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Research paper

Environmental pollution as engine of industrialization

Angelo Antoci[a], Marcello Galeotti[b,*], Serena Sordi[c]

[a] Dipartimento di Scienze Economiche e Aziendali, University of Sassari, via Muroni 25, 07100 Sassari, Italy
[b] Dipartimento di Statistica, Informatica, Applicazioni 'G. Parenti', University of Florence, viale Morgagni 59, 50134 Firenze, Italy
[c] Dipartimento di Economia Politica e Statistica, University of Siena, piazza San Francesco 7, 53100 Siena, Italy

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ABSTRACT

This paper analyzes the dynamics of a small open economy with two sectors (a farming sector and an industrial one), heterogeneous agents (workers and entrepreneurs) and free inter-sectoral labor mobility. Labor productivity in the first sector is negatively affected by environmental pollution generated by both sectors, whereas in the second sector it is positively affected by physical capital accumulated by entrepreneurs. Through a global analysis of the non-linear three-dimensional dynamic system of the model we derive conditions under which industrialization generates a decline in workers' revenues in both sectors.

1. Introduction

There is general agreement that industrialization is an integral part of the economic growth process in developing countries and that it produces improvements in the welfare of economic agents, exactly as happened in Europe in the nineteenth century due to the Industrial Revolution (see, e.g., Lewis [24]; Ranis and Fei [31]; Bade [3]; Lucas [29]). However, an increasing number of contributions in literature deals with the negative impact on welfare of environmental pollution and depletion of free access-natural resources which, in some cases, accompany industrialization processes. López [27] documented cases of structural changes triggered by the degradation of natural resources in Latin America and Sub-Saharan Africa. He introduced the term 'perverse structural change' to refer to structural changes of this type, which are characterized by a) environmental degradation and b) stagnant or declining wages of unskilled labor force in both farming and non farming sectors. The decline in the unskilled labor remuneration due to environmental degradation is documented by several works in literature (see, e.g., Bresciani and Valdés [6]). Environmental degradation lowers the opportunity cost to work in non agricultural sectors and may fuel a development process of the type described in this paper. Other examples of structural changes catalyzed by environmental degradation have been observed in regions that have grown at high rates in recent years. In several small or medium size rural areas of Africa, China and India, environmental degradation is becoming a key issue and citizens are forced to change their behavior to defend themselves against the pollution effects of the industrialization process (see Economy [13]; World Bank [42]; Dhamodharam and Swaminathan [11]; Boopathi and Rameshkumar [5]; Deng and Yang [10]; Holdaway [22]; Chuhan-Pole et al. [8]). This is well described by the case study of Reddy and Behera [33], where the economic costs of water pollution due to industrial activity in the rural communities located in the industrial belts in Andhra Pradesh, South India, is evaluated. The cost estimates revealed that the impact of industrial pollution on

* Corresponding author.
E-mail addresses: angelo.antoci@virgilio.it (A. Antoci), marcello.galeotti@dmd.unifi.it, marcello.galeotti@unifi.it (M. Galeotti), serena.sordi@unisi.it (S. Sordi).

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rural communities is quite substantial in monetary terms,[1] and is not compensated by the increase in the share of revenues deriving from the employment in industrial polluting sectors.

The purpose of this paper is to make a contribution to a better understanding of the interactions between environmental pollution, process of industrialization and workers' welfare. To this end, we analyze the dynamics of a small open economy where there are only two sectors (a farming sector, in short, 'F-sector', and an industrial sector, in short, 'I-sector'), free labor mobility and heterogeneous agents (farmers, 'F-agents', and industrial entrepreneurs, 'I-agents'). The latter are characterized as follows. F-agents are endowed only with their own working capacity and use it either in the F-sector, for the production of farming goods, or as employees of I-agents in the I-sector. In turn, only I-agents are able to accumulate physical capital, which is entirely employed in the I-sector to produce, jointly with the labor force provided by F-agents, industrial goods.

