossmann and Stadelmann (2011) and Cellini (2007) that provides a model on which developed countries may find optimal a larger brain drain flow than the one that would be chosen by a benevolent central planner.

1.2 Model

I consider a discrete time framework aiming to depict a developing economy. The economy under analysis is made up by two sectors which I label formal and informal. By formal sector is obviously intended the official economy while with the informal is meant all the economic activities done outside the former. Both sectors produce the same homogenous good according to two different production technologies. Firms acting in the formal sector produce employing a Cobb Douglas technology using as inputs both high and low skilled workers. Moreover, The productivity of formal firms is a function of the available amount of infrastructure and it is also affected by an education externality. Infrastructure is provided by the government thanks to resources collected from formal workers active the preceding period.

Conversely, informal firms produce employing a less productive technology which is modeled as a linear function of only low skilled workers. Informal workers do not contribute to infrastructure accumulation.

Two variants of the model are addressed. In the first one the model is defined leaving high skilled migration decisions exogenous to the model. Exogenous parameters set fertility, education and migration decisions which eventually determine the skill ratio law of motion. Subsequently, the model is enriched considering endogenous high skilled migration. Doing so the model is able to highlight the substance of the brain drain phenomena on the infrastructure accumulation process.

Increasing the tax rate will push more low skilled workers to the informal sector and when brain drain is accounted a larger fraction of individuals will find optimal to migrate. The dynamic implications are ambiguous depending on the effect, which can be positive or negative, on the formal sector productivity. In the following the model is solved and it will shed light on the dynamic implications of a restrictive fiscal policy.

I start with the set-up of the model. Subsequently theoretical insights suggested by the exogenous high skilled migration framework are discussed and finally the model is enriched fully accounting for endogenous high skilled migration.

1.2.1 Production

As already mentioned both the formal and the informal sector produce the same homogeneous good. Formal sector firms produce according to a Cobb-Douglas technology using as input high and low skilled workers. I define by H_t the amount of high skilled workers while $L_{f,t}$ is the quantity of low skilled workers acting in the formal sector. The formal sector output is given by:

$$Y_{f,t} = A_{f,t} H_t^\alpha L_{f,t}^{1-\alpha} \quad (.1)$$

α is merely the elasticity of formal sector output with respect high skilled labor. $A_{f,t}$ represent the state of the technology in the formal sector at time t . I assume that total factor productivity in the formal sector (TFP) is a function of both the available amount of infrastructure, k_t , in the formal sector (this variable will be defined afterwards) and of the skill ratio in the formal sector ($z_{f,t} = \frac{H_t}{L_{f,t}}$).¹²

$$A_{f,t} = A_0 z_{f,t}^\phi k_t^\varepsilon \quad (.2)$$

A_0 is a time independent scale factor. Moreover, it is hypothesized that TFP exhibits an elasticity with respect infrastructure equal to ε

I assume that the formal sector is populated by competitive firms which pay wages equal to individuals' marginal productivity. As a consequence high skilled and low skilled, respectively, receive a wage, net of taxes equal to:

$$w_{h,t} = (1 - \tau) \alpha A_0 z_{f,t}^{\alpha+\phi-1} k_t^\varepsilon \quad (.3)$$

$$w_{l,t} = (1 - \tau) (1 - \alpha) A_0 z_{f,t}^{\alpha+\phi} k_t^\varepsilon \quad (.4)$$

By assumption informal firms produce exploiting a less productive technology. I model it as a linear function of low skilled workers, in fact this study aims to depict a developing economy. This technological framework allows to highlight the fact that the informal sector helps low skilled to escape extreme poverty in developing countries¹³:

¹²I assume that the proportion of high skilled workers in the formal sector generates a positive externality on formal sector TFP, accounted through the parameter ϕ . This is a special case of Lucas (1988) model.

¹³Anyhow, the tax rate is one of the key variable which characterizes the size of the informal sector, hence the model accounts for tax-based informality as well

$$Y_{i,t} = BL_{i,t} \quad (.5)$$

$L_{i,t}$ ¹⁴ defines the amount of low skilled workers acting in the informal sector.

B is a scale factor which characterizes the available technology in the informal sector. I define it in terms of A_0 :

$$B = A_0\gamma \quad (.6)$$

Informal sector is populated by competitive firms as well. Moreover, informal workers are not subject to the government control, as a consequence informal sector workers earn a wage equal to B .

Low skilled workers join the informal sector if and only if informal sector firm can afford to pay a wage at least equal to the one available in the formal sector. It follows that output and employment in the informal sector can be defined by the following complementary slackness conditions:

$$\frac{w_{l,t}}{B} \geq 1, Y_{i,t} \geq 0 \left(\frac{w_{l,t}}{B} - 1 \right) Y_{i,t} = 0$$

Then the economy here considered can be characterized by two different equilibrium regimes¹⁵

- *Informal regime*: informal sector firms are active and low-skilled worker wage is equal to B in both sectors, which is simply the marginal productivity of labor in the informal sector.
- *Formal regime*: only formal firms produce. The low skilled wage exceeds the marginal productivity of labor in the informal sector.

It follows that the total workforce of the stylized economy here defined is simply equal to¹⁶:

$$N_t \equiv H_t + (L_{f,t} + L_{i,t})$$

Finally, for the sake of clarity, I define:

¹⁴If a production function such as $Y_{i,t} = BL_{i,t}k_i^\varepsilon$ were considered, with $\varepsilon < \varepsilon$, the qualitative results implied by the model will not change.

¹⁵the conditions that characterize which regime prevails will be deeply discussed afterwards

¹⁶The workforce is obviously defined by the individuals that remain in their native country after that the migration choice has been taken.

$$\begin{cases} \text{skill ratio on the economy} & z_t = \frac{H_t}{L_{f,t} + L_{i,t}} \\ \text{skill ratio in the formal sector} & z_{f,t} = \frac{H_t}{L_{f,t}} \end{cases} \quad (.7)$$

1.2.2 Individuals

The model is populated by two types of individuals: high skilled and low skilled. They differ by education attainment. High skilled individuals are individuals who got higher education whereas low skilled individuals represent the remaining workforce population. I label the two categories respectively by $\{h, l\}$ and depending on the education dimension different fertility and education parameters are hypothesized as in De la Croix and Docquier (2012). I define:

- n_h, n_l determine the exogenous fertility rates of respectively high skilled and low skilled.
- q specifies the exogenous low skilled childhood fraction who accomplishes higher education. Differently, I assume that all high skilled childhood obtain a university degree.

Furthermore, migration rates are linked to education as well:

- \bar{m}, m define the exogenous migration rates of respectively high skilled and low skilled. Consistent with findings of the empirical literature depending on the level of education individuals are characterized by distinct migration rates (Docquier and Rapoport (2012)).

First the model is solved leaving exogenous the brain drain phenomena to the model. Endogenous high skilled migration will be accounted afterwards.

1.2.3 Infrastructure

The government, each period, taxes the wage of individuals working in the formal sector at an exogenous rate equal to τ . These resources constitute the stock of infrastructure that will be available the next period. The amount of

infrastructure is specified in per capita terms and could be view as a monetary index which merely says how much infrastructure is available per worker in the country. For instance the definition can be associated with the amount of usable electricity, clean water, roads and telecoms lines per worker. Furthermore, the variable is lagged to highlight the fact that most of the investment in infrastructure are not immediately productive¹⁷. Mathematically:

$$k_{t+1} = \frac{\tau Y_{f,t}}{N_{t+1}} \quad (.8)$$

1.2.4 Formal and Informal Regimes

In the above defined framework low skilled individuals accept to work in the formal sector if and only if formal firms can afford to pay a wage at least equal to the one available in the informal sector. If it is the case, I assume that low skilled individuals join the informal sector until the point at which low skilled wage is equalized across sectors. After the unfolding of this process the skill ratio in the formal sector increases leading to an increase of the low skilled wage; conversely the high skilled wage will decrease.¹⁸. Differently, with formal regime is meant the case in which only formal firms produce. This is the case if the wage given by equation .4 plugging the overall skill ratio of the economy is larger than the one determined by equation .6. The formal reasoning is formally stated by the next lemma:

Lemma 1. *Low skilled individuals find it profitable to join informal firms if the wage that they may earn working for them is larger than the one available in the formal sector. This condition depends on the available amount of infrastructure which defines the productivity of formal firms.*

$$\begin{aligned} w_{l,i,t} &> (1 - \tau) w_{l,f,t} \\ &\longleftrightarrow \\ k_t &< k(z_{f,t}) \end{aligned}$$

¹⁷For instance the OECD reports that on average it takes 4-6 years to build a nuclear power plant without considering the time required for planning. Another good example is the Assuan Dam which become fully productive 16 years after the beginning of construction

¹⁸if $(\alpha + \phi < 1)$

which implies that the skill ratio in the informal sector is defined by:

$$z_{f,t} = \begin{cases} \text{Formal Regime} & z_{f,t} \equiv z_t \\ \text{Informal Regime} & z_{f,t} = \left(\frac{\gamma}{(1-\tau)(1-\alpha)} k_t^{-\varepsilon} \right)^{\frac{1}{\alpha+\phi}} \end{cases} \quad (.9)$$

The space $(z_{f,t}, k_t)$ is split in two regions by a decreasing curve (see fig 1.2.1), which from now on I call informality frontier, defined by the second equation of .9. Under this curve the economy is in the informal regime otherwise only formal firms exist. More clearly, if the skill ratio in the whole economy is larger than the threshold defined by .9, namely $(z_t > z_{f,t})$ the informal sector does not exist. Uneducated individuals have not incentive to join informal firms due to the fact that the technology there available does not allow informal firms to afford paying a wage which formal firms are willing to pay to uneducated individuals. Differently, if the skill ratio in the whole economy is lower than the threshold defined by the second of .9 both sectors coexist. Uneducated individuals join the formal sector up to the point the low skilled wage is equal in both sectors. It follows that in this model if both sectors exist uneducated individuals earn the same wage in both sectors namely the labor market is not segmented. The idea that identical workers earn the same wage in formal and informal sectors has consensus in the literature, see Amaral and Quintin (2006).

1.2.5 Dynamics, Intertemporal Equilibrium and Theoretical Implications

Given the assumptions made it is straightforward to define both the skill ratio law of motion and the workforce growth rate.

- Skill ratio law of motion¹⁹

$$z_{t+1} = \frac{H_{t+1}}{L_{t+1}} = \frac{(1-\bar{m})(nz_t + q)}{(1-q)(1-\underline{m})} \quad (.10)$$

- Gross growth rate of workforce

¹⁹ n is simply equal to the ratio $\frac{n_h}{n_l}$

$$g_t = \frac{\Gamma}{1 + z_t} \quad (.11)$$

where Γ is defined as follows:

$$\Gamma = n_h (1 - \bar{m}) z_t + n_l ((1 - \underline{m})(1 - q) + (1 - \bar{m}) q)$$

The equation which describes the infrastructure dynamics depends on the economy regime. The available amount of infrastructure depends on the amount of taxes raised by the government which is evidently a function of the formal sector size. Informal sector workers escape taxation, namely they do not participate to the accumulation of resources required to provide the stock infrastructure that will be available the subsequent period. Exploiting .1, .7, .8, and .11 it follows

- Formal Regime

$$k_{t+1} = \frac{\tau A_0 z_t^{\phi+\alpha} k_t^\varepsilon}{\Gamma} \quad (.12)$$

- Informal Regime

$$k_{t+1} = \frac{\tau A_0 z_{f,t}^{(\phi+\alpha-1)} k_t^\varepsilon z_t}{\Gamma} \quad (.13)$$

The two equations differ because the second one takes into account the fact that only a part of low skilled workers contribute to the next available stock of infrastructure. Before providing the intertemporal equilibrium I state the labor market clearing conditions:

$$\begin{aligned} H_t &= \bar{H}_t \\ L_{f,t} + L_{i,t} &= \bar{L}_t \end{aligned} \quad (.14)$$

Definition. Given an initial workforce size N_0 , an initial number H_0 of high skilled workers, an initial level of infrastructure per formal worker, k_0 , a set of parameters $\{\alpha, \phi, \varepsilon, \tau, n_h, n_l, q, \bar{m}, \underline{m}\}$ an intertemporal equilibrium consists: in a set of wages $\{w_{h,t}, w_{l,t}\}$, a sequence of skill ratio in the economy $\{z_t\}$, a sequence of skill ratio in the formal sector $\{z_{f,t}\}$ and a sequence of infrastructure levels $\{k_t\}$ such that depending on .9.

- Formal and Informal firms set competitive wages according to .3, .4 and .6
- Wages $\{w_{h,t}, w_{l,t}\}$ and aggregate quantities $\{\bar{H}_t, \bar{L}_t\}$ are such that market clear, .14 holds
- Skill ratio evolves according to .10
- Skill ratio in the formal sector is determined by .9
- Infrastructure depending on .9 evolves according either to .12 or .13.

Proposition 2. *Existence and Uniqueness of the temporary equilibria.*

Let consider an economy at time t with a given amount of workers, a given amount of high skilled individuals and a given amount of infrastructure. The economy is either under the formal regime or the informal regime depending on .9. The equations which set the wages in the economy are single valued and this suffices to prove the existence and the uniqueness of the temporary equilibria.

Corollary 3. *Existence and uniqueness of the intertemporal equilibria.*

Given a skill ratio in the overall economy at time t the subsequent level of the skill ratio is uniquely determined by .10. The infrastructure per capita level depending on the economy regime is uniquely determined either by .12 or .13. It follows that for a given initial pair of skill ratio and infrastructure levels $\{z_0, k_0\}$ the sequences $\{z_t, k_t\}_{t=0}^{\infty}$ are uniquely determined.

Before proving the uniqueness of the steady state the phase diagram equations need to be defined.

Lemma 4. *The skill ratio phase diagram is a straight line and its equation can be easily obtained applying the $z_t = z_{t+1}$ condition to .10. The equation is provided in the appendix. The infrastructure phase diagram is obtained applying the condition $k_t = k_{t+1}$ to both .12 and .13. The two equations cross the informality frontier at the same point. Proof and equations are provided in the appendix as well.*

The next lemma gives some simple comparative static results with respect the skill ratio phase diagram:

Lemma 5. *Exploiting equation (appendix .18) it is possible to show the following:*

- $\frac{dz^{ss}}{dm} < 0$, As long as the exogenous HS migration rate increases the skill ratio steady state decreases.
- $\frac{dz^{ss}}{dm} > 0$, As long as the exogenous LS migration rate increases the skill ratio steady state increases.
- $\frac{dz^{ss}}{dq} > 0$ if and only if $\frac{1-m}{1-\bar{m}} > n$

The next proposition proofs the uniqueness of the steady state.

Proposition 6. *The economy has only one (not trivial) steady state which lies either below the informality frontier or above it.*

The proof is by contradiction. I start assuming that the economy has a steady state that lies above the informality frontier. For the sake of clarity simple mixture of parameters are defined:

- $\Delta = \frac{\gamma}{(1-\alpha)(1-\tau)}$
- $\Theta = n_l ((1 - \underline{m})(1 - q) + (1 - \bar{m})q)$
- z^{ss} is the skill ratio steady state, (appendix, .18)

After some algebra and exploiting (appendix, .21 and .22)the following holds:

$$n_h (1 - \bar{m}) (z^{ss})^{\frac{\varepsilon - (\alpha + \phi)}{\varepsilon}} \Delta^{\frac{1-\varepsilon}{\varepsilon}} + \Theta (z^*) \Delta^{\frac{1-\varepsilon}{\varepsilon}} - \tau A_0 \leq 0 \quad (.15)$$

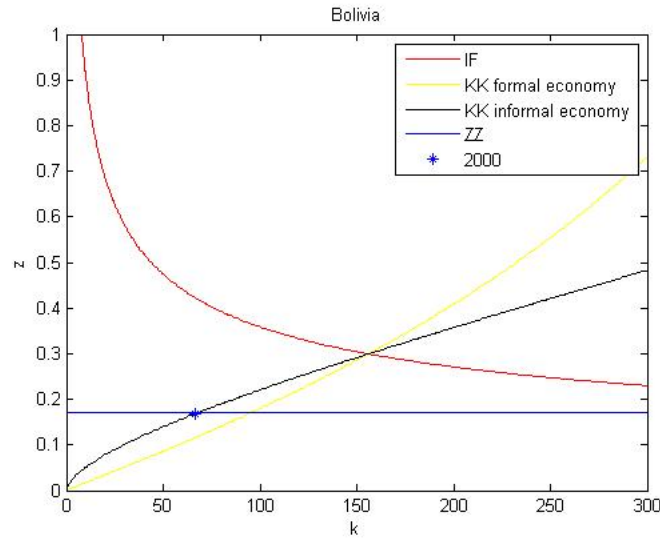
Hence if there exists a steady state without informal sector the former dis-equation should be satisfied. Let's assume otherwise that there exists a steady

state below the informality frontier, applying the same reasoning the following results:

$$n_h (1 - \bar{m}) (z^{ss})^{\frac{\varepsilon - (\alpha + \phi)}{\varepsilon}} \Delta^{\frac{1-\varepsilon}{\varepsilon}} + \Theta (z^*)^{-\frac{\alpha + \phi}{\varepsilon}} \Delta^{\frac{1-\varepsilon}{\varepsilon}} - \tau A_0 > 0 \quad (.16)$$

It is clearly evident that .15 and .16 cannot hold at the same moment. As a consequence the model has only one steady state which can be either with the informal sector or without. It holds that an economy with a steady state which lies below the informality frontier only a substantial change in the parameters can move the steady state above the informality frontier. In the next subsection the effect of exogenous variations in the tax rate are going to be object of study. The below picture helps to visualize the economy under analysis.

Figure 1.2.1: Exogenous Brain Drain



The red curve represents the informality frontier. If the pair of skill ratio and infrastructure is above to this curve the informal sector does not exists otherwise the economy is in the informal regime. The straight line parallel to the x axis represents the skill ratio phase diagram which does not depend on infrastructure. Finally, it should be noticed that the two curves which form the infrastructure phase diagram cross each other at the informality frontier.

1.2.5.1 Fiscal Policy

Tax rate policies do not affect education, fertility and migration choices having that these choices are kept exogenous to the model. Under this framework, the tax rate affects low skilled individuals in their choice in which sector to work.