In our formalization, the state of the economy is described by three variables which are defined as follows. $N \in [0, \bar{N}]$ ($\bar{N} - N$, respectively) represents the labor force employed in the F-sector (in the I-sector, respectively), P the stock of accumulated pollution, and K the aggregated stock of physical capital accumulated by I-agents. Labor productivity in the F-sector is negatively affected by the stock of pollution P, while in the I-sector it is positively affected by the physical capital stock K. The dynamics of K, N and P are represented by a three-dimensional dynamic system. The accumulation process of K is built on a Solow [39]-type capital accumulation mechanism (see, among the others, Guerrini [17]; Guerrini and Sodini [18]; Brianzoni et al. [7]). The labor allocation N evolves according to a payoff monotonic evolutionary dynamics (see Weibull [41]). More specifically, we assume that workers have to choose, in each instant of time, between two strategies: working in the F-sector or working in the I-sector. The payoff of the first strategy is the per capita output in the resource-dependent sector, while the payoff of the second strategy is the wage rate earned in the I-sector, which is assumed to coincide with the marginal productivity of $\bar{N} - N$. Finally, we assume that the accumulation of the stock P of pollution is positively affected by the production activities of both sectors.

We show that in such a framework environmental pollution can be the engine of the industrialization process. In fact, if the environmental impact of the I-sector is high enough relative to that of the F-sector, a self-reinforcing process of industrialization, driven by negative externalities, may be observed. The expansion of the I-sector generates a reduction in labor productivity in the F-sector via an increase in the stock of pollution and therefore leads workers to move from the resource-dependent sector towards the industrial one. The consequent further expansion of the I-sector generates an extra increase in the pollution level from which follows a further reduction in labor productivity in the F-sector, and so on. This expansion of the I-sector, at the expense of the F-sector, may be associated with a decrease in workers' revenues. When this happens – which requires a sufficiently small labor force and a polluting impact of the I-sector higher than that of the F-sector – the transition of labor from the natural resource-dependent sector towards the industrial sector can be classified as a perverse structural change, in the sense of López [27]; namely, a structural change associated with growing problems of environmental degradation, declining or stagnant wages and perpetuation of poverty.

The evolutionary dynamics of labor allocation driven by environmental degradation were also treated in Antoci et al. [2], although in a rather different context. In particular, in the latter contribution, labor productivity in the resource-dependent sector was assumed to be determined by the stock of a renewable natural resource and not to be negatively affected by pollution. Furthermore, the production technology in the resource-dependent sector was described by the function proposed by Schaefer [38], widely used in modeling production processes based on the exploitation of natural resources such as fishery and forestry. In the present paper, we assume a decreasing return technology which, in our opinion, is more suited to describe production processes in farming. This change in assumptions leads to quite different results in the dynamic analysis of the model. In particular, in Antoci et al. [2], the stationary state in which both sectors coexist can be attractive only if it corresponds to a structural change which improves workers' welfare. In the present paper we show that, if interior stationary states exist, one of them is always attractive, even if it corresponds to a structural change which reduces workers' welfare. Furthermore, differently from Antoci et al. [2], all the dynamic regimes that may be observed under the model analyzed in the present paper are fully described.

The structure of the paper is the following. Section 2 introduces the model. Section 3 contains a detailed analysis of the dynamics generated by the dynamic system of the model. A few concluding and summarizing results are finally given in Section 4. A mathematical appendix at the end of the paper supplies proofs for all lemmas and theorems.

2. Set up of the model

In the small open economy with two sectors we model in this paper, the prices of both goods are exogenously determined and, without loss of generality, we assume that they are both equal to unity.

The aggregated production functions of the F- and I-sectors are given, respectively, by:

$$Y_F = \frac{\alpha N^\beta}{(1+P)^\gamma} \qquad 1 > \beta > 0,\ \alpha, \gamma > 0 \tag{1}$$

$$Y_I = (\bar{N} - N)^\delta K^{1-\delta} \qquad 1 > \delta > 0,\ \bar{N} > 0 \tag{2}$$

[1] The costs of damage would be much higher if social and health costs were accounted for.

The production function (1) exhibits decreasing returns in N. This assumption is rather common in modeling the production activity of the farming sector. It is motivated by the fact that, among the other things, such activity depends on land endowment, which is a fixed factor of production. The case in which the production in the resource-dependent sector is not characterized by decreasing returns was analyzed in the work of Antoci et al. [2]. In this context, the production activity of the resource-dependent sector was modelled by the function proposed by Schaefer [38], which is widely used in modeling production processes based on the exploitation of natural resources such as fishery and forestry (not for the farming sector). As already stressed in the Introduction, this change in modeling the activity of the resource-dependent sector leads to rather different results in the dynamic analysis of the model.

For the sake of simplicity, we assume that only the productivity of the farming sector is affected by global pollution. In fact, pollution affects also the productivity in the industrial sector, for example by reducing workers' health (see, e.g., Graff Zivin and Neidell [16]). More in general, global effects of climate changes and pollution reduce the productivity of the whole economy. Several models on global warming take account of these effects (see, e.g., Golub and Toman [14]; Hackett and Moxnes [21]). Our conjecture is that the introduction of such effects in our model could lead to the following two scenarios. If the negative impact of pollution on the I-sector is lower than on the F-sector, then the dynamics and the basic results of our model would not change, although the results according to which, under specific conditions, industrialization produces a reduction in welfare would be strengthened. Vice-versa, if the negative impact of pollution on the I-sector is higher than the impact on the F-sector, then an increase in industrialization would generate an increase in the relative performance of the F-sector, leading to a contraction of the industrial sector; consequently, no self-reinforcing undesirable expansion of the industrial sector could be observed.

In our model, the economic activity in the F-sector is worked out by small firms (each of them owned by a household) and the output of each firm coincides with the average output. According to Ray [32], in the traditional sectors of developing countries the labor payment tends not to be based on marginal product, but on income sharing. Thus people working in these sectors receive the average product. Workers have to choose, in each instant of time, between two strategies: working in the F-sector or working in the I-sector. The payoff of the first strategy is the per capita output in the resource-dependent sector:

$$\frac{Y_F}{N} = \frac{\alpha N^{\beta-1}}{(1+P)^{\gamma}}$$

while the payoff of the second strategy is the wage rate w earned in the I-sector, which is assumed to coincide with the marginal productivity of $\bar{N} - N$:

$$w = \delta(\bar{N} - N)^{\delta-1} K^{1-\delta} \tag{3}$$

In addition, we assume that labor allocation N evolves according to the payoff monotonic evolutionary dynamics (see Weibull [41]):

$$\dot{N} = \lambda\left(\frac{Y_F}{N} - w\right) \tag{4}$$

where the parameter $\lambda > 0$ measures the speed of inter-sectoral mobility and a dot over a variable indicates its first derivative with respect to time. Eq. (4) represents an imitation-based learning mechanism according to which the better performing strategy spreads in the population of workers at the expenses of the other one, that is $\dot{N} > 0$ (respectively, < 0) if $(Y_F/N) - w > 0$ (respectively, < 0), for every $N \in (0, \bar{N})$.[2]

Finally, the coevolution of the variables N, K, and P is assumed to be given by the three-dimensional dynamic system:

$$\dot{K} = s\left[Y_I - w(\bar{N} - N)\right] - dK \tag{5}$$

$$\dot{N} = \lambda\left(\frac{Y_F}{N} - w\right) \tag{6}$$

$$\dot{P} = \varepsilon Y_F + \eta Y_I - \theta P \tag{7}$$

According to Eq. (5), physical capital is accumulated via a Solow-type mechanism [39]. The difference $Y_I - w(\bar{N} - N)$ measures the revenues of I-agents, and the parameters $s, d \in (0, 1)$ represent the propensity to save of I-agents and the depreciation rate of K, respectively. Eq. (5) is based on the assumption that workers in the I-sector do not accumulate physical capital (they consume all their revenues), and therefore the investment $s\left[Y_I - w(\bar{N} - N)\right]$ is a fraction of the revenues $Y_I - w(\bar{N} - N)$ of entrepreneurs. This assumption may appear simplistic, however it is not unusual in economic development literature (see, e.g., Goodwin [15]; López [28]). We have made the same assumption for workers in the F-sector. Empirical evidence

[2] The variable N could be considered as a control variable in an optimal control problem. For space constraints, in this paper we have focused on non optimal dynamics driven by negative externalities. Environmental externalities can affect economic activities especially in developing countries, where property rights tend to be ill-defined and ill-protected, environmental institutions and regulations are weak and natural resources are more fragile than in developed countries, which are located in temperate areas instead than in tropical and sub-tropical regions (see e.g. López [27]).

(see, e.g., Barbier [4]) suggests that in developing countries farmers are not able to sensibly modify their physical capital endowment in the short/medium run.

Eq. (7) models the accumulation process of pollution where the parameters $\varepsilon > 0$ and $\eta > 0$ represent the effects on the stock P of the aggregate outputs of the F- and I-sectors, respectively, whereas the parameter θ represents the natural decay rate of P. The effects of agricultural activity on pollution is mainly due to the use of pesticides, herbicides, fertilizers and other agrochemicals. Inappropriate uses of chemicals may produce several negative effects on the productivity of the agricultural sector (see, e.g., Conway and Pretty [9], Pimentel et al. [30], Rola and Pingali [34]): increased control expenses resulting from pesticide-related destruction of natural enemies of pests and from the development of pesticide resistance, costs related to crop pollination problems and honeybee losses, costs due to biodiversity loss (which negatively affects the stability and resilience of agricultural systems), productivity loss due to the negative impact of pesticides on farmers' health. The pollution effects of industrial activity, especially air and water pollution, are documented in several works dealing with the industrialization processes of emerging economies, as China and India (see, e.g., World Bank [42]; Liu et al. [25]; Ebenstein [12]; Reddy and Behera [33]).