Lemma 7. *An increase of the tax rate always move the informality frontier upwards. The Proof can be found in the appendix.*

For any tax rate increase the net wage that formal firms offer to uneducated individuals decreases. Hence, *ceteris paribus*, for any given pair of infrastructure level and skill ratio the wage available in the informal sector becomes more profitable.

The effect on the infrastructure phase diagram depends on the economy regime.

Proposition 8. *Under the formal regime an increase of the tax rate always boosts infrastructure accumulation (assuming no regime switching). Instead, under the informal regime, there is a positive effect on infrastructure accumulation if the initial tax rate satisfies the following condition:*

$$\tau < (\alpha + \phi) \tag{.17}$$

Assuming no regime switching as well. Proof provided in the appendix.

Let's start considering the formal regime. In such a case any increase in the tax rate has a positive effect on infrastructure accumulation. This is due to the fact that the curve represents the evolution of infrastructure when education, migration and fertility choices are not affected by the tax rate. Differently, under the informal regime two forces play a role after a tax rate increase. An increase in the tax burden makes the wage in the informal sector larger than the wage available in the formal sector. As a consequence uneducated individuals join the informal sector until the point the arbitrage condition is restored. At the same moment formal workers pay more taxes increasing infrastructure accumulation. These two forces cancel out each other when the tax rate is equal to the RHS

of .17. Conversely, if the tax rate is lower than this value the net effect on infrastructure will be positive.

Anyhow it is worth to analyze the impact of a restrictive fiscal policy on the welfare of both high skilled and low skilled individuals. If the economy is under the informal regime the low skilled wage will not be affected after a tax rate increase. In fact a fraction of individuals will leave the formal sector to join the informal sector and the wage will be equalized across sector. Differently high skilled individuals will be for sure worse off due to both the higher tax rate and the lowering in the skill premium implied by the flow of low skilled individuals to the informal sector.

The dynamics effects of a restrictive fiscal policy on wages depends both on the initial tax rate and if the increase (I assume that 8 holds) in the infrastructure accumulation will trigger a regime switching from the informal to the formal regime. The next proof defines the threshold values for which an increase in the tax rate will be welfare improving with respect high skilled welfare one period afterwards of the restrictive fiscal policy implementation

Proposition 9. *Let assume that the economy at time t is under the informal regime and that the next period there will not be a regime switching. Furthermore, assume that .17 holds. A restrictive fiscal policy at time t will increase the high skilled wage the subsequent period if the initial tax rate is such that:*

$$\tau < \frac{\varepsilon(\alpha + \phi)}{(\varepsilon + \phi + \alpha)} \quad (.18)$$

If the above equation is satisfied the gains due to the larger infrastructure accumulation overcomes the losses due to both the lower skill premium and the larger tax rate²⁰.

If the economy switches regime (from informal to formal) at time $t + 1$ the high skilled wage will increase if .17 holds. Proof provided in the appendix

In the next proposition the effects of a restrictive fiscal policy on the low skilled wage are analyzed

Corollary 10. *Let assume that the economy at time t is under the informal*

²⁰It suffices to exploit an easy an absurdum argument to show that .18 threshold is always lower than the one defined by 8

regime and that the next period there will be not a regime switching. In such a case the low skilled wage will remain constant and equal to .6. Let assume that there will be a regime switching the subsequent period (from informal to the formal regime). In such a case the effect on the low skilled wage will be always positive (otherwise there will not be a regime switching).

1.3 Endogenous high skilled migration

Here the model presented in the previous section is considered with endogenous brain drain. High skilled migrants bear a fixed cost, equal to everyone, when they leave their native country, moreover, I assume that high skilled individuals are heterogeneous with respect their migration taste as in Delogu et al. (2014). Low-skilled migration is kept exogenous to the model, a common assumption in the literature already discussed in the introduction. The reason is that low skilled migration rates are usually much lower than high skilled ones reports, and also more responsive to push-pull factors.

A rich-poor countries set-up is analyzed²¹ Furthermore, following a ordinary assumption, I assume that the migration taste is distributed across individuals according to a type 1 extreme value distribution function, with scale parameter μ .

Individuals' indirect utility is assumed being representable by a logarithmic function with respect the wage earned, it turns out that both the utility of staying and the utility of leaving can be easily determined. Clearly, it suffices to assume that individuals aim to maximize their own welfare to exploit the McFadden (1984) theorem. Before doing it, I define the probability that an high skilled individual native in the developing country as follows:

$$P_{ha} = [\ln(w_{h,t}) < \ln(w_t^*) + \ln(1 - x_t) + \bar{\eta}_{t,i}]$$

The first term represents the indirect utility of an high skilled individual i who does not migrate at time t , which is simply a function of the wage available in his native country. The second term represents the indirect utility of migrat-

²¹The extension with more than one destination country could be easily made. A two country set-up better suits the objective of the study given that I focus on the source country, effects at destination are disregarded.

ing. This term is a function of the wage available abroad net of migration costs and of the random component which defines the individual peculiar migration taste. Furthermore, perfect foresight is assumed. As already mentioned the random component follows the Type 1 extreme value distribution function:

$$F(\eta) = e^{-e(\frac{\eta}{\mu} - \gamma)} \quad \varepsilon \in \Re$$

γ is the Euler's Constant. All the ingredients needed to apply the McFadden theorem are available. It results

$$P_{ha} = \frac{e^{\frac{1}{\mu} \ln(w_t^*)}}{e^{\frac{1}{\mu} \ln(w_t^*)} + e^{\frac{1}{\mu} \ln((1-\tau)\alpha A_0 z_{f,t}^{\alpha+\phi-1} k_t^\varepsilon)}}$$

As a consequence the high skilled migration rate is equal to²²:

$$\bar{m}_t = \frac{w_t^*}{w_t^* + (1-\tau)\alpha A_0 z_{f,t}^{\alpha+\phi-1} k_t^\varepsilon} \quad (.19)$$

The above equation gives the endogenous migration rate. It turns out that it is negatively related with the available stock of infrastructure in the source country and positively related with the net wage available in the stylized destination country here considered. Moreover, it is inversely related with the skill ratio in the formal sector. The endogenous relationship between the latter and the high skilled migration rate will be clearer after the reading of the forthcoming section.

1.3.1 Implications

The endogeneity of the migration rate affects both the skill ratio and infrastructure law of motions. In fact, notice that from equation .19 results that the high skill migration rate is a function of the available stock of infrastructure which furthermore is a function of the total workforce that is clearly a function of the high skilled migration rate. As a consequence, both infrastructure and the skill ratio are not predetermined variable anymore. Anyhow, differences arise depending on the economy regime. Let consider with more attention equation

²²From now on the scale parameter, μ , is set equal to 1. This is a quite common assumption which will be discussed in the quantitative assessment part.

.19 which defines the HS migration rate. Under the formal regime is a function both of infrastructure and the skill ratio while in the informal regime; exploiting equation .9, it becomes only a function of the infrastructure available in the same period. It follows then that the law of motion of the skill ratio depends on the economy regime. The equations are provided in the appendix as well. Applying to these equations the condition $z_t = z_{t+1}$ the skill ratio phase diagrams equation are obtained. In the appendix is proved that the two curves cross each other at the informality frontier. The same is done with respect infrastructure. The equations which determine the infrastructure phase diagrams become implicit functions. Moreover, the one which characterize the formal regime is a function of both z_t and z_{t+1} while the one which characterize the informal regime is only a function of z_t due to the fact that the the skill ratio in the formal sector under the informal regime is determined by the arbitrage equation as just explained. The simultaneous determination of both skill ratio and infrastructure may suggest the existence of multiple equilibria. In the following I provide the definition of the intertemporal equilibrium and subsequently a proof which states the conditions for the uniqueness of the consistent migration rate at the temporary equilibrium. The phase diagrams below helps to visualize the economy under analysis²³

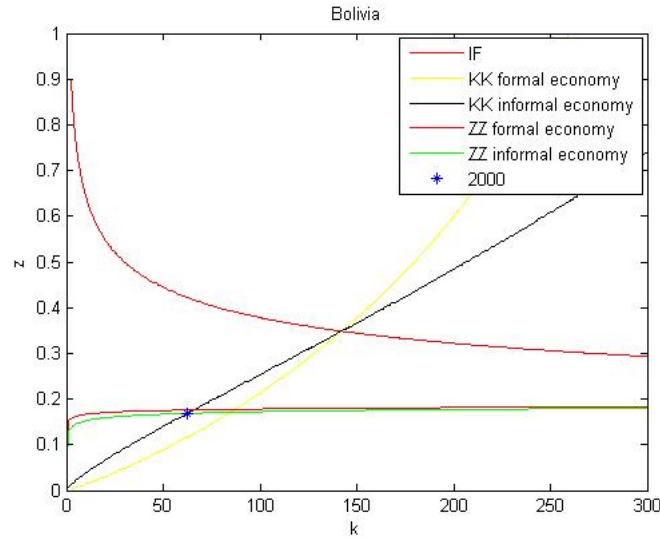
As showed by the above figure the skill ratio phase diagram is not a straight line anymore and it depends on the economy regime as well.

Definition. Given an initial workforce size, N_0 , an initial number H_0 of high skilled workers, an initial level of infrastructure per formal worker, k_0 , a set of parameters $\{\mu, \alpha, \phi, \varepsilon, \tau, n_h, n_l, q, \underline{m}\}$ an intertemporal equilibrium consists: in a set of wages $\{w_{h,t}, w_{l,t}, w_t^*\}$, a sequence of skill ratio in the economy $\{z_t\}$, a sequence of skill ratio in the formal sector $\{z_{f,t}\}$, a sequence of infrastructure levels $\{k_t\}$ and a sequence of migration rates $\{\bar{m}_t\}$ such that depending on .9.

- Formal and Informal firms set competitive wages according to .3, .4 and .6
- High skilled migration rate is determined by .19
- Skill ratio evolves depending on .9 evolves according to .23

²³the graph is drawn exploiting a myopic expectations assumptions ($z_t = z_{t+1}$) and ($k_t = k_{t+1}$)

Figure 1.3.1: Endogenous Brain Drain



- Infrastructure depending on θ evolves according to (24)

Proposition 11. *Let assume perfect foresight. The following holds:*

1. In the informal regime there is only one consistent migration rate.
2. Under the formal regime there is no multiplicity of equilibria if $(\alpha + \phi) < 1$. Otherwise if the former condition is not satisfied by the parameters two consistent migration rates exist: one which lies between zero and 1 and a second one equal to 1 (all high skilled individuals leave their home country).

The proof is provided in the appendix. The intuition behind the proof can be easily explained. It comes from a simple fixed point reasoning. I Start considering the equation which defines the high skilled migration rate, eq (19). Afterwards, I plug into the former the value of the skill ratio in the formal sector. This value depends on the economy regime as summarized by equation (9). Furthermore, consider equation (8), which defines the infrastructure. The latter is exploited not forgetting that the total workforce is a function of the

high skilled migration rate as well. Hence, the migration rate will appear in both side of the equations. The right hand side is studied and it is showed that is always a decreasing function in the interval $[0, 1]$ in the informal regime. From this follows that it crosses a 45 degree line only one time. The same holds in the formal regime if $(\alpha + \phi) < 1$. In the formal regime, as long as this condition is not satisfied, when the migration rate approaches 1 the RHS becomes equal to 1 proving that the a migration rate equal to 1 is an equilibrium.

An analysis of the equations which define the skill ratio phase diagrams allows to state the next lemma:

Lemma 12. *After a tax rate increase the skill ratio phase diagrams always move downwards.*

In fact a sufficient condition for the formal regime is given by the following equation

$$z_t < \frac{q}{(1-q)(1-m) - n} \quad (.20)$$

Which is always met (it is simply the skill ratio that would characterize an economy without HS migration). In the informal regime the movement is downwards if the tax rate is below one.

$$\tau < 1 \quad (.21)$$

As a consequence of the former lemma a positive effect on the economy steady state may come only through an upwards displacement of the infrastructure phase diagrams.

1.4 Quantitative Assessment

A model which highlights the connections that relate informality, infrastructure accumulation and high skilled migration has been set in the first part of the chapter. Keeping aside high skilled migration the model predicts that, under condition .17, a restrictive fiscal policy could help to boost infrastructure accumulation. I start defining a calibration strategy which suits both frameworks. Subsequently, the calibration strategy is exploited and the effectiveness of different development policies through numerical experiments will be eventually assessed.

1.4.1 Parametrization

The model is calibrated assuming that each period lasts 25 years. The parametrization strategy is designed to match labor force growth between 1975 to 2000 and it is made under the assumption that infrastructure and skill ratio in year 2000 are at their steady state levels. In the next section both data sources and the calibration strategy are explained in detail.

1.4.1.1 Calibration Strategy

I calibrate the model as to match the estimated informal sector size, skill premium data and formal sector production. Doing so the economic situation in year 2000 of the considered countries is almost perfectly matched allowing to assess the dynamics implications of infrastructure development policies in the next section. I start discussing the calibration of the parameters, distinguishing between structural and country specific ones.

- *Structural Parameters* ($\tau, \varepsilon, \phi, n, \mu$)

I start setting the tax rate equal to 0.05 for all countries. I acknowledge that it is very rough measure, anyhow it is quite close from Dobbs (2013) estimates on infrastructure investments in developing countries. Using a perpetual inventory method the former paper estimated the infrastructure spending in between 1992-2001. The country which invested more is China with total spending estimated equal to 8.5 % of GDP. In Africa and Middle East the investment has been equal, on average, to 3.6% of GDP.

The value of the two key elasticities of the model (ε, ϕ) is calibrated ex-

exploiting recent insights of the literature. The value of the education externality parameter, namely the elasticity of productivity to human capital in the formal sector which has been labeled with the greek letter ϕ , is set equal to 0.277 as in De la Croix and Docquier (2012). They obtained this estimate through a regression using a large sample of developing countries (142). Following the same paper, I recall that there is still a debate about the size of the education externality. In fact, Acemoglu and Angrist (2001) finds no productivity increase due to increased schooling whereas Iranzo and Peri (2009) using US state data propose a value equal to 0.44. As noticed by the authors, the value of 0.277 might be considered conservative with respect the Iranzo and Peri estimate.

The other key elasticity is the one which relate total productivity with infrastructure. As already mentioned in the introduction a huge literature focused on the subject. As a recent IMF (2014) paper I set this value to the one suggested by the meta analysis conducted by Bom and Ligthart (2013). They considered 68 studies showing that much of the difference comes from, among others, the definition of infrastructure, the econometric specification defined and the sample coverage. They suggest that elasticity of core infrastructure installed by the national government is equal to 0.17. This estimate is going to be used both in the parametrization and in the simulation sections.

The fertility differential, n , which is simply the ratio between the high skilled and low skilled fertility rate is assumed invariant across countries as well. This data is taken from De la Croix and Docquier (2012) as well²⁴. They took data on fertility rates from Kremer and Chen (1999) which allowed them to compute differential fertility in 1985-1989 for 26 developing countries. It results that the correlation between country-specific fertility differentials and the human capital of women is enough low to assume that fertility differential are independent from the level of development. They obtained an average fertility rate equal to 0,605 and here the same value is going to be used. The last structural parameter is μ which is set, as in the theoretical part, equal to 1. This choice implies an elasticity of the high skilled migration rate with respect the wage ratio equal to 1.

- *Country Specific Parameters*

²⁴High skilled are individuals with more than 10 years of education and respectively low skilled workers are the ones with less than 10 years of education.

I start parametrizing the low skilled fertility rate. This step is accomplished exploiting equation .11 with respect the time span 1975-2000. Data on size and education structure of the labor force for both 1975 and 2000 are obtained from Defoort (2008) and Docquier et al. (2009). Moreover, the latter provides data on migration rates at the source country level differentiated with respect the education dimension as well. That allows to determine both the left hand side of .11 and the skill ratio in year 2000. Solving for n_l the just mentioned equation an estimate of the fertility rate is obtained.

A key feature of the paper is that it relates productivity to the skill ratio in the formal sector. Data on the size of the informal labor force can be found in Schneider (2012). He collected data from both OECD and World Bank datasets. A key assumption made on estimating that size is that the informal sector labor force is the same in rural areas is at least as high as in the cities, which is clearly a conservative assumption. As Schneider did I report briefly the main results of Jutting et al. (2009) study, published by the OECD, that focused on the size of the informal sector. It has been estimated that without considering the informal sector over 900 million of workers can be considered informal. The estimate reaches the amazingly high value of 2000 million when the agricultural sector is incorporated. Hence, The OECD study comes to the conclusion that informality is really the norm in developing countries, in fact it results that 1.8 billion of people work in informal jobs compared to 1.2 billion who benefit from formal contracts and social security protection. In my calibration strategy I consider only Developing and Transition countries. Data is available, with respect year 1998 for 33 African countries. The average size of the informal labor force with respect total labor force is found equal to 54.2 %. Nine Asian countries are considered with an average size equal to 46.5%. With respect Latin and south America data is available for nine countries, with an average of 49.6. Moreover also 9 European transition countries are considered. In this case, again the size of the informal labor force is high, being the estimate a bit lower than 50%. Hence data is available to calibrate the model for 60 countries. The skill ratio in the formal sector is obtained as follows:

$$z_{f,00,j} = \frac{H_{00,j}}{(L_{f,00,j} + L_{i,00,j}) - s_{98,j} (L_{00,j} + H_{00,j})} \quad (.22)$$

Where s_{98} is the size of the informal labor force reported by Schneider (2012).

The next parameter which is going to be calibrated is α , namely the elasticity of official GDP with respect the stock of high skilled labor leaving aside the

total factor productivity. This is accomplished exploiting the wages defined by the model and computing the wage ratio. The wage ratio between high skilled and low skilled us determined using both data on returns to schooling and average years of education. Mincerian returns to schooling can be found in Hendricks (2004) with respect 54 countries around the year 2000. For the same countries using Barro and Lee (2013) data allows to know the difference in years of schooling in year 2000. Let define as $MR_{00,j}$ the mincerian return and by $DY_{00,j}$ the difference in years of schooling. It follows that the wage ratio is simply equal to $wr_{00,j} = (1 + MR_{00,j})^{DY_{00,j}}$. For countries where data is not available the wage ratio is taken from Delogu et al. (2014)²⁵. For the sake of clarity α is determined exploiting the following equation:

$$\alpha_j = \frac{wr_{j,00}z_{j,00,f}}{1 + (wr_{j,00}z_{j,00,f})} \quad (.23)$$

The last parameters are determined exploiting the assumptions that each economy is at the steady state and in the informal regime. A difference arises between exogenous and endogenous high skilled migration frameworks. In fact, with respect the second case, it is needed to calibrate also the wage net migration cost that can be earned moving to a developing country. In the exogenous case $A_{0,j}$, q_j , $k_{00,j}$, γ_j are determined simultaneously solving numerically for each country the system determined by equations .1,.9,.18,.22. Data on GDP in USD for year 2000 are obtained from the WorldBank (2012). Calibrating the model with endogenous brain drain requires to add to the former system one more equation and one more unknown, namely I solve also for w^* , considering equation .19 as well.