We assume that the two categories of economic agents take aggregate outputs Y_F and Y_I of the two sectors as exogenously given. Thus, in our model, both sectors produce environmentally negative externalities that agents are not able to internalize due to coordination problems. This assumption plays a crucial role in determining the results of the model, much more than the behavioral assumption about the accumulation process of physical capital. In fact, environmental externalities play a crucial role in conditioning economic growth dynamics, especially in developing countries, where environmental resources tend to be less protected and more fragile than in developed countries (López [26,27]).

Given equations (1), (2) and (3), the dynamic system (5)–(7) can be re-written as:

$$\begin{aligned} \dot{K} &= s(1-\delta)(\bar{N}-N)^{\delta}K^{1-\delta} - dK \\ \dot{N} &= \lambda\left[\frac{\alpha N^{\beta-1}}{(1+P)^{\gamma}} - \delta(\bar{N}-N)^{\delta-1}K^{1-\delta}\right] \\ \dot{P} &= \varepsilon\frac{\alpha N^{\beta}}{(1+P)^{\gamma}} + \eta(\bar{N}-N)^{\delta}K^{1-\delta} - \theta P \end{aligned} \tag{8}$$

In what follows, the dynamic system (8) will be studied in a positively invariant box $\mathcal{B} = \left\{(K,N,P) \in \left(0,\bar{K}\right) \times \left(0,\bar{N}\right) \times \left(0,\bar{P}\right)\right\}$ after a suitable re-scaling.[3] This rescaling is just a change of the measure unities of the variables K, N, P, which allows to reduce the number of parameters and simplify the equations. Specifically, thanks to it, the stock of capital is measured per number of workers and the revenues of workers in the two sectors are defined by the same unity of measure. First of all, we set $K = aK'$ such that $s(1-\delta) = da^{\delta}$, implying, in particular, by renaming K' as K, $\bar{K} = \bar{N}$. Then, we pose $K = bK'$, $N = bN'$, in such a way that $\alpha b^{\beta-1} = \delta$. Finally we re-scale the time t so as to obtain $\theta = 1$. Hence, maintaining the original symbols for the variables, system (8) becomes:

$$\begin{aligned} \dot{K} &= lK^{1-\delta}\left[\left(\bar{N}-N\right)^{\delta} - K^{\delta}\right] \\ \dot{N} &= m\left[N^{\beta-1}(1+P)^{-\gamma} - \left(\bar{N}-N\right)^{\delta-1}K^{1-\delta}\right] \\ \dot{P} &= qN^{\beta}(1+P)^{-\gamma} + r\left(\bar{N}-N\right)^{\delta}K^{1-\delta} - P \end{aligned} \tag{9}$$

where $l, m, q, r, \gamma, \bar{N} > 0$, $1 > \beta, \delta > 0$. In fact, in terms of the original parameters, we have:

$$l = \frac{d}{\theta},\ m = \frac{\lambda\delta}{\theta},\ q = \frac{\varepsilon\alpha^{\frac{2}{1-\beta}}}{\delta^{\frac{\beta}{1-\beta}}\theta},\ r = \frac{\eta\alpha^{\frac{1}{1-\beta}}d^{\frac{1-\delta}{\delta}}}{[\varepsilon(1-\delta)]^{\frac{1-\delta}{\delta}}\delta^{\frac{1}{1-\beta}}\theta} \tag{10}$$

while the new $\bar{N}$ is equal to the former one multiplied by $b^{-1} = (\delta/\alpha)^{1/(1-\beta)}$.

Hence, in the box $\mathcal{B}$, $\bar{K} = \bar{N}$. In order to determine $\bar{P}$, we proceed as follows. Consider the equation $\dot{P} = f(K,N,P) = 0$. Then, as $\frac{\partial f}{\partial P} < 0$ for any positive triad (K, N, P), it follows that $f(K,N,P) = 0$ defines an implicit function $P(K, N)$ on the open square $\left(0,\bar{N}\right)^2$. But it is easily checked that $P(K, N)$ can be continuously extended to the closed square $\left[0,\bar{N}\right]^2$. Therefore we define

$$\bar{P} = \max_{\left[0,\bar{N}\right]^2} P(K,N)$$

In this way, it is easily seen that when $(K,N) \in \left(0,\bar{N}\right)^2$, then $f\left(K,N,\bar{P}\right) \le 0$ and $f\left(K,N,\bar{P}+\varepsilon\right) < 0$ for any arbitrarily small $\varepsilon > 0$. So the box $\mathcal{B} = \left(0,\bar{N}\right)^2 \times \left(0,\bar{P}\right)$ satisfies our requirements.

[3] *Positively invariant* means that the trajectories starting in $\mathcal{B}$ cannot leave it.

3. The dynamics of the model

We are now in the position to describe the basic mathematical results about the dynamics generated by the nonlinear three-dimensional system (9). Starting from a local analysis, we will be able to fully characterize the dynamics at a global level.

3.1. Local analysis

Let us consider the function:

$$\varphi(K) = q(\bar{N} - K) + rK - (\bar{N} - K)^{\frac{\beta-1}{\gamma}} + 1 \tag{11}$$

defined for $K \in [0, \bar{N})$. Then, the local analysis results are summed up in the following theorem.

Theorem 1. *Consider the above function $\varphi(K)$. Then:*

1. *If:*

$$\varphi(0) = q\bar{N} - \bar{N}^{\frac{\beta-1}{\gamma}} + 1 > 0 \tag{12}$$

there exists exactly one stationary state in $\mathcal{B}$, which is a sink.

2. *If $\varphi(0) < 0$, there may exist, generically, two or zero stationary states in $\mathcal{B}$. In the former case, named $Q^* = (K^*, N^*, P^*)$ and $\tilde{Q} = (\tilde{K}, \tilde{N}, \tilde{P})$, $0 < K^* < \tilde{K} < \bar{N}$, the two stationary states, Q^* is a saddle endowed with a two-dimensional stable manifold and $\tilde{Q}$ is a sink. In the bifurcation case $Q^* = \tilde{Q}$, the stationary state is a saddle-node.*
3. *If $\varphi(0) = 0$ and $\varphi'(0) > 0$, there exists exactly one sink in $\mathcal{B}$; if, instead, $\varphi(0) = 0$ and $\varphi'(0) \leq 0$, there is no stationary state in $\mathcal{B}$.*

Proof. See Appendix A.1 □

From the above theorem it follows that no Hopf bifurcation can occur. Moreover, given the expressions for q and r in (10), we know that their values are (*ceteris paribus*) positively proportional to ε (the parameter measuring the environmental impact of the F-sector) and η (the parameter measuring the environmental impact of the I-sector), respectively. According to condition (12) in Theorem 1, one (and only one) stationary state exists if (*ceteris paribus*) either the size $\bar{N}$ of the population of workers or the parameter q are high enough. A necessary condition for the existence of two stationary states (a saddle and a sink) is $q < r$; such a condition is satisfied if (*ceteris paribus*) the parameter ε is low enough with respect to the parameter η. In the following subsection we will see that, when two stationary states exist, there are two generic dynamic regimes (depending on initial conditions), and the one converging to the disappearance of the industrial sector leads to a higher workers' welfare. The existence of these two regimes is due to the fact that, when $q < r$, the expansion of the industrial sector is self-reinforcing: an increase in the share of workers employed in such a sector increases the pollution level and, therefore, (*ceteris paribus*) increases the relative productivity of labor in the industrial sector. This further stimulates labor migration towards the industrial sector and so on. In the opposite case, when $q > r$, the expansion of the industrial sector generates a reduction in pollution, and consequently (*ceteris paribus*) an increase in the relative performance of the agricultural sector. In such a context, at most one stationary state exists. It represents a mixture of traditional and industrial activities where the workers attain the highest welfare, to which all the trajectories converge (see Theorem 2 and Corollary 1).

The parameters q and r play a crucial role also in determining the coordinates of the sink $\tilde{Q}$, when it exists. An increase either in the value of parameter q or in the value of parameter r generates an increase in capital accumulation $\tilde{K}$ and a decrease in the employment level $\tilde{N}$ in the F-sector. The mechanism giving rise to such a result is rather intuitive: an increase in either q or r increases (*ceteris paribus*) the pollution level P reducing labor productivity in the F-sector; this, in its turn, has the effect of increasing labor employment and capital accumulation in the I-sector. In this sense, we can say that, in our model, environmental degradation can be an engine of industrialization, i.e., of a structural change. In the next section we will give a complete classification of the regimes that can be observed under the dynamic system (9). These global analysis results will allow us, among other things, to illustrate the crucial role which is played by the initial value of the pollution level P in determining the future evolution of the economy.