The above defined calibration strategy is going to be exploited in the next section to provide forecasts, namely as to estimate the effectiveness of different policies.

1.4.1.2 Comments

The infrastructure levels at the steady state obtained from the above defined calibration strategy are the same for both models. Moreover also the other parameters are equal between variants, the only difference between the two frameworks is that in the endogenous one is calibrated also w^* . The table below shows the calibrated values with respect four countries on which I will focus in

²⁵They obtained wage ratio estimates through a simple OLS regression

Table 1.4.1: Calibrated Values

Country	q	n_l	α	k_{ss}
Croatia	0,056	1,60	0,62	255,57
El Salvador	0,063	2,45	0,44	125,88
Sierra Leone	0,013	1,57	0,14	11,38
Sri Lanka	0,021	1,47	0,16	50,28

the simulation part as well ²⁶. I choose to take one country per continent: Croatia for Europe, El Salvador for Latin America, Sierra Leone for Africa and Sri Lanka for Asia. The choice has not been made randomly, I prefer to focus on countries on which the brain drain is substantial and larger than the region average in order to highlight its impact on infrastructure development policies. Anyhow, in the appendix data and comments for the remaining countries are provided.

1.4.2 Simulations

Here the calibration strategy above defined is exploited to provide forecasts from year 2025 up to year 2100. Having that the model is calibrated at the steady state²⁷ my attempt consists in perturbing some parameters and then to simulate the model in order to quantify how these change affect the evolution of key variables, highlighting the changes in infrastructure and in the high skilled migration rate. Furthermore, for both models the same policy change is considered. As a consequence it is possible to determine how the brain drain affects the infrastructure accumulation process. Before moving to the description of the results I briefly explain the strategy followed in producing these numerical experiments. In the exogenous high skilled migration the program starts exploiting the law of motion which characterize the skill ratio evolution. Subsequently, the program computes the amount of infrastructure available per worker. Finally, exploiting equation .9, the program ascertains which regime (formal and informal) characterizes the economy. The same is done for all the subsequent periods.

The task that the program has to solve is more complex when endogenous brain drain is accounted. First, exploiting proposition 11, the high skilled mi-

²⁶Data with respect the remaining countries are provided in the appendix.

²⁷It is worth to mention that the model does not show endogenous per capita growth at the steady state

gration rate that would characterize the economy if it were under the formal regime is determined. Afterwards, the skill ratio is determined and eventually the respective amount of infrastructure per capita is determined. The same is done assuming that the economy were under the informal regime. Finally the condition which characterize the informal sector existence, lemma 1, is checked and are kept the result consistent with.

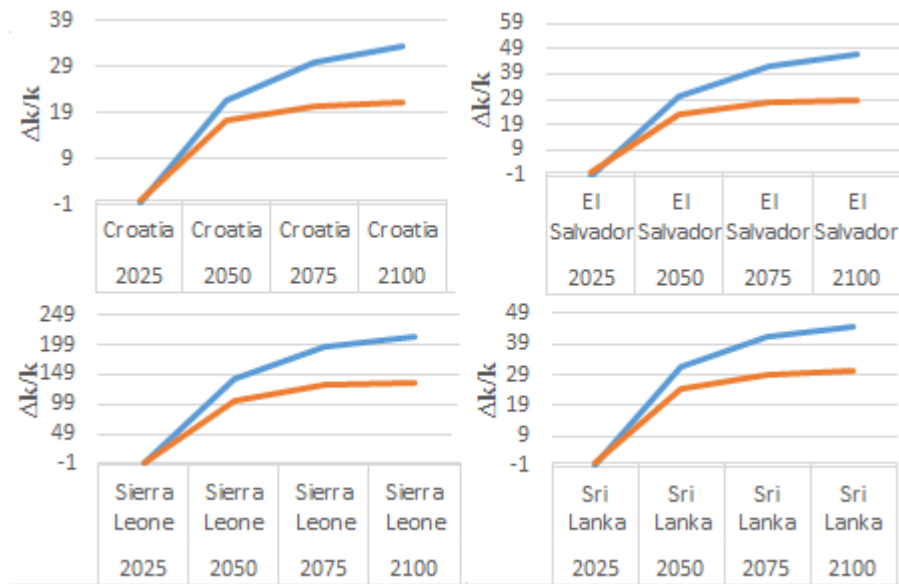
In the following two policies changes are considered. First, I assume that the government is able to remove informality, namely the parameter which defines the informal sector productivity, γ , is set equal to zero from year 2025 onwards. The second experiment checks the quantitative implications of an increase in the tax rate. A small increase is considered, τ will be set equal to 0.06 from year 2025 up to year 2100.

1.4.2.1 Removing Informality

As Docquier et al. (2014) I estimate the model assuming a sudden banning of the informal sector. This policy option clearly could be not accomplished, anyhow it is a useful experiment which permits to appraise how the informal sector is related with both infrastructure accumulation and the high skilled migration rate. Hence in this numerical experiment from year 2025 all the workers will participate to the formal economy. Here I discuss the results with respect the selected four countries. First I describe the evolution of infrastructure, secondly the brain drain is assessed and eventually a discussion on the evolution of the wages is provided. Let's focus on the infrastructure measure:

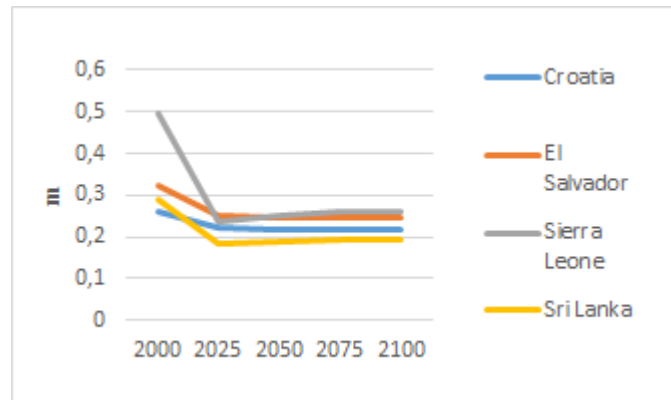
The blue curve represent the percentage change in infrastructure under the endogenous HS migration scenario. As it is evident from the above figure the effect is always larger in such a case. This is due to the fact that removing informality would have a beneficial effect, with respect the source country, in terms of brain drain. Moreover, it should be noticed that in the exogenous scenario the level of infrastructure remains constant in year 2025. Differently under the endogenous scenario there would be a small decrease in the short run due to the response of the high skilled migration rate to the new economic scenario. Despite this tiny decrease in the short run the foresee benefits would be much more larger in the long run. Let consider each country separately. Croatia is the country which starts with the largest infrastructure stock, $k_{ss} = 255,57$. Removing informality without considering high skilled migration would lead

Figure 1.4.1: $\Delta k_t/k_{ss}$



to an increase of the stock of infrastructure in the long run equal to 33, 59% while the model with exogenous migration predicts gains equal to 21, 18%. The same pattern is found for the poorest country of the subset here considered, Sierra Leone. As showed by table 1.4.1, this country has the lowest initial stock of infrastructure, equal to 11, 38. Removing informality would more than triplicate the infrastructure stock in the long run. Not accounting for the brain drain would lead to underestimate the gains, even if they remain quite large. In such a case the the model predicts only a double of the k . The evolution of infrastructure would follow the same pattern for both El Salvador and Sri Lanka. Moreover it makes sense to look at the foresee evolution of the high skilled migration rate in this counterfactual scenario:

From the above figure it is evident that the model predicts a huge decrease of the high skilled migration rate in the short run. Given that High skilled individuals constitute only a small fraction of the population in developing country this huge drop has almost no effect on the infrastructure measure. Considerable gains in productivity will appear in the long run due to both the assumed formal sector production technology and the education externality. As time pass by the high skilled migration rate will converge to a substantial lower equilibrium. The same pattern is found for all the countries under analysis (see appendix).

Figure 1.4.2: \bar{m}_t Table 1.4.2: $\Delta w_{l,t}/w_{l,ss}$

Country	$w_{l,ss}$	2025		2100	
		end \bar{m}_t	ex \bar{m}_t	end \bar{m}_t	ex \bar{m}_t
Removing Informality					
Croatia	12173,58	-72,79	-74,02	-69,54	-73,15
El Salvador	5966,32	-37,54	-41,85	-27,57	-39,29
Sierra Leone	982,47	-28,69	-39,94	-6,21	-30,48
Sri Lanka	1723,22	-10,3	-15,53	-0,36	-11,66

A kind of welfare analysis can be done looking at the effect on wages. Removing informality has different effects on both low skilled and high skilled wages. In the short run the low skilled wage surely will surely shrinks. In fact the informal sector is not at work anymore and the low skilled wage will be freely determined in the formal economy labor market. The increase in the low skilled supply will decrease the skill ratio implying the described decrease. In the long run the effect is ambiguous. In fact the disappearing of the informal economy boosts infrastructure accumulation and this positively affects the low skilled wage

The above table provides the numerical results obtained with respect the four selected countries. In all countries the negative impact in the short run is larger than in the long run. Moreover, for all the countries the positive effect due to infrastructure accumulation will not cover the loss borne by low skilled due to the removing of informality even after 75 years. The losses in the endogenous scenario are lower, but for most of the countries the difference is negligible.

Table 1.4.3: $\Delta w_{h,t}/w_{l,ss}$

Country	2000	2025		2100	
	$w_{h,ss}$	end \bar{m}_t	ex \bar{m}_t	end \bar{m}_t	ex \bar{m}_t
	Removing Informality				
Croatia	38597,37	16,61	17,38	21,71	21,28
El Salvador	18742,99	19,81	23,53	24,24	28,97
Sierra Leone	4692,51	60,61	104,91	74,44	137,17
Sri Lanka	7685,88	15,01	24,57	16,07	30,27

The discrepancy between the two models comes from the decrease in the high skilled migration which has a negative impact on the skill premium. In the four countries under analysis this effect is sizable in Sierra Leone due to huge foreseen drop in the high skilled migration rate.

Instead, removing informality would have a positive effect on the high skilled wage both in the short and long run. The increase in the stock of low skilled workers in the formal economy would enlarge the skill premium. This effect is stronger than the negative one due to the reduction in the skill ratio. This positive effect is reinforced in the long run due to the increase in the available stock of infrastructure. The next table shows the results with respect the four countries under analysis.

As it is evident from the table the effect on the high skilled wage is always positive and increases over time. For most of the countries gains are lower in the endogenous scenario. The endogeneity of high skilled migration affects in two ways, one positive and one negative, the high skilled wage. A decline of the skill ratio in the formal sector has an adverse effect on the high skilled wage. The negative effect comes through the skill premium due to the decrease in the high skilled migration that leading a larger stock of high skilled increases then the skill premium. Moreover, the increase in the stock of low skilled workers in the formal sector reinforces the positive effect on the skill premium. This mechanism explains why in some countries, such as Croatia, the positive effect on wages is larger with endogenous brain drain than in the model without (the positive effect on the skill premium overcomes the negative on TFP)

Summarizing, I have shown that removing informality would boost infrastructure and that larger gains would be attained in the long run. Anyhow, in the short run a banning of the informal sector deeply hurt low skilled workers whereas the high skilled will get considerable monetary gains.

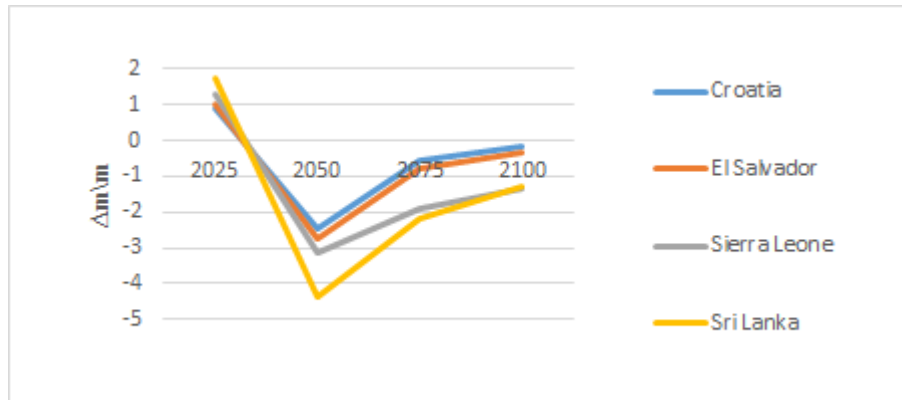
Table 1.4.4: $\Delta k_t/k_{ss}$

Country	year	Endogenous \bar{m}	Exogenous \bar{m}
Croatia	2025	0,03	0,00
	2050	19,44	19,85
	2075	24,34	24,05
	2100	25,99	24,86
El Salvador	2025	0,05	0,00
	2050	18,88	19,51
	2075	25,02	24,64
	2100	27,62	25,89
Sierra Leone	2025	0,01	0,00
	2050	16,72	18,23
	2075	27,75	26,61
	2100	36,08	30,21
Sri Lanka	2025	0,02	0,00
	2050	17,50	18,36
	2075	26,99	26,43
	2100	32,88	29,73

1.4.2.2 Fiscal Policy

One of the theoretical implications of the model, without endogenous HS migration, is that if the tax rate is lower than an endogenous threshold a restrictive fiscal policy will increase the infrastructure stock (proposition 8). It is easy to conjecture that such a threshold must be lower when brain drain is accounted into the model. In fact, doing so it is given the opportunity to escape taxation to high skilled individuals as well. In this section I compare the two frameworks perturbing the model at the steady state assuming a one percentage point increase in the tax rate. The next table shows the results obtained with respect the four selected countries:

As the above table shows there are only slightly differences between the estimates of the two frameworks. In the endogenous brain drain scenario, after the tax rate increase, infrastructure per capita shows a small growth due to a declining high skilled migration rate. Anyhow, in the long run the predicted gains are only slightly different between the two scenarios. Among the selected countries the only country for which there is a substantial, in percentage terms, difference between the two frameworks, in the long run, is Sierra Leone. Anyhow, it should be not forgotten that Sierra Leone starts from a very low level of infrastructure, hence in real terms the difference is not large at all. The next

Figure 1.4.3: $\Delta m_t/m_t$ Table 1.4.5: $\Delta w_{h,t}/w_{h,ss}$

Country	2000 $w_{h,ss}$	2025		2100	
		end \bar{m}_t	ex \bar{m}_t	end \bar{m}_t	ex \bar{m}_t
		$\uparrow \tau (+0, 01)$			
Croatia	38597,37	-1,171	-0,012	3,159	3,086
El Salvador	18742,99	-1,448	-0,015	4,200	4,049
Sierra Leone	4692,51	-2,510	-0,025	9,569	8,606
Sri Lanka	7685,88	-2,399	-0,024	8,320	8,058

graph shows the percentage variation over time of the high skilled migration after the increase in the tax rate:

In the short run a restrictive fiscal policy always implies an increase in the brain drain. By assumption, the infrastructure stock is determined by the tax collection done the earlier period. Subsequently the positive effect due to increase accumulation of infrastructure prevails. The reason is that due to the increase in tax rate the stock of infrastructure increases, boosting wages, and then reducing the incentive to migration. In the long run the high skilled migration rate approaches to a new equilibrium. Given the small size of the tax increase studied the low skilled wage will always be determined by the informal sector technology, namely it remains at its 2000 level. The reason is that no country exhibits a regime switching behavior, from informal to formal regime. Differently the high skilled wage after the change in fiscal policy change. The next table shows the results with respect the four selected countries:

From the table above is evident that a restrictive fiscal policy will always

negatively affects the high skilled wage in the short run. The negative effect is predicted larger when brain drain is accounted. The reason is that when brain drain is not accounted the tax policy change does not affect the skill ratio in the formal sector, hence the skill premium remains at the 2000 level. Differently when brain drain is accounted, infrastructure per capita is determined simultaneously with brain drain. The brain drain slightly increases in the short run as well as the size of the informal sector have a detrimental effect on the skill premium. The decrease in the stock of population led by the increase in the global migration rate increases the stock of available infrastructure. The numerical experiments show that this effect does not prevail in the short run, namely the decrease in the high skilled wage is larger when brain drain is accounted. In the long run the positive effect due to infrastructure prevails and gains are slightly larger when brain drain is accounted.

1.5 Conclusions

This chapter provides a macroeconomic framework able to highlight the links between infrastructure accumulation, informal sector and brain drain. In the first part the set up of the model has been described, highlighting the role of brain drain. Theoretical implications suggest that a restrictive fiscal policy could be the right tool to boost infrastructure accumulation in developing countries in the long run.

In the second part theory has been confronted with data. First a calibration strategy has been developed with respect year 2000. Subsequently the model has been simulated in order to assess the dynamic implications of two policy shocks: a banning of the informal sector from year 2025 and a small tax rate increase with respect the same initial point as well.

The numerical experiments show that the informal sector protect from extreme poverty low skilled. The implied boost in infrastructure accumulation is not enough to restore the former level of the low skilled wage even in the long run. Differently, the high skilled wage increases both in the short run and long run even if the brain drain declines. I acknowledge that fertility and education are both kept exogenous at their calibrated level in year 2000. The change in the relative high skilled wage would increase the incentives to acquire education and that it is likely that will affect the results here obtained. A direction for future research could be an extension of the framework here defined accounting for both education and fertility decisions.

The fiscal policy here analyzed does not affect low skilled wage given that no country exhibits a regime switching behavior. Differently, the restrictive fiscal policy slightly harm high skilled in the short run. In the long run the positive effect due to infrastructure prevails with gains in between 5% to 10% with respect the steady state high skilled wage. The result is consistent with the brain drain mechanism accounted. It comes out that a slightly restrictive fiscal policy in the short run, in developing countries, could imply substantial productivity gains in the long run and eventually help developing countries to achieve development.

Chapter 2

An OLG Model of Sovereign Debt Crisis

2.1 Introduction

Wright (2011) points out that when an international banker signs a contract with an independent country, it faces an additional problem compared to the situation when the same contract is traded with a private borrower. In fact, when dealing with a private borrower, the costs of a default are determined by the country legal system and devoted institutions governing bankruptcy. Wright refers to the Hobbes primary work, the Leviathan, where the author argues that an agent who makes the laws is not bounded by those laws. Despite this, most of the economic models, including the seminal paper of Diamond (1965) have been set under the assumption that a government always repays its debt.

This assumption used to be reasonable for developed countries, though the current economic crisis in Greece, Ireland, Italy, Portugal and Spain has raised doubts on the ability of rich EU countries to repay their debts as well.

Reinhart and Rogoff (2008) provides a full review of all financial crises occurred between 1350 and 2006, showing that serial default is a universal policy, implemented primarily by countries struggling to change their status from emerging to advanced countries. The authors also observed that defaults were usually spaced years apart and suggest that the relative long time between defaults may lead investors and policy-makers to think that “this time is different” and in so doing it seems that the bankers forgive that a government can default and over-borrow when the economic situation is favorable. For in-

stance, between the historical period 1350-2006 Reinhart and Rogoff found that all countries under study have defaulted at least once and many several times on external debts; for instance Spain defaulted 8 times between the 1350-1647. The default policy was common among European countries and as Reinhart and Rogoff recall, quoting the french policy maker Abbe Terray, who served from 1768-1773, that a “government should defeated at least once every 100 years to restore equilibrium”. The authors claim that a country tends to over borrow in good times leaving them to inevitable downturns when the economy is hit by damaging shocks. Between 1800 and 2006, both the emerging of international capital markets and the emerging of new countries generated an increase in sovereign default. Such stylized facts suggest that government’s default is not a sporadic phenomena and for these reasons is necessary to deal rigorously with the research defining the relationship between the government and its lenders.

First a brief review of the literature on the subject is provided. Initially, I focus on the nature of default’s cost. The literature tackles the problem assuming that after defaulting a governments incurs reputational costs that directly affect the economy fundamentals. Subsequently, a discussion of overlapping generations model on which agents are allowed to invest on government’s bonds is provided. Finally, models with endogenous sovereign debt crisis are discussed in light of the model afterwards developed.

In the second part of the chapter an OLG model with endogenous government’s default choice is developed. First the model set-up is developed and afterwards its implications are discussed. It is obtained that there exists threshold values of the initial level of capital for which the government is indifferent to refuse its debt or roll over. These amounts can be interpreted as levels of capital which separate countries in between developed and developing ones. As a consequence the model is coherent with the stylized facts reported by Reinhart and Rogoff.

2.2 Review of the literature

2.2.1 Default's costs

In the next part, in developing the model, it will be assumed that if a country defaults on its debt it incurs some costs, namely a permanent drop in productivity. Here I present the main papers that made the same or a similar assumptions. As already mentioned in the introduction, it does not exist any legal mechanism which compels a government to repay its debt. Yet, there exists a literature that defines the incentives that may explain why a country will repay its debt. The literature differentiates the bankrupt's costs as follows:

1. Real costs that affects the economy, which sometimes are modeled either as a decrease in productivity or as a one time cost which affects the country's budget constraint.
2. Access lost to the international capital market

For the first source of costs, defined in the literature as direct costs, I can give as an example the possible damages that a domestic banking crises may have on the output of a country. Another mechanism that explains why a government's bankrupt should have a negative effects on the output is the one that links the default with a decline in foreign trade, which may arise from the imposition of trade restrictions.

The costs denoted in the second point may arise from two sources. The first explanation comes from the threats of creditors to retaliate against a creditor in default by denying them access to new credit.

Eaton and Gersovitz (1981) is the first paper where this idea has been formalized, more specifically they addressed the problem of lending to poor countries. In their work, the authors defined an intertemporal model where the government can freely default on its debt. In so doing the government loses access to the capital market and eventually incurs a default cost. For instance, one may think to an internal banking crises which affects the whole economy, namely once the government defaults it loses the banker's confidence and cannot smooth consumption across periods issuing bonds. These costs define a credit ceiling above which the bankers are not willing to finance the government. As a consequence, the need to keep a good reputation prevents a government to bankrupt.

Bulow and Rogoff (1988) challenges such formalization by showing that the worst thing that can happen to a country which bankrupts on its debt is that, in the future, signing reputational contract will not be allowed anymore. Key in their paper is the idea of reputational contracts. The authors show that in equilibrium such punishing behavior are not sustainable. The idea is that are available cash-in advance contract. The former can be defined just as conventional contracts under which a country makes a payment up front in return for a state contingent no-negative payment. Then Bulow and Rogoff claim that the only way to enforce debt contracts is to develop a legal mechanism which enforce the debt repayment. Cole and Kehoe (1998) argue that if the Bulow Rogoff argument is the relevant one the government must be enforced to honor contracts by either the direct use of force or by sanctions. These mechanism give at the government the incentives to repay its debt. Anyhow, the authors reports that there is no empirical evidence of such means of enforcement, especially in the previous century. In the same paper they develop a model on which the reputation argument is revived. In such paper, they argue that the government's actions in the debt arena should spill over to different arenas in which trust is crucial. As a consequence if a government is viewed untrustworthy in one arena it will be viewed untrustworthy in other relationships and this way of thinking revives the reputational argument.

2.3 A brief review of overlapping generations model with debt

De la Croix and Michel in their book (De La Croix and Michel (2002) point out that public debt should be used either to implement transfer policies or to finance public spending. Hence, the overlapping generations framework suits to study the implications of different kinds of debt policies. One of the main result that stands out in the Diamond (1965) framework is that the ricardian equivalence¹ does not hold. The former defined neutrality does not hold in the Diamond's framework because of unborn generations whose interests are not taken into account.

In the next subsection the results of both Weil (1987) and Dávila (2008)

¹The ricardian equivalence merely says that only the quantity of government purchases, not the division of the financing of these purchases between taxes and bonds, affects the economy

are going to be discussed. In both models the agents can invest their savings either in capital or in another asset. The other asset can be defined as a bubble. Tirole (1985) defines a bubble as an asset which does not give any dividend in any periods. If the agents act rationally, namely they evaluate each asset by its market fundamental², the market value of a bubble should be equal to zero. Do exists assets in the real world which satisfy the bubble's definition? A natural answer is money but Tirole (1985) points out that even zero coupon bond satisfy the definition. It follows that such definition is satisfied for one period public debt modeled as zero coupon bond. Furthermore, De la Croix and Michel (2002) report three differences between public debt and the bubble's concept:

1. Public-debt is a pre-determined variable while a bubble is a forward looking variable. The difference comes from the fact that a bubble cannot be related to any initial condition.
2. A bubble cannot be negative if the asset can be freely disposed of. If the debt is negative it means that the government holds some capital in the economy.
3. With public debt is possible to use the concept of both temporary and intertemporal equilibrium. In the bubble case the walrasian auctioneer determines the prices of the bubble for all periods at the same time.

Anyhow both the results of Weil (1987) and Dávila (2008) are appealing. Both papers develop economies with bubbles that may crash each period with a given exogenous probability. Especially Dávila (2008) can be interpreted as a model where the government's probability of default is left exogenous.

Olg models with public can help to analyze welfare effects even if individuals are considered heterogeneous along some dimensions. Sargent and Wallace (1981) defines a model, a manna from heaven kind, where the "poors" can transfer wealth across periods only by using money. The former is a useful reference in building an OLG model dealing with both public debt and heterogeneity of individuals. For instance, one possible idea to address the question of debt crises could be done by developing a model where only a small fraction of individuals hold the bonds issued by the government. As a consequence, if the government

²In finance theory the market fundamental is defined by the expected discounted value the stream of returns of the asset.

gives a different weight to the different types of agents different optimal policies will be obtained.

2.3.1 Weil (1987), Davila (2008)

Weil (1987) proposes two models. The first one is characterized by the fact that the agents receive each period an exogenous stream of output, manna from heaven. The author allows the rational individuals considered in the model to invest in a stochastic bubble in the exchange economy. In the same paper the bubble is implemented in a Diamond (1965) framework. The agent believes that the price of money is random and that it follows a random Markov process. Furthermore, it is hypothesized that if today the value of the money is zero it will stay equal to zero in the future as well.

Here I report the matrix which defines the evolution of money in the Weil paper:

$$\begin{array}{ccc}
 & p_t = p_t > 0 & p_t = 0 \\
 p_{t+1} = p_{t+1} > 0 & q & 0 \\
 p_{t+1} = 0 & 1 - q & 1
 \end{array}$$

In this model the value of the bubble is a function of its past realization. More clearly the bubble's value depends on:

- The state of nature prevailing the subsequent period
- The today value of the bubble

Weil (1987) proves that a stationary stochastic bubble can exist in the exchange economy only if the probability of a crash is not too high and if the no-bubble economy is not efficient.

³In the same paper the concept of no-stationary stochastic bubble is addressed by the author. An equilibrium where the agents find optimal to invest a part of their savings in the bubble asset exists if two conditions are met: the

³In such a case there exists another feasible sequence of the capital stock where all the generations may consume an amount at least as large as the previous sequence. Indeed, there must exist at least one generation which may consume an amount larger than the one defined by the equilibrium path.

bubble should be not too risky and depending on the initial amount of the bubble. In fact, if the initial value of the bubble should be lower than a threshold the economy will be demonetized before the bubble burst. If the initial condition coincides with the threshold value the bubble is stationary. Differently if the initial value is larger than the threshold the derived paths are infeasible.

Weil (1987) address the same problem in an OLG competitive production framework. The agents can invest their savings either in the bubble or in physical capital. It is proved that the agent finds profitable to invest in the bubble if:

$$q \frac{p_{t+1}}{p_t} \geq 1 + r_{t+1}$$

Namely, the bubble must give an expected return at least as large as the one given by the physical capital.

The model is solved exploiting the assumption of logarithmic preferences which implies that the propensity to save is independent with respect the asset return. Indeed, the asset return affects only the portfolio composition. Furthermore, assuming that the Diamond's economy is not efficient the production economy can be analyzed in a similar fashion than the pure exchange one.

Another interesting proof of the paper is the one which claim that if the probability that the bubble does not burst the subsequent period is larger than a threshold value there exists an equilibrium on which the agent invest a share of their saving in the bubble. It is worth to keep in mind that if the Diamond economy is dynamically inefficient then a stationary monetary equilibrium does not exist. Moreover, in the inefficient case only not too risky bubbles may exist.

Reversing the assumption, namely considering a dynamically inefficient economy it is proved that if the level of confidence is enough high the existence of an equilibrium where the representative agents invests in the bubble depends on the initial value of the bubble:

- if $b_0 < u$ the economy converges to the no monetary steady state before that the bubble burst
- if $b_0 = u$ the economy converges to the monetary steady state
- if $b_0 > u$ the resulting path is infeasible

In both models the economy will be demonetized in a finite time.

Dávila (2008) allows individuals to invest their savings in a bubble as well as in physical capital. Differently from Weil (1987) here is assumed that once that bubble burst (which happen in a finite time as well as in Weil) the economy will not become demonetized but the old bubble will be substituted by a another one which starts right after the burst of the former. More clearly, the author assumes that if the bubble burst the resources that the old individuals hold in the bubble will be transferred to the young individuals. Hence investing in the bubble is risky, in fact the individuals face a positive probability to loose a part of their savings.

The representative individual defines her portfolio before uncertainty is resolved. As a consequence an excess return in money is required otherwise no individual will invest in money. It is proved that at the competitive steady state, assuming a dynamically inefficient economy, the agents invest in the bubble if the probability that the bubble burst is small enough. The best attainable steady state, using both risky money and remunerating factors at the marginal productivity, does not coincides with the best one if the probability that the bubble burst is positive.

Furthermore, the author determines the combination of taxes and subsidies that would allow a benevolent central planner to implement a steady state on which the representative agent utility is maximized. Obviously, labour and capital will be remunerated at their marginal productivity values.

2.3.2 Comments

The model that will be presented in the second part of this chapter has two important differences with respect the previous papers. In the paper previously presented the bonds are a possible investment opportunity for the representative agent. Differently, in mine model the representative agent can not invest in bonds, only the international banker act in the debt arena. Indeed, the mechanism which may lead to a crash is different. In fact neither Weil (1987) nor Dávila (2008) provide a theory that explains the burst of the bubble. In both models the crash may happen each period with a probability equal to an exogenous given constant. A possible suggestion for future research is the development of a model where the probability of the default is truly endogenous and related both with both the production sector and the agents' expectations.

2.4 Models of Sovereign default

In this section the implications of the most important papers which dealt with model on which the default choice is endogenous are presented. First Calvo (1988) model and results are discussed. Subsequently, models that try to link debt crises to agents' expectations are reported. Such a kind of model generate self-fulfilling debt crises. As before, at the end of the section comments which relate the literature with the model proposed in the second part are made.

2.4.1 Self-fulfilling debt crises

Calvo (1988) is a benchmark model in the sovereign default literature. The author develops a model where the existence of government bonds is the source of multiple equilibria. The paper provides a two periods model where the agents are the government and the representative consumer.

In the first period the representative consumer is allowed to invest in capital or government bonds while during the second period it only consumes the returns of the investments done. Moreover, the government finances an exogenous amount of public expenditure aiming to maximize the representative consumer's welfare. Public expenditure can be financed by the government in two ways. The first solution is by taxing the exogenous endowment of the representative agent, the second way consists in issuing bonds.

Taxes generates distortionary costs which affects the representative individual budget constraint. Indeed, the government in the second period is allowed to default on a share of the debt previously released. The bankrupt choice is not free of costs, in fact in such case the government incurs reputational costs which affect its budget constraint. Furthermore, these costs are proportional to the repudiated amount of debt.

In a perfect foresight equilibrium if the representative individual expects a government's default a risk premium on the bonds is asked. It is proved that if a condition on the optimal tax level is satisfied the model will be characterized by two equilibrium. In one the government will repudiate a share of the debt and a the bonds are issued considering a risk premium. In the second equilibrium the government will not default and, as a consequence, the bonds are issued without paying any risk premium.

This model shows that the individual's expectations about the future play a crucial role in the determination of the equilibrium.

Alesina et al. (1989) developed a similar model but considering a framework

characterized by infinitely many periods. Anyhow the assumption that raising taxes generates distortionary costs is kept. The agents of the model are infinitely lived and identically individuals, international bankers and the government and the economy is assumed small with respect the rest of the world. Each period the representative individual receives a unit of a no storable good and on this taxes are raised by the government which, as already mentioned, implies distortionary costs. Furthermore, the representative individual may trade in the international capital market at the international risk free rate. In this paper the government starts with an exogenous amount of outstanding debt. The government can choose either to repay this amount issuing bonds or by raising taxes. It easily follows that the optimal policy is to roll over the debt for ever. The default choice is allowed but in doing so the government incurs reputational costs which are not related on the share of debt that it refuses to reimburse. As a consequence, in equilibrium the government repays the whole debt or default on its all debt. Government and individuals play a game which is characterized by the following timing:

1. The government defines both the bonds' prices and the maximum amount which it is going to release.
2. The representative individual chooses how many bonds to buy given her expectations on the government's repayment behavior
3. Government chooses if bankrupt or not and how much taxes to raise.

Exploiting the sequentially Nash equilibrium concept the equilibrium of the model is derived. The authors prove that the prevailing equilibrium depends on the cost of default and repudiation threshold values are computed.

It is proved that the government defaults during the first period if the default costs would be lower than a threshold, if the default costs are larger than another threshold the government will never default and the bonds are sold without any risk premium.

If the default costs lies in between the two values the model exhibits two equilibria. In the first the government roll over its debt without paying any risk premium. Differently, in the other one, the representative individual's expectations brings the government to a bad equilibria where bonds are not bought.

Indeed, it is proved that issuing long maturity bond shrinks the interval characterized by multiple equilibria and where a confidence crisis may happen.

In Cole and Kehoe (2000) the bankers' expectations about a country's default are driven by a sunspot variable⁴. The model is able to replicate the Mexican debt crisis of 1994-1995. The Mexican debt crisis is characterized by two features: first, the Mexican outstanding debt was not too large; secondly, the government fiscal behavior was accountable. The authors explain the arise of the crisis arguing that its emergence can be somewhat arbitrary, namely the banker expectations could be self-fulfilling (the international bankers' beliefs that the government will not repay the debt makes become optimal defaulting).

The agents of the model are: perfectly identically infinitely lived consumers, the government and international bankers. The international bankers are characterized by risk neutral preferences, namely they are willing to invest in assets which give, in expected terms, at least the international interest factor. The government is, as usual, benevolent aiming to maximize the utility of the representative agent. The government provides a public good which is financed both by raising taxes and issuing bonds. Furthermore, the government chooses how many bonds to issue and is free to default on its debt. The default is not free from costs and in doing so the productivity of the economy drops for all subsequent periods.

Each period is realized a sunspot variable which is assumed independently and uniformly distributed on $[0, 1]$. This variable is crucial, in fact it drives the banker's expectations about the country's default.

The timing protocol of the model is the following:

1. At the begin of each period is realized the sunspot variable that defines the state of the economy
2. The government chooses how much debt to sell
3. The government chooses if defaults and how much public good to provide
4. The representative consumer chooses how much to consume. Such a choice defines the capital available in the future

⁴A sunspot variable is a variable that is not linked to any real characteristics of the economy.