3.2. Global analysis

In the previous paragraph we have seen that three *local* configurations are generically possible, namely:

a) *There is no stationary state in the box $\mathcal{B}$*;
b) *There is exactly one stationary state in $\mathcal{B}$, which in that case is a sink*;
c) *There are two stationary states in $\mathcal{B}$, precisely a sink and a saddle endowed with a two-dimensional stable manifold.*

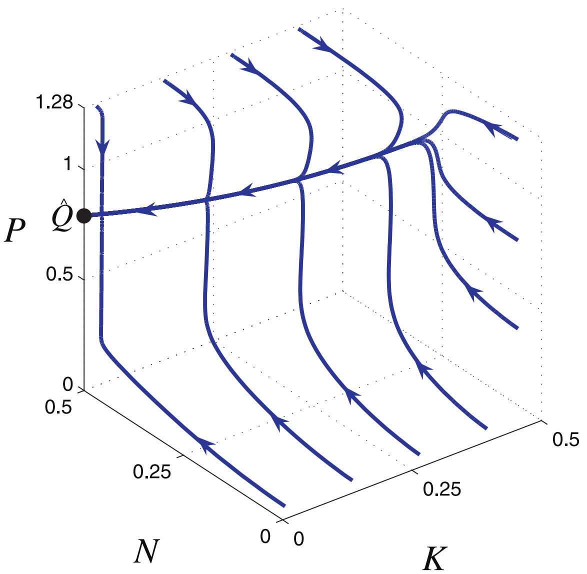

Fig. 1. Example of regime 1 (with $l = 1$, $m = 1$, $\beta = 0.5$, $\gamma = 0.5$, $\delta = 0.06$, $\bar{N} = 0.5$, $q = 1.5 > r = 1.4$ such that $\varphi(0) = -0.25 < 0$).

Now we will prove that there are precisely three global dynamic patterns corresponding to such local configurations. In fact, in case (a) all the trajectories starting in $\mathcal{B}$ converge to a boundary stationary state, $\hat{Q} = (0, \bar{N}, \hat{P})$; in case (b) all the trajectories starting in $\mathcal{B}$ converge to the sink $\tilde{Q} = (\tilde{K}, \bar{N}, \tilde{P})$; finally in case (c) the stable manifold of the saddle $Q^* = (K^*, N^*, P^*)$ (a two-dimensional surface) separates the trajectories converging to the boundary stationary state $\hat{Q} = (0, \bar{N}, \hat{P})$ from those converging to the sink $\tilde{Q} = (\tilde{K}, \bar{N}, \tilde{P})$.

Such an exhaustive description of the system's global dynamics is achieved through three lemmas, whose statements and proofs are provided in the Appendix A.2. By naming $\mathcal{A}$ the region filled by trajectories converging to the boundary stationary state $\hat{Q}$ (which in case (c) coincides with the whole box $\mathcal{B}$) and $\mathcal{C}$ the region filled by trajectories converging to the sink $\tilde{Q}$ (which coincides with the whole box $\mathcal{B}$ in case (b)), the final result of the global analysis can be stated as follows:

Theorem 2. *System (9), defined in the open box $\mathcal{B}$, can exhibit (generically) at most three regimes. More precisely:*

1. *There exists a positively invariant region $\mathcal{A} \subseteq \mathcal{B}$ whose trajectories tend, as $t \to +\infty$, to a boundary stationary state $\hat{Q} = (0, \bar{N}, \hat{P})$.*
2. *There exists a positively invariant region $\mathcal{C} \subseteq \mathcal{B}$ whose trajectories tend, as $t \to +\infty$, to a sink $\tilde{Q} = \left(\tilde{K}, \bar{N}, \tilde{P}\right)$.*
3. *There exists a positively invariant two-dimensional manifold $\mathcal{T} \subset \mathcal{B}$ whose trajectories tend, as $t \to +\infty$, to a saddle $Q^* = (K^*, N^*, P^*)$.*

The three possible dynamic regimes of system (9) coexist if and only if the system exhibits two stationary states in B, i.e. the saddle Q^* and the sink $\tilde{Q}$. In this case, the stable manifold of Q^* separates the basins of attraction of $\tilde{Q}$ and the boundary stationary state $\hat{Q}$.[4]

The limit boundary point $\hat{Q} = (0, \bar{N}, \hat{P})$, where $\hat{P}$ is the solution of the equation $q\bar{N}^{\beta} - P(1+P)^{\gamma} = 0$, coincides with the unique (globally attractive) stationary state of the one-sector dynamics that would be observed in the absence of the industrial sector. In this case, $K = 0$ and $N = \bar{N}$ and the time evolution of P would be described by the equation:

$$\dot{P} = q\bar{N}^{\beta}(1+P)^{-\gamma} - P \tag{13}$$

Along every trajectory of the three-dimensional system (9) approaching $\hat{Q}$ as $t \to +\infty$, the economy tends (asymptotically) to become specialized in the resource-dependent sector.