The authors prove that the prevailing equilibria depends on the outstanding level of debt:

- If the outstanding level of debt is enough low for any possible realization of the sunspot variable the international banker anticipates that the government will never default. Hence a debt crisis is not possible
- If the level of debt is high enough the government defaults even if the international banker will finance the government public expenditure.
- In between these two thresholds there is a continuum of debt levels for which the default's choice depends on the realization of the sunspot variable. In such a case the international bankers ask a risk premium on the government's bonds. The government may choose to follow a policy to run down the debt, in a finite time, as to go out from the crisis zone

Summarizing in Cole and Kehoe the debt crises can be self-fulfilling and it depends both on the outstanding amount of debt and in some unexplained banker's beliefs. Conesa and Kehoe (2012) modifies the previous model as to address the debt crises that hit Portugal and Greece during the previous years. The model assumes that the economy starts with a low productivity level namely is in recession. The economy's productivity may increase each period with a given exogenous probability and once the economy has recovered it will keep this level of productivity for all the subsequent periods. A key result of this paper is that for the government is sometimes optimal to gamble for redemption. Namely, it could be optimal to follow a martingale strategy which sends the economy in the crisis zone if the economy's recover does not happen soon enough. As a consequence if the outstanding level of debt puts the economy in the crisis zone, defined close to Cole and Kehoe (2000) it could be not optimal to run down the debt as to exit from the crisis zone. The former sentence means that, if the economy is in the crisis zone, could be optimal to increase the stock of debt if the economy has not yet recovered.

Chamon (2007) challenges Cole and Kehoe (2000) paper. In this paper the economy is similar to the one developed by Cole and Kehoe (2000). A difference is that each period the economy is hit by iid shocks which affect the economy's productivity. Along this framework the author proves that self-fulfilling crises

do not arise if the investors can condition their bids on the amount that will be bought by the other investors.

Arellano (2008) is another paper which deserves a mention. A model able to predict a crisis similar to the one that hit Argentina in 2001 is provided. In this paper the risk premium is endogenous and is also a positive function of the outstanding debt.

2.4.2 Comments

In the following a mechanism similar to the one developed by Cole and Kehoe (2000) will be embodied in an overlapping generations framework. Anyhow the international banker's behavior will not be driven by a sunspot variable. In fact in Cole and Kehoe the extraneous beliefs of the international bankers are the main force that triggers a debt crisis, more clearly if the outstanding amount of debt belongs to the defined interval the debt crisis may happen with a positive and exogenous probability. Likewise the former the production technology will be affected by the government bankrupt relying on the idea that after a default an economy will bear reputational cost.

2.5 The Model

The aim of this model is to try to develop an OLG model where the government bankrupt on its debt. The agents of the model are: producing firms, investing firms, individuals, international banker and the government. The strategic player of the model are the government and the international banker. Each agent's behavior is going to be analyzed in the next subsections. The economy is populated by identically agents that live two periods. I assume that the growth rate of population is zero, furthermore population is normalized to 1. Differently from Cole and Kehoe (2000), the government issues and repays the debt at the same moment in time. I start the analysis assuming that the government can freely bankrupt the debt each period in any case. Subsequently, the model is solved assuming the existence of a commitment technology at period zero. It results that government opportunistic behavior is strike out from the model. It is assumed perfect foresight, hence the international banker is able to anticipate which will be the government's debt policy. The government is able to predict the international banker's behavior as well. Afterwards the individuals, international banker, firms and government distinctive feature are shown. The representative individual is not allowed to invest on bonds. As a consequence in such model there exists only external debt.

2.6 Basic Assumptions and parameters

It is assumed that the economy starts at time zero with a given amount of capital, which is owned by the individuals born old. Another initial condition is a given positive amount of outstanding debt B_{-1} , which is held by the international banker. Having that the population is assumed constant and normalized at 1 all the model is written in per capita terms. Here we assume that $z_{-1} = 1$, that is a way to hypothesize that the economy before time zero has never defaulted

2.6.1 Individuals

As already stated, each individual lives two periods. In the first period, the individual supplies inelastically one unit of labour to the producing firms and in exchange receives a wage. The individual in the first period pays a proportional wage tax, which is constant and exogenous. When young, the agent consumes the public good provided by the government. The tax rate is kept constant and

exogenous to the model. The remaining amount of the wage is invested in the capital market and it is given back plus an interest in the second period. When old the individual only consumes. The agent's utility is assumed to be linear. The individuals discount the time at a discount factor equal to β . The agent in the first period consumes the amount provided by the government which is equal to the raised taxes plus the bond issued and less the outstanding bonds, if the government is going to repay its debt. It is also assumed that the economy starts with a generation 'born' old which owns the capital stock.

$$U(g_t^y, d_{t+1}^o) = g_t^y + \beta d_{t+1}^o \quad (.1)$$

where:

$$d_{t+1}^o = s_t R_{t+1}$$

$$g_t^y = \theta w_t + q_t b_t - b_{t-1}$$

where

$$s_t = (1 - \theta) w_t$$

The representative old individual alive at the initial date $t = 0$ consumes:

$$d_0 = R_0 k_0$$

2.6.2 Production Function

This section shows the technology available to each producing firm. A similar production function can be found in Cole and Kehoe (2000) as well.

This production function is assumed to be Cobb Douglas:

$$Y_t = Z^{1-z_t} L_t^{1-\alpha} K_t^\alpha$$

where

$$0 < Z < 1$$

$$z_t \in \{0, 1\}$$

z_t represents the government's default choice. If the government defaults this variable is equal to 0. As a consequence the productivity suddenly decreases from t onward. The drop in productivity is measured by the parameter Z which is exogenous. This is a way to model the cost of a government's bankrupt. In the review part justifications of such assumptions have been provided. Indeed, by assumption, after a bankrupt the government loses the access to international capital market and cannot issue bonds for all the subsequent periods.

It follows, in intensive form:

- $y_t = Zk_t^\alpha$ If the government defaults or has already defaulted in the past
- $y_t = k_t^\alpha$ If the government does not default and has not yet defaulted in the past

2.6.3 Producing firms

The homogenous consumption good is produced by homogenous firms which use the production technology already described. Each firm has an installed capital stock equal to k_t . The competitive firms hire labour in order to produce the homogenous capital good which will be sold in the good-market. Each firm follows the objective to maximize its profits, then:

$$\pi_t = \max_{L_t} f(k_t, z_t) = Z^{1-z_t} k_t^\alpha - w_t L_t$$

Computing the f.o.c. I obtain:

$$w_t = \begin{cases} (1 - \alpha) k_t^\alpha & \text{if } z_t = 1 \\ (1 - \alpha) Z k_t^\alpha & \text{if } z_t = 0 \end{cases}$$

The capital is owned by the old-individuals which receive the producing firm's profits:

$$\pi_t = \begin{cases} \alpha k_t^\alpha & \text{if } z_t = 1 \\ \alpha Z k_t^\alpha & \text{if } z_t = 0 \end{cases}$$

Now it is possible to define of the old-agents consumption:

$$d_t = \begin{cases} \alpha k_t^\alpha ((1 - \alpha)(1 - \theta)) k_{t+1}^{\alpha-1} & \text{if } z_t = 1; z_{t+1} = 1 \\ \alpha Z k_t^\alpha ((1 - \alpha)(1 - \theta)) k_{t+1}^{\alpha-1} & \text{if } z_t = 1; z_{t+1} = 0 \\ \alpha Z^2 k_t^\alpha ((1 - \alpha)(1 - \theta)) k_{t+1}^{\alpha-1} & \text{if } z_t = 0; z_{t+1} = 0 \end{cases}$$

Three cases arise needs to be considered. The case $z_t = 0, z_{t+1} = 1$ is not investigated due to assumption that the productivity drop after the government's bankrupt is permanent. Finally the consumption of the agents born old is equal to:

$$d_0 = \begin{cases} \alpha k_0^\alpha & \text{if } z_0 = 1 \\ \alpha Z k_0^\alpha & \text{if } z_0 = 0 \end{cases}$$

2.6.4 Investing firms

These firms play the role to invest the saving of the individuals young at time t in order to deliver a return available at time $t + 1$ when they will be old. Moreover, exploiting the assumption of a constant population normalized to 1 it results that the total investment of the economy is equal to the savings, hence:

$$I_t = s_t$$

2.6.5 Capital accumulation

The capital accumulation is driven by the savings of the young representative individual:

$$k_{t+1} = s_t \tag{.2}$$

where

$$s_t = (1 - \theta) w_t$$

Exploiting these formulas:

$$k_{t+1} = (1 - \theta) (1 - \alpha) Z^{1-z_t} k_t^\alpha \tag{.3}$$

given k_0

The former equation provides the law of motion of physical capital. The closed formed solution is presented in the appendix .

2.6.6 International banker

Here I assume that an international banker can freely invest on the government's bonds and its behavior is driven by the bank's expectations on the government's default choice and by the international interest rate r^* , which is taken as exogenous. The international banker, by assumption of the model, buys the discounted bonds only if the government repays the outstanding debt. Let define by q_t the price at which the international banker is willing to buy the

bonds.

Here I model the international banker's behavior in such a way:

if

$$E[z_t] E[z_{t+1}] \dots E[z_\infty] = 1$$

then

$$q_t = \omega = \frac{1}{1 + r^*}$$

is the discounted price at which the banker is willing to buy bonds which repay 1 in the following period.

Otherwise, if

$$E[z_t] E[z_{t+1}] \dots E[z_\infty] = 0$$

then

$$q_t = 0$$

which means that if the international banker expects that the government defaults in the current period or will default in the future then it is not willing to buy any amounts of bonds at a positive price. The two equations determine the banker's behavior.

As a consequence the bond's price pattern is defined as follows:

$$\begin{cases} q_t = \omega \text{ if } E[z_t] E[z_{t+1}] \dots E[z_\infty] = 1 \\ q_t = 0 \text{ if } E[z_t] E[z_{t+1}] \dots E[z_\infty] = 0 \\ q_t = 0 \text{ if } z_{t-1} = 0 \end{cases} \quad (.4)$$

One of the key assumption which characterize the model is that the banker is not willing to lend resources once that the government defaults. The government loses the access to the capital market and cannot release bonds anymore. Furthermore, if the banker rationally expects that at some point in time in the future the government will not reimburse the debt the banker does not finance the government.

2.7 Government

The government is the central agent of this model. It is assumed benevolent, namely it aims to maximize discounted sum of utilities of all generations. The government discounts the utility of the individuals belonging to different generations using a given discount factor. It makes the main choices of the model: the debt issuing decision and the decision if to repay or not the debt previously financed by the international banker. Moreover, the individual are constrained when young by the government policy since they consume only the public good provided by the government. The international banker behavior is determined by the government's default choice. Each period, at the same moment, the government makes two choices: it chooses whether and how many bonds to release, and it chooses also whether bankrupt or not. Once the government has defaulted it finances the public good only by raising taxes on the wages of individuals. Anyhow, doing so, it loses the possibility to smooth consumption across generations by issuing bonds.

Before all else, I assume that the government can either commit to roll over the outstanding debt or to keep constant the initial debt GDP ratio. In another section I study a commitment credible solution, meaning that the government will not have the incentive to default for any period subsequent to zero. In the following subsections the government's problems are defined presenting the government's issuing decision and default decision.

2.7.1 Government's issuing decision

The government's issuing decision defines the amount of bonds that the government issues. It is required to analyze the issuing behavior for each possible combinations of the default choice. The assumption of perfect foresight of both the government and the international banker directly implies that both agents are able each to perfectly foresee their behavior each other. Thus, given the assumptions made in the previous subsections the government chooses the amount of debt which maximizes:

$$\max_{b_t} \sum_{t=t'}^{t=\infty} \gamma^{t-t'} ((g_t^y) + \beta (d_{t+1}^y))$$

s.t.

$$g_t = \theta Z^{1-z_t} (1 - \alpha) k_t^\alpha + E [q_t] z_{t-1} b_t - E [z_t] z_{t-1} b_{t-1}$$

$$d_{t+1} = (1 - \theta) (1 - \alpha) \alpha Z^{1-z_t} k_t^\alpha k_{t+1}^{\alpha-1}$$

$$k_{t+1} = (1 - \theta)(1 - \alpha) Z^{1-z_{t+1}} k_t^\alpha$$

given

$$k_t, b_{t-1}, z_{t-1}$$

Computing the f.o.c with respect to b_t :

$$\gamma^{t-t'} z_{t-1} E[q_t] = \gamma^{t+1-t'} z_t E[z_{t+1}] \quad (.5)$$

$$E[q_t] = \gamma E[z_{t+1}] z_t \text{ if } z_{t-1} = 1$$

The first order condition says that the government's issuing decision at any point in time is determined by its default past behavior, its expectations about the price at which the international banker is willing to pay the bond and by the future choice about the default. If it defaults the price at which the banker is willing to buy the bonds becomes equal to zero.

2.7.2 Government's default choice

In this section the government's default choice is defined. Moreover, when the government makes this choice already knows k_t , and b_{t-1} . It is able to compute k_{t+1} which depends on its default choice and to foresee q_t which is a function of its default choice as well. As a consequence the government determines its optimal choice comparing the utility that representative individuals get if it chooses to default or otherwise. It follows that the government prefers to default on its debt if and only if it allows to attain a larger discounted sum of intertemporal utilities. Let define the value function

$$V_t(z_t, \{b_t\}) = \sum_{t'=t}^{t=\infty} \gamma^{t-t'} ((g_t^y) + \beta (d_{t+1}^y)) \quad (.6)$$

where:

β is the individual's discount fact: $0 < \beta < 1$

γ is the government's discount factor $0 < \gamma < 1$

Hypothesizing that government has not yet defaulted at time t' chooses to repay its debt if and only if

$$V_t(1, \{b_t^*\}_{t=t'}^{t=\infty}) \geq V_t(0, 0) \quad (.7)$$

Where $\{b_t^*\}_{t=t'}^{t=\infty}$ is a debt sequence which maximizes the value function

2.8 Government's choice under commitment

In this section is defined the government's optimal debt policy assuming that there exists a commitment technology available at time zero. Namely at time zero the government is able to define the whole debt path. Here we allow the government to choice between four options:

1. Default. The government does not reimburse the outstanding debt
2. The government reimburses all the outstanding debt but does not release any amount of debt for all subsequent periods
3. The government can credible commit to roll over the outstanding debt for all subsequent periods. In such a case the debt sequence is defined by

$$\{b_t = b_{t-1} \forall t\} \quad (.8)$$

4. The government can credible commit to keep the initial ratio debt/gdp constant. In such a case the debt sequence is defined by

$$\left\{ b_t = [(1 - \theta)(1 - \alpha)]^{\frac{\alpha - \alpha^{t+1}}{1 - \alpha}} k_0^{\alpha^t} b_{-1} \forall t \right\} \quad (.9)$$

Proposition 13. *Under the assumption that the government's discount rate is lower than the discounted price at which the banker is willing to buy the bonds the government never issues bonds. Moreover there exists a threshold value of the initial capital level. If the initial level of capital is larger than the threshold the government find it profitable to repay the outstanding debt*

Proof

In such a case the government never finds profitable to release bonds. Such a result comes from .5. In fact, the rate at which the market allows to smooth consumption is too small from the government point of view. Moreover, any positive debt sequence has a negative expected discounted value. It follows that a benevolent government never releases bonds in such a case. As a consequence the government's choice shrinks in the choice of bankruptcy at time zero or to reimburse all the outstanding debt. In fact exploiting .7 at the initial period I obtain:

$$\begin{aligned}
 & \alpha k_0^\alpha (1 - Z) + \sum_{t=0}^{t=\infty} \gamma^t b_t (q_t - \gamma) \\
 & \sum_{t=0}^{t=\infty} \gamma^t \left(\left((1 - \alpha) \left([(1 - \theta)(1 - \alpha)]^{\frac{\alpha - \alpha^t}{1 - \alpha}} k_0^{\alpha^t} \right)^\alpha \right) \left(1 - Z^{\frac{1 - \alpha^t}{1 - \alpha}} \right) \right) \\
 & \sum_{t=0}^{t=\infty} \gamma^t \left(\left((1 - \alpha) \left([(1 - \theta)(1 - \alpha)]^{\frac{\alpha - \alpha^{t+2}}{1 - \alpha}} k_0^{\alpha^t} \right)^\alpha \right) \left(1 - Z^{\frac{1 - \alpha^{t+2}}{1 - \alpha}} \right) \right) \\
 & \geq b_{-1} \tag{.10}
 \end{aligned}$$

It follows that the optimal debt sequence is $\{b_t = 0 \forall t\}$. The right-hand side of .10 is monotonically increasing in k_0 . Then there exists a threshold level of capital such that the benefits of not defaulting are equal to the outstanding debt at time zero. The government can reimburse the outstanding debt if the taxes that it raises at time zero at least enough to reimburse the debt.

$$\theta (1 - \alpha) k_0^\alpha \geq b_{-1} \tag{.11}$$

It results that exists a k^* which satisfies both .10 and .11. If k_0 is larger than k^* the government reimburses the outstanding debt and does not release bonds anymore. Otherwise the government defaults

Lemma 14. *If the government's discount rate is lower than the price at which the international banker is going to purchase the bonds the government has the incentive to issue as many bonds as possible.*

Due to the linearity assumption on the utility function the government is indifferent between γ units at t and -1 at $t + 1$. The international banker allows the government to exchange ω units at t versus -1 at $t + 1$. As a consequence the government has the incentive to release as many bonds as possible.

Lemma 15. *If the initial capital stock is larger than the threshold value defined by:*

$$\sum_{t=0}^{t=\infty} \gamma^t \left([(1-\theta)(1-\alpha)]^{\frac{\alpha-\alpha^t}{1-\alpha}} k' \right)^{\alpha^t} = \frac{1}{1-\gamma} \quad (.12)$$

The government prefers the debt path defined by .9 with respect the debt path defined by .8

Proof

If $k_0 > k'$ the debt sequence defined by .9 has a larger discounted value than the one defined by .8

Exploiting Lemma2 and Lemma3 the following proposition is stated:

Proposition 16. *Under the assumption of lemma1 and lemma2 the optimal government policy depends on the initial level of capital.*

Proof.

Let .10 for the debt path defined by .8 there exists a threshold value of the initial level of capital such that if the initial level of capital is larger than the threshold, the government then prefers to start the constant debt policy. Let define such threshold level of capital as k_0^{b-1} . A similar threshold exists for the debt path which keeps constant the debt GDP ratio. Let define such threshold as k_0^{dg} . The thresholds above defined are lower than the one defined by proposition1 because under the assumption that the discounted price of the bonds is lower than the government's discount factor any debt sequence as a positive value. If k_0^* satisfies .10 and k_0^{dg} satisfies as well as .10 indeed they allow to attain a larger discounted utility. Then in such a case the government never follows the policy to reimburse the debt and never release the bonds anymore. Under the results of this proposition I disentangle the following cases:

- $k_0 < k_0^{b-1} < k_0^{dg}$ the government optimal policy at time zero in such a framework is to default on its debt
- $k_0^{b-1} < k_0 < k_0^{dg}$ the government follows the debt path defined by .8
- $k_0^{dg} < k_0 < k_0^{b-1}$ the government follows the debt path defined by .9
- if $k_0 > k_0^{dg}$ and $k_0 > k_0^{b-1}$ and $k_0 < k_0'$ the government follows the debt path defined by .8

- if $k_0 > k_0^{dg}$ and $k_0 > k_0^{b-1}$ and $k_0 > k_0'$ the government follows the debt path defined by .9

2.8.1 Comments

In proposition 13 I have proved the existence of a threshold value of capital when the discounted price of bonds is lower than the government discount factor: if the initial level of capital is larger than the threshold the government never defaults, even if it can not release bonds (it finds such a policy not profitable). Yet, reversing the reasoning, we may think that countries repay their debentures even if the international banker is not willing to finance the debt anymore. Moreover, if the capital level is seen as a proxy of the development of a country, the results that I have obtained does not contradict Reinhart and Rogoff (2008) stylized facts. Namely, as previously stated in the introduction, the authors find empirical evidence that most of the countries have defaulted several times on external debt during their emerging economic phase.

In proposition 16, when the government find profitable to release bonds, it has been proved that if the outstanding debt is too large with respect the initial level of capital, the government prefers to default even if it should be optimal to release bonds. This may be the case when the government never reimburses the outstanding debt, because the available policies plus the benefits of not defaulting have a discounted value lower than the outstanding debt. Moreover , the benefits of not defaulting are increasing with respect k_0 . Then if the initial level of capital is large enough the government repays the outstanding debt and commits to follow the policy which gives the larger discounted value. Furthermore, the comments done for proposition 13 holds for proposition 16 as well. Finally, the main result here obtained is that in such a framework the government's default behavior is driven by the initial level of capital.

2.8.2 Government's constant debt path with "credible" commitment

The debt paths defined in the previous section may be not credible. In fact, if the initial level of capital is larger than the steady state level of capital then the government should find optimal to default if it can re optimize later on, because the capital stock decrease over time. Furthermore, if the initial level of capital is lower than the steady state level the government should find it

profitable to increase the debt stock without having any incentive to default. Here I try to define a constant debt path which is credible in the sense that since zero onward the government has not incentive to default anymore. Namely, the government states in its constituency at time zero an upper bound which does not make profitable to bankrupt in any future period. The upper bounds here defined depend on the position of the initial capital stock with respect to the steady state level of capita.

From .29 the steady state level of capital without defaulting is

$$k^{ss} = [(1 - \theta)(1 - \alpha)]^{\frac{1}{1-\alpha}} \quad (.13)$$

Let define by t' the period where the economy reaches the steady state. As a consequence the maximum debt per capita for which the government has not incentive to default from t' onwards is given by:

$$\sum_{t=t'}^{t=\infty} \gamma^{t-t'} \left((1 - \alpha) \theta \left([(1 - \theta)(1 - \alpha)]^{\frac{1-\alpha t}{1-\alpha}} (k^{ss})^{\alpha t} \right)^{\alpha} \right) \left(1 - Z^{\frac{1-\alpha t}{1-\alpha}} \right) + \quad (.14)$$

$$\begin{aligned} \sum_{t=t'}^{t=\infty} \gamma^{t-t'} \left((1 - \alpha) \theta \left([(1 - \theta)(1 - \alpha)]^{\frac{\alpha - \alpha t + 2}{1-\alpha}} (k^{ss})^{\alpha t + 2} \right)^{\alpha} \right) \left(1 - Z^{\frac{1-\alpha t + 2}{1-\alpha}} \right) + \\ = b(k^{ss}) \left(\frac{1 - \omega}{1 - \gamma} \right) \end{aligned}$$

Lemma 17. *if $k_0 > k^{ss}$ the constant debt sequence for which the government can credible commit at time zero is given by*

$$\{b_t = b(k^{ss}) \forall t \geq 0\} \quad (.15)$$

If the above defined constant debt sequence satisfies .10 the government prefers to follow .15 rather than choose to bankrupt at time zero.

Proof.

In fact for any $0 < t < t'$ the benefits of not defaulting are larger than the ones of the right hand side of .14. Such a result comes from the monotonicity of the benefits of not defaulting with respect the capital stock. If .10 is satisfied a

benevolent government prefers to follow .14 instead of defaulting at time zero. Otherwise the optimal choice is the bankruptcy.

If the initial level of capital is lower than the steady state level the benefits of not defaulting are increasing over time. As a consequence if I define an optimal debt sequence at time 1, such a constant debt path is not optimal at time 2. Anyhow, the government finds not profitable to default in such a case because the benefits of not defaulting are increasing from period 1 on.

Let define

$$\sum_{t=1}^{t=\infty} \gamma^{t-1} \left((1-\alpha) \theta \left([(1-\theta)(1-\alpha)]^{\frac{1-\alpha^t}{1-\alpha}} (k^{\alpha^t}) \right)^\alpha \right) \left(1 - Z^{\frac{1-\alpha^t}{1-\alpha}} \right) + \quad (.16)$$

$$\begin{aligned} \sum_{t=1}^{t=\infty} \gamma^{t-1} \left((1-\alpha) \theta \left([(1-\theta)(1-\alpha)]^{\frac{\alpha-\alpha^{t+2}}{1-\alpha}} (k_0)^{\alpha^{t+2}} \right)^\alpha \right) \left(1 - Z^{\frac{1-\alpha^{t+2}}{1-\alpha}} \right) + \\ = b(k_0^1) \left(\frac{1-\omega}{1-\gamma} \right) \end{aligned}$$

Then we can state:

Lemma 18. *If $k_0 < k^{ss}$ then the credible constant debt path is given by:*

$$\{b_t = b(k_0^1) \forall t \geq 0\} \quad (.17)$$

If the above constant debt sequence satisfies .10 then the government starts the constant debt path defined by .17. Otherwise it defaults at time zero.

Proof. In such a case the benefits of not defaulting are increasing over time. If the government finds it optimal not to default at time 0 for the debt path defined by .17 it will never find it optimal to default following .17 anymore. Otherwise the government prefers to default at time zero.

2.8.3 Comments

The model is built under some arguable assumptions, such as the one that

constraints the representative individual behavior. In such a model the accumulation of capital is determined by the tax rate which is exogenous. The former assumption has been made to highlight the mechanism which links the capital with the government's default choice. Developing a model where the savings of the agent are endogenously determined and depends on the government's default choice may be a subject for future research.

Another arguable assumption is the linearity of the utility function. I made this assumption in order to simplify the government's behavior, but as the one discussed previously it is fairly reasonable to discuss on it. The international banker's behavior is questionable as well, since I assume that the international banker is not willing to finance any amount if rationally expects that a default may happen in an period in the future. But, as Reinhart and Rogoff (2008) argued, it seems that policy makers and investors sometimes are not so focused about the future. Their expectations are loosely speaking, a little bit too clouded, leading at the phenomenon "this time is different", in which the investors seem to assume that the default's option is not available. Therefore, it will be interesting to assess the results of the model considering not fully rational agents.

2.9 Conclusions

The aim of this chapter was to develop an Olg model with a government's default choice endogenous.

In the first part of this chapter I have presented the papers that have inspired this work. It has been discussed the nature of default costs presenting models with exogenous crisis in an Olg framework and how these papers are related with the model here developed. Finally, I have presented models that defines default mechanism in an infinite lived household framework. Suggestions and insight for possible extensions are showed in light of the model developed in the second part have been provided.

In the second part this objective has been accomplished and the main result obtained says that countries with a lower initial level of capital may find optimal optimal to bankrupt for a lower threshold value of the outstanding debt. Furthermore, constand debt for which the government can credible commit that it will never default for all the subsequent periods have been determined.

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Appendix

Chapter 1

First proofs and equations recalled in the theoretical part are provided. Subsequently, data and numerical experiments for the remaining countries analyzed are given.

Model

Exogenous HS migration

Lemma 4. *Proof*

To obtain the skill ratio phase diagram it suffices to apply the $z_t = z_{t+1}$ condition to .10. Easily, it results:

$$z^{ss} = \frac{(1 - \bar{m})q}{(1 - q)(1 - \bar{m}) - n(1 - \bar{m})} \quad (.18)$$

The infrastructure phase diagram equation depends on the economy regime. If the economy is under the formal regime and applying the condition $k_t = k_{t+1}$ to .12

$$k_t = \left(\frac{\tau A_0 z_t^{\phi+\alpha}}{\Gamma} \right)^{\frac{1}{1-\varepsilon}} = \varphi(z_t) \quad (.19)$$

Doing the same with respect .13

$$k_t = \left(\frac{\tau A_0 \left(\Delta^{\frac{\phi+\alpha-1}{\alpha+\phi}} \right) z_t}{\Gamma} \right)^{\frac{\alpha+\phi}{\alpha+\phi-\varepsilon}} = \xi(z_t) \quad (.20)$$

Let assume that $z_{t,f}$ is given by the second of .9. After some algebra it results that at this point both .19 and .20 coincide. As a consequence the skill ratio which solve the equation stated below is the same for both phase diagrams proving that the two curves cross each other at the informality frontier.

$$z_t^{-\frac{\phi+\alpha}{\varepsilon}} \Gamma^{-1} = \Delta^{-\frac{1}{\varepsilon}} \tau A_0$$

Lemma 3. *Proof*

Its result can be easily obtained by solving the RHS of .18 for the considered parameters.

Proposition 6. Here are provided the equations which define the infrastructure steady state. The skill ratio steady state under exogenous HS is simply given by the RHS of .18. Plugging this value in both .19 and .20 I obtain

- Infrastructure Steady State, if .15 holds

$$k_{for}^* = \left(\frac{\tau A_0 (z^{ss})^{\phi+\alpha}}{n_h (1-\bar{m}) z^{ss} + \Theta} \right)^{\frac{1}{1-\varepsilon}} \quad (.21)$$

- Infrastructure Steady State, if .16 holds

$$k_{inf}^* = \left(\frac{\tau A_0 \left(\Delta^{\frac{\phi+\alpha-1}{\alpha+\phi}} \right) z^{ss}}{n_h (1-\bar{m}) z^{ss} + \Theta} \right)^{\frac{\alpha+\phi}{\alpha+\phi-\varepsilon}} \quad (.22)$$

Lemma 7. Proof

Let consider the equation which defines the skill ratio in the formal sector under the informal regime, the second of .9. Computing the derivative with respect the tax rate I obtain:

$$\frac{dk_t}{d\tau} = \left(\frac{\gamma}{(1-\tau)^2 (1-\alpha)} \right) z_{f,t}^{-\frac{\alpha+\phi}{\varepsilon}}$$

which is clearly always positive.

Proposition 8. Proof

Let consider equation .19. Clearly an increase in the tax rate has a positive effect on the right hand side. As a consequence larger is the tax rate larger will be the amount of infrastructure in the next period. The same does not hold under the informal regime, in such a case after a tax increase some low skilled workers will find profitable to join the informal sector. That flow of workers from the formal economy to the informal shrinks the tax base. Let consider equation .13 and compute the derivative of its RHS with respect τ . It results:

$$\frac{dk_{t+1}}{d\tau} = \frac{A_0 z_t k_t^{\frac{\varepsilon}{\alpha+\phi}}}{\Gamma} * \left(\Delta^{\frac{\phi+\alpha-1}{\alpha+\phi}} \left(1 + \frac{\tau(\frac{\phi+\alpha-1}{\alpha+\phi})}{1-\tau} \right) \right)$$

The first term of the RHS is clearly positive. Hence I focus on the second term and collecting for the tax rate I obtain:

$$\Delta \frac{\phi + \alpha - 1}{\alpha + \phi} \left(\frac{1 - \tau \left(\frac{1}{\alpha + \phi} \right)}{1 - \tau} \right)$$

The first term is always positive whereas the second is not negative if and only if condition .17 is satisfied.

Proposition 9. Proof

Let consider equation .3 for period $t + 1$. In the case of no regime switching infrastructure is set by equation .13 and the skill ratio in the formal sector is determined by the second of .7. Computing the derivative with respect tax rate of $w_{h,t+1}$ I obtain:

$$\frac{\varepsilon}{\alpha + \phi} (\tau^{-1}) - \frac{(1 - \phi - \alpha)\varepsilon + (\alpha + \phi)}{(\alpha + \phi)^2} (1 - \tau)^{-1} > 0$$

which after a bit of algebra reduces to .18. In order to obtain the second result it suffices to consider .12 in place of .13.

Endogenous HS migration

I start providing the equations which characterize the law of motions of both infrastructure and skill ratio. First I focus on the skill ratio law of motion. Let equation .10 and substitute the exogenous high skilled rate with the one given by .19. Then

$$(1 - \bar{m}_t) = \frac{(1 - \tau) \alpha A_0 z_{f,t}^{\alpha + \phi - 1} k_t^\varepsilon}{w(1 - x) + (1 - \tau) \alpha A_0 z_{f,t}^{\alpha + \phi - 1} k_t^\varepsilon}$$

Where $z_{t,f}$ is determined by .9 as usual.

- Skill ratio law of motion:

$$z_{t+1} = \frac{(1 - \bar{m}_{t+1})(nz_t + q)}{(1 - \bar{m})(1 - q)} \tag{.23}$$

As a consequence, the equation which determines the skill ratio law of motion depends on the economy regime. Moreover from .23 is evident that the skill ratio is not a predetermined variable anymore.

It remains to provide the equations which determine the evolution of infrastructure. As before plugging .19 into the law of motions obtained in the exogenous HS migration case allows to obtain the law of motions under the endogenous migration. Doing so

- Infrastructure law of motion:

$$k_{t+1} = \frac{\tau A_0 z_{f,t}^{\phi+\alpha} k_t^\varepsilon}{(1 - \bar{m}_{t+1})(z_t n_h + n_l q) + n_l ((1 - \underline{m})(1 - q))} \quad (.24)$$

As previously $z_{t,f}$ is determined by .9.

Applying to .23 the phase diagram condition, for both the formal and the informal regime I obtain:

- Skill Ratio Phase Diagram, Formal Region

$$k_{t+1} = \left(\frac{\Lambda}{(1 - \tau) \alpha A_0 (q z_t^{\alpha+\phi-2} + z_t^{\alpha+\phi-1} (n - (1 - q)(1 - \underline{m})))} \right)^{\frac{1}{\varepsilon}} = \lambda(z_t) \quad (.25)$$

- Skill Ratio Phase Diagram, Informal Region

$$k_{t+1} = \left(\frac{\Lambda}{(1 - \tau) \alpha A_0 (B)^{\frac{\alpha+\phi-1}{\alpha+\phi}} (q z_t^{-1} + (n - (1 - q)(1 - \underline{m})))} \right)^{\frac{\alpha+\phi}{\varepsilon}} = \gamma(z_t) \quad (.26)$$

where

$$\Lambda = w_t (1 - x_t) (1 - q) (1 - \underline{m})$$

It is evident that the skill ratio phase diagrams can be drawn in the space $\{k_{t+1}, z_t\}$.

Let consider the evolution of infrastructure. Before showing the equation which define the phase diagram I provide the equation which characterize the fraction of stayers in the formal regime.

$$(1 - \bar{m}_{t,f}) = \frac{(1 - \tau) \alpha A_0 z_t^{\alpha+\phi-1} k_t^\varepsilon}{w_t (1 - x_t) + (1 - \tau) \alpha A_0 z_t^{\alpha+\phi-1} k_t^\varepsilon}$$

- Infrastructure Phase Diagram, Formal Region

$$n_l ((1 - \underline{m})(1 - q)) k_t + (1 - \bar{m}_{t,f}) k_t * (n_h z_t + n_l q) - \tau A_0 z_t^{\phi + \alpha} k_t^\varepsilon = 0 \quad (.27)$$

- Infrastructure Phase Diagram, Informal Region

$$\left(\frac{(1 - \tau) \alpha A_0 \left(\Delta^{\frac{\alpha + \phi - 1}{\alpha + \phi}} \right) k_t}{w_t (1 - x_t) + (1 - \tau) \alpha A_0 \left((\Delta)^{\frac{\alpha + \phi - 1}{\alpha + \phi}} \right) k_t^{\frac{\varepsilon}{\alpha + \phi}}} \right) (n_h z_t + n_l q) + \left(k_t^{\frac{\alpha + \phi - \varepsilon}{\alpha + \phi}} n_l ((1 - \underline{m})(1 - q)) \right) = \tau A_0 \Delta^{\frac{\alpha + \phi - 1}{\alpha + \phi}} z_t \quad (.28)$$

The comparison of .27 with .28 reveals that the formal region phase diagram is a function of both z_t and z_{t+1} while the phase diagram in the informal region can be drawn in the space $\{k_{t+1}, z_t\}$

Proposition 11. *Proof*

I start proving the statement made with respect the informal regime. Let consider equation .19. By assumption the economy is under the informal regime hence the skill ratio in the formal sector is determined by the second of .9. Let consider also .8. Infrastructure is a function of the workforce, hence it depends on the migration rate. As to highlight the relationship between high skilled migration rate consider:

$$k_t = \frac{\tau Y_{f,-1}}{((1 - \bar{m}_{t,i})(n_h H_{-1} + q n_l L_{-1})) + n_l (1 - \underline{m})(1 - q) L_{-1}}$$

Exploiting both the arbitrage equation and the infrastructure definition the high skilled migration equation becomes equal to:

$$\bar{m}_{t,i} = \frac{w_t (1 - x_t)}{w_t (1 - x_t) + (1 - \tau) \alpha A_0 \Delta^{\frac{\alpha + \phi - 1}{\alpha + \phi}} (k_t)^{\frac{\varepsilon}{\alpha + \phi}}} = f(\bar{m}_{t,i})$$

It follows that a consistent high skilled migration rate should equalize the LHS and the RHS of the above equation. Namely, I am looking for a fixed point, the one at which a 45 degree line crosses the function defined by the RHS of the above equation. The next step is to prove that the above equation has one solution and that it is unique. Let consider the limits of $f(\bar{m}_{t,i})$