Moreover, assume two interior stationary states exist, the saddle Q^* and the sink $\tilde{Q}$, with $0 < K^* < \tilde{K} < \bar{N}$. Then it follows from straightforward computations that $\hat{P} < P^* < \tilde{P}$; that is, the pollution level P in the boundary point $\hat{Q}$ is lower than in the internal stationary states, Q^* and $\tilde{Q}$, when existing.

The numerical simulations illustrated in Figs. 1–3 show all possible phase portraits that can be generically observed in the box $\mathcal{B}$ under the dynamic system (9). Fig. 1 illustrates the case in which only the regime 1 (of the above theorem) is observed in the box $\mathcal{B}$. Analogously, Fig. 2 illustrates the case in which only the regime 2 occurs in the box $\mathcal{B}$. Finally, Fig. 3 illustrates the case in which all regimes take place in $\mathcal{B}$, depending on the initial values of the state variables. In this

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4 It is worth stressing that, in our model, multiplicity of stationary states is due to the existence of environmental negative externalities, differently from Krugman [23] and related works, where the multiplicity of equilibrium paths is due to the existence of positive external economies in production.

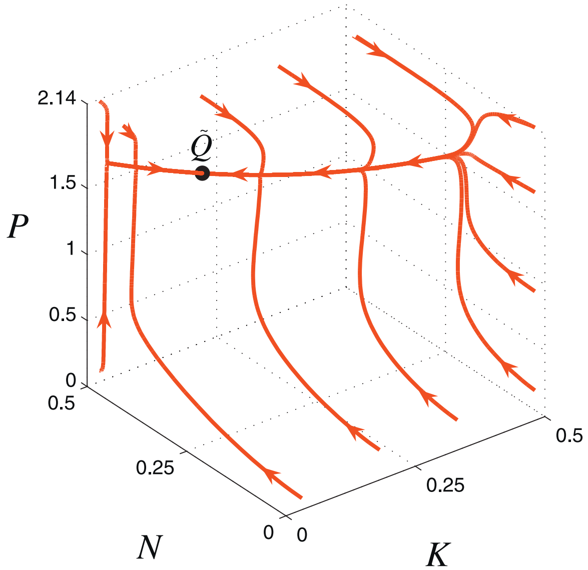

Fig. 2. Example of regime 2 (with $l = 1$, $m = 1$, $\beta = 0.5$, $\gamma = 0.5$, $\delta = 0.06$, $\bar{N} = 0.5$, $q = 4 > r = 1.4$ such that $\varphi(0) = 1 > 0$).

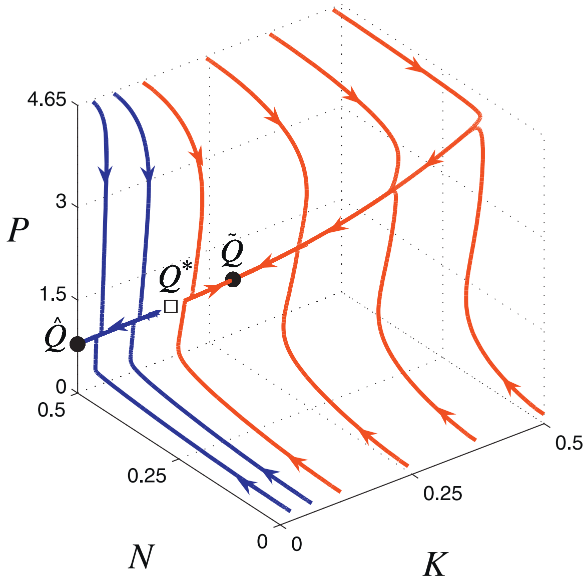

Fig. 3. Example of regime 3 (with $l = 1$, $m = 1$, $\beta = 0.5$, $\gamma = 0.5$, $\delta = 0.06$, $\bar{N} = 0.5$, $q = 1.5 < r = 9$ such that $\varphi(0) = -0.25 < 0$).

latter case, the two-dimensional stable manifold of the saddle Q^* (along which regime 3 occurs) separates the trajectories of regime 1 from those of regime 2.