- $\lim_{\bar{m}_t \rightarrow 0} f(\bar{m}_t) = \frac{w_t^*}{w_t^* + (1-\tau)\alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} \left(\frac{\lim_{\bar{m}_t, i \rightarrow 0} k_t}{\bar{m}_t, i \rightarrow 0} \right)^{\frac{\varepsilon}{\alpha+\phi}}} = < 1$
- $\lim_{\bar{m}_t \rightarrow 1} f(\bar{m}_t) = \frac{w(1-x)}{w_t^* + (1-\tau)\alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} \left(\frac{\lim_{\bar{m}_t, i \rightarrow 1} k_t}{\bar{m}_t, i \rightarrow 1} \right)^{\frac{\varepsilon}{\alpha+\phi}}} = < \lim_{\bar{m}_t \rightarrow 0} f(\bar{m}_t) < 1$

In fact is evident that the denominator of the limit when \bar{m}_t approaches 1 is lower. Moreover, it should be noticed that the both limits are lower than 1. Having that .19 is clearly continuous it is proved that $f(\bar{m}_{t,i})$ crosses at least one time the 45 degree line. Proving that $f(\bar{m}_{t,i})$ is decreasing in the interval $[0, 1]$ is enough for uniqueness.

$$\frac{df(\bar{m}_{t,i})}{d\bar{m}_{t,i}} = \frac{-\frac{\varepsilon}{\alpha+\phi} w_t^* (1-\tau) \alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} (k_t)^{\frac{\varepsilon}{\alpha+\phi}}}{\left(w_t^* + (1-\tau) \alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} (k_t)^{\frac{\varepsilon}{\alpha+\phi}} \right)^2} \cdot \frac{(n_h H_{-1} + q n_l L_{-1})}{((1-\bar{m}_{t,i})(n_h H_{-1} + q n_l L_{-1})) + n_l (1-\bar{m})(1-q)}$$

which ends up the proof for the first case which is always negative in the interval $[0, 1]$

Let consider the formal regime. In such a case the skill ratio in the formal sector is not determined by the arbitrage equation but is defined by .23 with $z_{t,f}$ equal to z_t .

$$z_t = \frac{(1-\bar{m}_{t,f})(nz_{-1} + q)}{(1-q)(1-\bar{m})}$$

Then It follows that the consistent HS migration rate under the formal regime must satisfy:

$$\bar{m}_{t,f} = \frac{w_t^*}{w_t^* + (1-\tau) \alpha A_0 (z_t)^{\alpha+\phi-1} (k_t)^\varepsilon} = f(\bar{m}_{t,f})$$

As before let consider the limit of the RHS of the above equation when the HS approaches zero and when the HS migration rate tends to 1. The first limit can be easily computed:

- $\lim_{m_t \rightarrow 0} f(\bar{m}_t) = \frac{w_t^*}{w_t^* + (1-\tau)\alpha A_0 \left(\frac{\lim_{\bar{m}_t, f \rightarrow 0} z_t}{\bar{m}_t, f \rightarrow 0} \right)^{\alpha+\phi-1} \left(\frac{\lim_{\bar{m}_t, f \rightarrow 0} k_t}{\bar{m}_t, f \rightarrow 0} \right)^\varepsilon} = C < 1$

The limit of $f(\bar{m}_{t,f})$ as m_t approaches 1 is not straightforward as before. Two cases needs to be considered:

- $\alpha + \phi < 1$

$$\lim_{\bar{m}_{t,f} \rightarrow 1} \left(\frac{(1 - \bar{m}_{t,f})(nz_{t-1} + q)}{(1-q)(1-\bar{m})} \right)^{\alpha + \phi - 1}$$

The above limit is an undetermined form which can be solved through the following substitution

$$e^{(\alpha + \phi - 1) \ln \left(\frac{(1 - \bar{m}_{t,f})(nz_{t-1} + q)}{(1-q)(1-\bar{m})} \right)} = +\infty$$

The former result holds if and only if $\alpha + \phi < 1$. Hence

$$\lim_{\bar{m}_{t,f} \rightarrow 1} f(\bar{m}_t) = \frac{w_t^*}{w_t^* + \infty} = 0$$

Hence the function goes from a number lower than 1 to zero in the interval $[0, 1]$. Let's compute the derivative and show that it is decreasing.

$$\begin{aligned} \frac{df(\bar{m}_{t,i})}{\bar{m}_{t,i}} &= \frac{-(1-\tau)\alpha A_0(\alpha + \phi - 1) \left(\frac{nz_{t-1} + q}{(1-q)(1-\bar{m})} \right) \left(\frac{(1 - \bar{m}_{t,f})(nz_{t-1} + q)}{(1-q)(1-\bar{m})} \right)^{\alpha + \phi - 2} (k_t)^\varepsilon}{\left(w_t^* + (1-\tau)\alpha A_0(z_t)^{\alpha + \phi - 1} (k_t)^\varepsilon \right)^2} \\ &- \varepsilon \frac{\left((1-\tau)\alpha A_0 \left(\frac{(1 - \bar{m}_{t,f})nz_{t-1} + q}{(1-q)(1-\bar{m})} \right)^{\alpha + \phi - 1} \right) (k_t)^\varepsilon \frac{(n_h H t_{-1} + q n_l L_{t-1})}{(((1 - \bar{m}_{t,f})(n_h H t_{-1} + q n_l L_{t-1})) + n_l(1 - \bar{m})(1 - q)L_{t-1})}}{\left(w_t^* + (1-\tau)\alpha A_0(z_t)^{\alpha + \phi - 1} (k_t)^\varepsilon \right)^2} \end{aligned}$$

In such a case the derivative is made up by two components. The second is always negative while the first one is negative if $\alpha + \phi < 1$ otherwise the sign is ambiguous. This proves the uniqueness when the latter condition on parameters holds.

- $\alpha + \phi > 1$

As before in order to obtain the limit the former substitution, it follows:

$$e^{(\alpha + \phi - 1) \ln \left(\frac{(1 - \bar{m}_{t,j})(nz_{t-1} + q)}{(1-q)(1-\bar{m})} \right)} = 0$$

which implies

$$\lim_{\bar{m}_{t,f} \rightarrow 1} f(\bar{m}_t) = \frac{w_t^*}{w_t^*} = 1$$

Under the formal regime if $\alpha + \phi > 1$ it follows that an high skilled migration rate equal to 1 is always a fixed point. As already shown before in such a case the sign of the derivative is ambiguous, hence *there may* be multiplicity of solutions with at least one fixed point such that $m_t = 1$

Proposition 12. *Proof*

In order to prove the statement it suffices to compute the derivative of both .25 and .26. I start with .25. Computing the derivative of the latter with respect the tax rate I obtain:

$$\frac{d\lambda(z_t)}{d\tau} = \frac{1}{\varepsilon} \left(\frac{\Lambda}{(1-\tau)\alpha A_0 \Xi} \right)^{\frac{1-\varepsilon}{\varepsilon}} \frac{(\Lambda\alpha A_0 \Xi)}{((1-\tau)\alpha A_0 \Xi)^2}$$

where

$$\Xi = (qz_t^{\alpha+\phi-2} + z_t^{\alpha+\phi-1}(n - (1-q)(1-\underline{m})))$$

Ξ can be negative. If it is the case the derivative is not defined. After some algebra is possible to show that this is the case only if .20 does not hold. Doing the same with respect .26 it results:

$$\frac{d\gamma(z_t)}{d\tau} = \frac{\alpha + \phi}{\varepsilon} \left(\frac{\Lambda}{(1-\tau)\alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} \Pi} \right)^{\frac{\alpha+\phi-\varepsilon}{\varepsilon}} \frac{\frac{1}{\alpha+\phi} \left((1-\tau)^{\frac{1-\alpha-\phi}{\alpha+\phi}} \right) \Upsilon}{\left((1-\tau)\alpha A_0 \Delta^{\frac{\alpha+\phi-1}{\alpha+\phi}} z_t^{-2} \Pi \right)^2}$$

where

$$\Pi = qz_t^3 + (n - (1-q)(1-\underline{m}))$$

and

$$\Upsilon = qz_t^{-1} + (n - (1-q)(1-\underline{m}))$$

which is positive if both .25, .26 are satisfied.

Quantitative Assessment

The next table provides the calibrated values of q, n_l, α, k_{ss} for all countries considered⁵

Country	q	n_l	α	k_{ss}
Angola	0,020	2,02	0,24	22,35
Benin	0,009	2,35	0,28	19,21
Bolivia	0,061	2,13	0,46	62,62
Botswana	0,018	2,79	0,27	148,97
Brazil	0,035	2,14	0,54	170,82
Bulgaria	0,080	1,21	0,60	106,88
Burkina Faso	0,002	2,81	0,23	20,67
Cameroon	0,012	1,88	0,20	44,34
Chile	0,070	2,12	0,48	227,02
China	0,011	1,32	0,05	55,98
Colombia	0,048	2,42	0,56	105,56
Congo, Dem. Rep.	0,007	1,96	0,25	7,23
Congo, Rep.	0,032	2,15	0,32	64,03
Cote d'Ivoire	0,016	2,48	0,31	37,17
Croatia	0,056	1,60	0,62	255,57
Ecuador	0,081	2,58	0,68	61,98
El Salvador	0,063	2,45	0,44	125,88
Ethiopia	0,010	2,04	0,21	7,95
Gabon	0,052	2,81	0,56	201,42
Gambia, The	0,010	2,44	0,08	29,61
Georgia	0,076	1,40	0,47	39,87
Ghana	0,013	2,27	0,13	14,58
Guatemala	0,035	2,13	0,42	123,14
Guinea	0,009	2,34	0,36	21,58
India	0,021	1,93	0,30	25,26
Indonesia	0,022	2,01	0,26	38,74
Kazakhstan	0,073	1,38	0,52	80,05
Kenya	0,013	2,49	0,09	25,19
Kyrgyzstan	0,068	1,36	0,49	22,75
Lesotho	0,006	1,74	0,07	36,07
Liberia	0,027	2,03	0,16	11,50
Madagascar	0,012	2,10	0,23	15,40
Malawi	0,004	2,17	0,05	10,32

⁵Mali and Chad have been deleted from the sample because the calibrated value of k_{ss} is different across models. Armenia has been deleted because is the only country for which the condition $(\alpha + \phi < 1)$ is not satisfied.

Country	q	n_l	α	k_{ss}
Mauritania	0,010	1,88	0,14	36,60
Mongolia	0,012	2,06	0,18	25,37
Namibia	0,030	1,96	0,28	145,57
Nepal	0,011	1,82	0,36	14,91
Niger	0,002	2,48	0,05	10,15
Nigeria	0,016	2,13	0,24	25,55
Pakistan	0,016	2,07	0,18	31,96
Paraguay	0,032	2,63	0,31	69,12
Peru	0,095	2,39	0,70	104,50
Philippines	0,113	1,79	0,60	79,22
Romania	0,046	1,28	0,38	105,74
Russia	0,082	1,39	0,42	96,87
Rwanda	0,004	1,79	0,09	18,14
Senegal	0,012	2,17	0,22	30,42
Sierra Leone	0,013	1,57	0,14	11,38
Slovenia	0,064	1,51	0,28	536,84
Sri Lanka	0,021	1,47	0,16	50,28
Sudan	0,011	2,15	0,14	20,98
Tanzania	0,010	2,28	0,14	17,64
Togo	0,012	2,46	0,14	14,91
Tunisia	0,029	2,49	0,38	96,35
Uganda	0,009	2,03	0,09	21,33
Yemen	0,016	2,92	0,21	29,14

The calibrated values of infrastructure seem reasonable for most of the countries. Doubts on the results obtained arise looking to big state such as Russia, China and India for which the infrastructure proxy look like way too small. The country which has the largest estimated stock of infrastructure is Slovenia with $k_{ss} = 536,84$ whereas the Democratic Republic of Congo exhibits the lowest level, with a per capita infrastructure estimated equal to, only, almost seven dollars.

Numerical Experiments

In the following table is showed the infrastructure evolution for all the countries considered with respect the different shocks considered

$$\Delta k_t / k_{ss}$$

	Removing Informality			$\uparrow \tau (+0,01)$		
	2025	2100	2100	2025	2100	2100
Country	end \bar{m}	end \bar{m}	ex \bar{m}	end \bar{m}	end \bar{m}	ex \bar{m}
Angola	-0,09	34,24	31,39	0,00	28,63	28,17
Benin	-0,16	148,39	123,56	0,00	28,62	27,48
Bolivia	-0,34	38,43	33,95	0,02	26,16	25,79
Botswana	-0,14	46,74	40,98	0,01	28,36	27,67
Brazil	-0,04	18,97	18,05	0,00	25,34	25,24
Bulgaria	-0,42	29,13	24,40	0,02	25,31	24,93
Burkina Faso	-0,10	106,25	91,38	0,00	29,36	28,35
Cameroon	-0,22	105,66	85,39	0,01	30,52	28,74
Chile	-0,25	22,92	20,23	0,01	25,90	25,59
China	-0,04	24,24	22,83	0,00	33,90	33,30
Colombia	-0,24	24,90	19,88	0,01	25,68	25,16
Congo, Dem. Rep.	-0,12	182,57	156,24	0,00	29,07	27,94
Congo, Rep.	-0,56	63,49	43,26	0,02	28,94	27,01
Cote d'Ivoire	-0,10	67,69	61,57	0,00	27,56	27,09
Croatia	-0,54	33,59	21,18	0,03	25,99	24,86
Ecuador	-0,15	9,20	7,60	0,02	24,94	24,61
El Salvador	-1,13	47,17	28,83	0,05	27,62	25,89
Ethiopia	-0,12	94,55	82,86	0,00	29,74	28,70
Gabon	-0,13	25,32	22,97	0,01	25,35	25,13
Gambia, The	-0,47	417,82	246,87	0,01	41,43	31,88
Georgia	-0,21	21,61	19,76	0,01	25,92	25,70
Ghana	-0,51	231,99	155,77	0,01	36,12	30,54
Guatemala	-0,51	48,20	32,00	0,02	27,55	26,07
Guinea	-0,09	117,57	104,89	0,00	27,13	26,60
India	-0,12	50,89	46,31	0,01	27,71	27,25
Indonesia	-0,07	34,20	32,01	0,00	28,13	27,81
Kazakhstan	-0,08	18,48	17,67	0,01	25,47	25,37
Kenya	-0,35	73,84	49,61	0,01	36,93	31,60
Kyrgyzstan	-0,06	20,43	19,77	0,00	25,61	25,54
Lesotho	-0,14	60,96	47,21	0,01	35,71	32,25
Liberia	-0,73	60,56	35,15	0,03	34,73	29,72
Madagascar	-0,10	76,14	68,52	0,00	28,90	28,20
Malawi	-0,10	100,66	80,54	0,00	37,20	33,24
Mauritania	-0,11	53,31	46,31	0,01	31,54	30,25
Mongolia	-0,10	53,72	48,30	0,00	30,14	29,32
Namibia	-0,60	64,31	42,85	0,02	29,84	27,50

Nepal	-0,07	108,47	100,13	0,00	26,93	26,56
Niger	-0,03	87,67	79,03	0,00	35,17	33,43
Nigeria	-0,16	59,15	50,74	0,01	29,16	28,19
Pakistan	-0,29	104,64	86,49	0,01	30,95	29,20
Paraguay	-0,16	36,92	33,84	0,01	27,49	27,12
Peru	-0,06	3,96	3,48	0,02	24,71	24,52
Philippines	-0,36	10,37	7,70	0,04	25,43	24,95
Romania	-0,42	37,32	29,51	0,02	27,24	26,39
Russia	-0,14	31,93	30,82	0,01	26,14	26,03
Rwanda	-0,13	247,14	191,56	0,00	36,07	31,84
Senegal	-0,24	105,66	83,96	0,01	30,20	28,42
Sierra Leone	-0,50	212,41	136,34	0,01	36,08	30,21
Slovenia	-0,61	31,92	26,95	0,03	28,35	27,53
Sri Lanka	-0,41	44,62	30,13	0,02	32,88	29,73
Sudan	-0,18	60,22	48,59	0,01	32,10	30,08
Tanzania	-0,13	56,28	47,86	0,01	31,56	30,08
Togo	-0,20	53,52	42,24	0,01	32,28	30,11
Tunisia	-0,35	58,20	46,33	0,01	27,30	26,38
Uganda	-0,29	125,73	89,31	0,01	36,96	31,67
Yemen	-0,46	135,61	94,15	0,01	31,73	28,56
Zimbabwe	-0,43	36,68	25,80	0,02	28,99	27,39

From the previous table stands out that the result obtained either with or without the brain drain mechanism are quite close. Especially the restrictive fiscal policy results show only slightly changes between the two frameworks. Removing informality implies over than average gains, in percentage terms, with respect the poorest countries considered

The next table provides the foresee migration rates due to the policy changes studied.

Country	\bar{m}_t				
	2000	Removing Informality		$\uparrow \tau (+0, 01)$	
		2025	2100	2025	2100
Angola	0,052	0,033	0,032	0,053	0,049
Benin	0,139	0,062	0,057	0,141	0,132
Bolivia	0,076	0,054	0,052	0,077	0,074
Botswana	0,084	0,052	0,050	0,085	0,080
Brazil	0,022	0,017	0,017	0,022	0,021

Bulgaria	0,105	0,085	0,082	0,106	0,102
Burkina Faso	0,110	0,064	0,061	0,112	0,104
Cameroon	0,181	0,089	0,088	0,185	0,171
Chile	0,067	0,052	0,051	0,068	0,064
China	0,041	0,025	0,025	0,043	0,037
Colombia	0,111	0,090	0,088	0,113	0,108
Congo, Dem. Rep.	0,126	0,049	0,044	0,129	0,120
Congo, Rep.	0,264	0,171	0,171	0,268	0,254
Cote d'Ivoire	0,062	0,035	0,033	0,063	0,059
Croatia	0,263	0,223	0,216	0,265	0,257
Ecuador	0,098	0,091	0,090	0,099	0,095
El Salvador	0,323	0,250	0,248	0,326	0,314
Ethiopia	0,107	0,052	0,050	0,109	0,101
Gabon	0,049	0,039	0,037	0,050	0,047
Gambia, The	0,682	0,309	0,369	0,689	0,653
Georgia	0,049	0,038	0,037	0,050	0,048
Ghana	0,459	0,204	0,225	0,465	0,433
Guatemala	0,244	0,176	0,174	0,246	0,236
Guinea	0,074	0,036	0,033	0,075	0,071
India	0,060	0,036	0,034	0,061	0,057
Indonesia	0,039	0,025	0,024	0,040	0,037
Kazakhstan	0,023	0,018	0,018	0,023	0,022
Kenya	0,400	0,230	0,251	0,406	0,372
Kyrgyzstan	0,017	0,013	0,013	0,017	0,016
Lesotho	0,248	0,145	0,152	0,253	0,226
Liberia	0,461	0,304	0,327	0,467	0,439
Madagascar	0,078	0,041	0,039	0,080	0,074
Malawi	0,260	0,124	0,131	0,266	0,235
Mauritania	0,112	0,063	0,062	0,115	0,104
Mongolia	0,079	0,044	0,043	0,081	0,074
Namibia	0,294	0,189	0,191	0,298	0,282
Nepal	0,052	0,025	0,022	0,053	0,050

Niger	0,113	0,052	0,051	0,117	0,101
Nigeria	0,109	0,062	0,061	0,111	0,103
Pakistan	0,172	0,084	0,082	0,175	0,161
Paraguay	0,054	0,036	0,035	0,055	0,051
Peru	0,065	0,063	0,062	0,066	0,063
Philippines	0,141	0,127	0,125	0,142	0,137
Romania	0,139	0,098	0,096	0,140	0,133
Russia	0,024	0,017	0,016	0,024	0,023
Rwanda	0,307	0,111	0,116	0,313	0,283
Senegal	0,190	0,097	0,095	0,194	0,180
Sierra Leone	0,496	0,237	0,262	0,503	0,471
Slovenia	0,126	0,088	0,087	0,128	0,119
Sri Lanka	0,291	0,185	0,193	0,296	0,273
Sudan	0,178	0,100	0,101	0,182	0,166
Tanzania	0,130	0,072	0,072	0,133	0,121
Togo	0,190	0,112	0,114	0,194	0,177
Tunisia	0,143	0,093	0,090	0,145	0,138
Uganda	0,392	0,197	0,216	0,399	0,365
Yemen	0,330	0,171	0,175	0,335	0,314
Zimbabwe	0,209	0,144	0,145	0,212	0,199

From the previous table comes out that removing informality would generate a huge drop in the high skilled migration rate in the short run. Subsequently, the migration rate will remain almost equal to the new attained level. Fiscal policy affects consistently the brain drain in the long run. In fact the long run high skilled migration rate are almost equal in the two policy shocks considered.

The next tables shows the effects of the policy changes under analyst on the wages. The first table is with respect the low skilled wage. As already explained in the text fiscal policy does not affect the low skilled wage when the economy is under the informal regime. Given the small policy change considered no country exhibited a regime switching behavior

	$\Delta w_{l,t}/w_{ss}$		
	2000	2025	2100

Country	$w_{l,ss}$	end \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t
	Removing Informality				
Angola	1061,97	-20,57	-21,36	-15,39	-17,63
Benin	2728,97	-55,52	-57,59	-45,02	-51,38
Bolivia	3597,43	-48,05	-48,90	-43,66	-46,30
Botswana	10862,61	-27,88	-29,22	-21,21	-24,97
Brazil	7213,98	-46,69	-46,87	-44,80	-45,35
Bulgaria	4606,58	-72,95	-73,45	-70,83	-72,45
Burkina Faso	1602,69	-40,69	-42,15	-28,53	-35,40
Cameroon	3411,08	-34,72	-37,96	-22,35	-31,10
Chile	9738,91	-38,08	-38,79	-34,75	-36,84
China	1741,17	-7,49	-7,99	-3,37	-4,72
Colombia	5493,28	-52,23	-53,15	-49,03	-51,68
Congo, Dem. Rep.	1064,20	-56,59	-58,49	-45,49	-51,29
Congo, Rep.	3684,76	-31,47	-36,12	-20,62	-32,09
Cote d'Ivoire	3247,69	-43,06	-44,01	-36,46	-39,25
Croatia	12173,58	-72,79	-74,02	-69,54	-73,15
Ecuador	3782,10	-70,35	-70,56	-69,53	-70,19
El Salvador	5966,32	-37,54	-41,85	-27,57	-39,29
Ethiopia	650,67	-35,73	-37,55	-25,60	-30,81
Gabon	14724,34	-59,13	-59,49	-56,95	-58,04
Gambia, The	6184,54	-26,47	-44,46	4,01	-31,38
Georgia	1119,42	-35,19	-35,72	-32,12	-33,72
Ghana	1980,30	-31,30	-41,13	-8,27	-30,94
Guatemala	5880,11	-37,67	-41,20	-29,08	-38,36
Guinea	3005,45	-64,16	-65,06	-57,59	-60,53
India	1402,35	-34,39	-35,32	-28,31	-31,00
Indonesia	1848,52	-22,99	-23,58	-18,21	-19,89
Kazakhstan	2193,87	-41,08	-41,29	-39,00	-39,64
Kenya	1817,48	-10,02	-17,87	3,48	-12,05
Kyrgyzstan	640,90	-39,49	-39,65	-37,26	-37,78
Lesotho	1611,97	-12,21	-16,06	-0,86	-10,36
Liberia	530,78	-8,00	-17,60	4,98	-13,27
Madagascar	1156,59	-35,37	-36,66	-27,05	-30,79
Malawi	837,37	-16,93	-21,40	-2,43	-13,09
Mauritania	1900,51	-18,31	-20,09	-9,93	-14,75
Mongolia	1510,87	-22,50	-23,79	-14,95	-18,51
Namibia	7477,10	-25,97	-31,43	-14,21	-27,15
Nepal	1662,89	-63,81	-64,46	-58,03	-60,01
Niger	913,48	-19,05	-20,77	-7,71	-12,53

Nigeria	1610,31	-28,42	-30,24	-20,20	-25,20
Pakistan	2674,79	-32,54	-35,58	-20,07	-28,38
Paraguay	4234,93	-28,73	-29,50	-23,59	-25,92
Peru	5420,01	-69,42	-69,49	-69,10	-69,32
Philippines	1962,56	-34,58	-35,47	-32,10	-34,65
Romania	3029,57	-32,09	-34,08	-25,64	-31,12
Russia	3550,26	-40,68	-40,96	-37,32	-38,20
Rwanda	2248,28	-34,15	-39,84	-12,60	-27,84
Senegal	2623,33	-36,37	-39,73	-23,92	-33,15
Sierra Leone	982,47	-28,69	-39,94	-6,21	-30,48
Slovenia	17483,00	-20,36	-22,12	-13,85	-18,89
Sri Lanka	1723,22	-10,30	-15,53	-0,36	-11,66
Sudan	1288,03	-18,28	-21,31	-8,19	-15,83
Tanzania	1150,03	-18,93	-21,07	-10,03	-15,65
Togo	955,65	-15,96	-19,12	-6,20	-14,12
Tunisia	6967,23	-43,97	-46,02	-36,53	-42,41
Uganda	1695,91	-18,75	-26,62	-1,03	-18,21
Yemen	3624,87	-34,94	-41,37	-18,39	-34,37
Zimbabwe	1762,45	-18,72	-22,21	-10,59	-19,12

From the previous table is evident that informality help low skilled to escape from poverty in developing countries. Removing informal sector would have a tremendous negative impact on the low skilled wage in the short run. Only a small fraction of the losses will be retrieved in the long run due to the increase in the available infrastructure.

The next table show the effect of removing informality on the high skilled wage, both in the short run and in the long run.

Country	$\Delta w_{h,t}/w_{ss}$				
	2000	2025		2100	
	$w_{h,ss}$	end \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t
Removing Informality					
Angola	4386,41	24,34	25,57	29,16	31,53
Benin	12684,93	88,35	95,63	110,35	124,30
Bolivia	7171,37	26,74	27,61	32,80	34,11
Botswana	51882,63	31,06	33,17	37,27	41,17
Brazil	43075,32	14,74	14,84	18,06	18,13
Bulgaria	6536,75	19,57	19,98	24,44	24,52

Burkina Faso	7654,88	67,60	71,84	78,07	91,89
Cameroon	16292,21	58,28	67,35	69,49	85,86
Chile	25606,36	16,13	16,61	19,69	20,32
China	2631,81	17,36	18,71	20,14	22,93
Colombia	25237,48	15,82	16,33	19,69	19,97
Congo, Dem. Rep.	5082,91	110,53	119,20	140,06	157,23
Congo, Rep.	17599,35	28,56	34,96	34,18	43,47
Cote d'Ivoire	15511,81	47,44	49,21	58,64	61,89
Croatia	38597,37	16,61	17,38	21,71	21,28
Ecuador	9489,38	6,24	6,30	7,80	7,63
El Salvador	18742,99	19,81	23,53	24,24	28,97
Ethiopia	3107,79	60,35	65,44	73,37	83,31
Gabon	47374,78	18,59	18,82	22,94	23,07
Gambia, The	28272,70	71,64	182,19	102,05	248,64
Georgia	2510,37	15,85	16,23	19,30	19,85
Ghana	6949,12	73,83	118,87	88,37	156,75
Guatemala	32591,43	22,73	26,06	27,87	32,15
Guinea	14354,79	79,24	81,90	100,45	105,49
India	5725,22	35,88	37,36	43,81	46,53
Indonesia	7450,42	25,20	26,06	30,50	32,16
Kazakhstan	5574,08	14,41	14,54	17,61	17,75
Kenya	8661,14	19,54	39,94	21,66	49,86
Kyrgyzstan	1603,36	16,13	16,24	19,71	19,86
Lesotho	7699,20	27,04	38,06	27,87	47,44
Liberia	2469,01	11,11	28,56	12,96	35,32
Madagascar	5524,16	51,53	54,54	63,00	68,88
Malawi	4031,66	46,12	63,69	50,83	80,98
Mauritania	8779,43	33,08	37,35	38,20	46,53
Mongolia	6808,99	36,07	38,91	42,97	48,54
Namibia	35712,58	26,52	34,64	31,22	43,06
Nepal	7514,72	76,57	78,37	97,47	100,70
Niger	4363,02	55,36	62,54	64,49	79,45
Nigeria	7691,25	37,33	40,82	44,56	51,00
Pakistan	8041,25	59,09	68,17	69,98	86,97
Paraguay	11775,16	26,50	27,52	32,01	34,00
Peru	12966,20	2,88	2,90	3,56	3,50
Philippines	6229,84	6,11	6,39	7,67	7,74
Romania	10182,48	22,04	24,07	26,51	29,65
Russia	4914,45	24,83	25,12	30,45	30,97
Rwanda	10871,09	108,15	144,13	126,99	192,84

Senegal	12529,71	57,33	66,27	68,25	84,42
Sierra Leone	4692,51	60,61	104,91	74,44	137,17
Slovenia	25972,56	19,65	22,02	22,54	27,07
Sri Lanka	7685,88	15,01	24,57	16,07	30,27
Sudan	6151,94	31,99	39,14	36,13	48,84
Tanzania	5492,85	33,48	38,57	38,57	48,10
Togo	4564,44	27,13	34,17	30,02	42,45
Tunisia	26933,02	34,65	37,37	42,23	46,56
Uganda	8100,08	42,72	70,29	48,33	89,80
Yemen	16753,91	55,87	73,92	66,08	94,68
Zimbabwe	8417,91	16,96	21,10	19,59	25,92

The informal sector depresses the high skilled wage in all countries. The long run gains are larger due to the infrastructure mechanism accounted

Finally the effect of the tax policy change on the high skilled wage are presented for all the countries considered.

Country	$\Delta w_{h,t}/w_{ss}$				
	2000	2025		2100	
	$w_{h,ss}$	end \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t	ex \bar{m}_t
		$\uparrow \tau (+0, 01)$			
Angola	4386,41	-2,04	-0,02	6,08	6,35
Benin	12684,93	-1,87	-0,02	5,58	5,62
Bolivia	7171,37	-1,43	-0,01	3,87	3,95
Botswana	51882,63	-1,91	-0,02	5,66	5,82
Brazil	43075,32	-1,28	-0,01	3,33	3,43
Bulgaria	6536,75	-1,19	-0,01	3,11	3,15
Burkina Faso	7654,88	-2,08	-0,02	6,39	6,54
Cameroon	16292,21	-2,17	-0,02	6,96	6,96
Chile	25606,36	-1,38	-0,01	3,68	3,77
China	2631,81	-3,18	-0,03	11,26	12,43
Colombia	25237,48	-1,26	-0,01	3,33	3,36
Congo, Dem. Rep.	5082,91	-1,98	-0,02	6,02	6,10
Congo, Rep.	17599,35	-1,74	-0,02	5,30	5,15
Cote d'Ivoire	15511,81	-1,77	-0,02	5,07	5,23
Croatia	38597,37	-1,17	-0,01	3,16	3,09
Ecuador	9489,38	-1,10	-0,01	2,82	2,86

El Salvador	18742,99	-1,45	-0,01	4,20	4,05
Ethiopia	3107,79	-2,16	-0,02	6,73	6,91
Gabon	47374,78	-1,25	-0,01	3,26	3,33
Gambia, The	28272,70	-2,88	-0,03	12,51	10,60
Georgia	2510,37	-1,41	-0,01	3,76	3,87
Ghana	6949,12	-2,58	-0,03	9,85	8,99
Guatemala	32591,43	-1,50	-0,02	4,32	4,22
Guinea	14354,79	-1,65	-0,02	4,63	4,74
India	5725,22	-1,81	-0,02	5,22	5,39
Indonesia	7450,42	-1,95	-0,02	5,70	5,96
Kazakhstan	5574,08	-1,32	-0,01	3,45	3,56
Kenya	8661,14	-2,82	-0,03	10,95	10,26
Kyrgyzstan	1603,36	-1,36	-0,01	3,59	3,71
Lesotho	7699,20	-2,96	-0,03	11,08	11,07
Liberia	2469,01	-2,39	-0,02	8,80	8,04
Madagascar	5524,16	-2,04	-0,02	6,17	6,38
Malawi	4031,66	-3,17	-0,03	12,33	12,35
Mauritania	8779,43	-2,52	-0,03	8,33	8,65
Mongolia	6808,99	-2,31	-0,02	7,28	7,59
Namibia	35712,58	-1,87	-0,02	5,87	5,65
Nepal	7514,72	-1,64	-0,02	4,56	4,70
Niger	4363,02	-3,21	-0,03	11,79	12,60
Nigeria	7691,25	-2,04	-0,02	6,22	6,36
Pakistan	8041,25	-2,28	-0,02	7,41	7,46
Paraguay	11775,16	-1,78	-0,02	5,07	5,25
Peru	12966,20	-1,08	-0,01	2,72	2,77
Philippines	6229,84	-1,19	-0,01	3,14	3,16
Romania	10182,48	-1,59	-0,02	4,50	4,54
Russia	4914,45	-1,50	-0,01	4,03	4,18
Rwanda	10871,09	-2,87	-0,03	10,88	10,56
Senegal	12529,71	-2,10	-0,02	6,64	6,61
Sierra Leone	4692,51	-2,51	-0,03	9,57	8,61
Slovenia	25972,56	-1,87	-0,02	5,56	5,68
Sri Lanka	7685,88	-2,40	-0,02	8,32	8,06
Sudan	6151,94	-2,48	-0,02	8,37	8,46
Tanzania	5492,85	-2,48	-0,02	8,22	8,46
Togo	4564,44	-2,49	-0,02	8,44	8,49
Tunisia	26933,02	-1,59	-0,02	4,51	4,53
Uganda	8100,08	-2,83	-0,03	11,01	10,34
Yemen	16753,91	-2,13	-0,02	7,12	6,76
Zimbabwe	8417,91	-1,84	-0,02	5,60	5,53

Small losses in the short run become positive gains in the long run. The results is consistent with the brain drain mechanism accounted given the slightly differences between the results reported in columns 5 and 6.

Chapter 2

Here is provided the closed form solution of the law of motion of capital with respect time zero:

Let's start assuming that the government never defaults. Then

$$k_{t+1} = [(1 - \theta) (1 - \alpha)] k_t^\alpha$$

given k_0

then

$$k_1 = [(1 - \theta) (1 - \alpha)] k_0^\alpha$$

$$k_2 = [(1 - \theta) (1 - \alpha)] k_1^\alpha =$$

$$(1 - \theta) (1 - \alpha) [(1 - \theta) (1 - \alpha) k_0^\alpha]^\alpha = [(1 - \theta) (1 - \alpha)]^{\alpha+1} k_0^{\alpha^2}$$

$$k_3 = [(1 - \theta) (1 - \alpha)] k_2^\alpha =$$

$$(1 - \theta) (1 - \alpha) \left[[(1 - \theta) (1 - \alpha)]^{\alpha+1} k_0^{\alpha^2} \right]^\alpha$$

$$[(1 - \theta) (1 - \alpha)]^{\alpha^2+\alpha+1} k_0^{\alpha^3}$$

I suppose that a close form solution of the law of motion of capital is given by the following equation:

$$k_t = [(1 - \theta) (1 - \alpha)]^{\frac{1-\alpha^t}{1-\alpha}} k_0^{\alpha^t} \quad (.29)$$

I prove that the former formula is correct exploiting the induction principle. First I show that the formula is correct for $t = 1$:

$$k_1 = [(1 - \theta) (1 - \alpha)] k_0^\alpha = [(1 - \theta) (1 - \alpha)]^{\frac{1-\alpha}{1-\alpha}} k_0^{\alpha^1}$$

Secondly is needed to show that the “domino effect” applies:

Let

$$k_t = [(1 - \theta)(1 - \alpha)]^{\frac{1-\alpha^t}{1-\alpha}} k_0^{\alpha^t}$$

then

$$k_{t+1} = [(1 - \theta)(1 - \alpha)]^{\frac{1-\alpha^{t+1}}{1-\alpha}} k_t^{\alpha^{t+1}}$$

it follows

$$k_{t+1} = [(1 - \theta)(1 - \alpha)] \left([(1 - \theta)(1 - \alpha)]^{\frac{1-\alpha^t}{1-\alpha}} k_0^{\alpha^t} \right)^\alpha$$

then

$$k_{t+1} = [(1 - \theta)(1 - \alpha)]^{\frac{1-\alpha^{t+1}}{1-\alpha}} k_t^{\alpha^{t+1}}$$

Which proves that the solution is correct.