The following corollary of Theorems 1 and 2 illustrates welfare properties of stationary states.

Corollary 1. *In the context in which there exist two stationary states in $\mathcal{B}$, the saddle Q^* and the sink $\tilde{Q}$, at the boundary attractor $\hat{Q}$*[5] *the revenues of the workers employed in the F-sector are higher than at the sink $\tilde{Q}$, where they are equal to those of the I-sector. The opposite holds when there is exactly one stationary state in $\mathcal{B}$, the sink $\tilde{Q}$ (which implies that the boundary point $\hat{Q}$ is not attracting).*

Proof. It follows from the above theorems that a region $\mathcal{A}$ whose trajectories tend to the boundary point $\hat{Q}$ exists if and only if $\varphi(0) < 0$ or $\varphi(0) = 0$ and $\varphi'(0) \le 0$. In the former case, we have $\varphi(0) = 1 + q\bar{N} - \bar{N}^{\frac{\beta-1}{\gamma}} < 0$, which implies, as it

[5] Remember that such point corresponds to the unique (globally attractive) stationary state of the one-sector dynamics (13) that would be observed in absence of the industrial sector.

is easily checked, $\hat{P} > q\bar{N}$, where $\hat{P}$ is the solution of the equation $q\bar{N}^{\beta} - P(1+P)^{\gamma} = 0$. Hence, replacing in the previous equation the first P by $q\bar{N}$, we get $\bar{N}^{\beta-1} - (1+\hat{P})^{\gamma} > 0$, i.e. $\bar{N}^{\beta-1}/(1+\hat{P})^{\gamma} > 1$. In other words, at the boundary attractor $\hat{Q}$, the revenues of the workers employed in the F-sector are higher than at the possible sink, where the wage rate is, by the equilibrium conditions, equal to 1. Vice-versa, when $\hat{Q}$ is not attracting, which implies $\varphi(0) \geq 0$, we get $\bar{N}^{\beta-1}/(1+\hat{P})^{\gamma} \leq 1$ (the strict inequality holding if $\varphi(0) > 0$). □

In our model, workers' welfare is measured by the wage in the I-sector and by the average product in the F-sector. The results of the global dynamics described above shed light on the evolution of these two magnitudes. In particular, the following cases occur. If the dynamic system exhibits two stationary states (which necessarily are a sink and a saddle), then the stable manifold of the saddle is a two-dimensional surface separating two regimes. In one of them the trajectories tend to a boundary state where the industrial sector disappears. Along such trajectories the workers' wage in the I-sector and the average output in the F-sector tend to the same final value $\alpha\bar{N}^{\beta-1}\left(1+\bar{P}\right)^{-\gamma}$, except that eventually the industrial workers will disappear. Vice-versa, in the other regime all the trajectories tend to a stationary state where the two sectors coexist and all the workers receive the same revenue. However such common revenues are lower than what the traditional workers obtain in the former regime once the industrial sector has disappeared. In such a context, the expansion of the I-sector, at the expense of the F-sector, can be classified as a perverse structural change, in the sense of López [27]; namely, a structural change associated with growing problems of environmental degradation, declining or stagnant wages and perpetuation of poverty. The situation is different if the system exhibits exactly one stationary state, which is necessarily a sink. In that case all the trajectories in the open prism converge to the sink, where, as above, the two sectors coexist and all the workers receive the same wage. The boundary stationary state with no industrial activity still exists, although it is no more attractive, but the traditional workers' revenues in such a state are lower than those perceived by workers of both sectors in the sink (then in such case the possible structural change produces an increase in workers' welfare). Finally, in the case of no interior stationary state, all the trajectories are shown to converge to the boundary state with no industrial activity; but in that case no comparison is really possible, as the industrial sector tends from all the initial situations to disappear. Clearly, what are compared above are asymptotic situations: the way they are reached by depends of course (in a three-dimensional model) on the initial conditions. So, along a trajectory both the wage in the I-sector and the average product in the F-sector, or only one of them, can increase or decrease, depending on their initial levels, but eventually they will tend to a same value, and in the case these asymptotic values are more than one they can be compared, as recalled above.

The following theorem gives a further insight about the global dynamics of system (9). Remember that the three possible dynamic regimes of system (9) coexist if and only if the system exhibits two stationary states in $\mathcal{B}$, i.e. the saddle Q^* and the sink $\tilde{Q}$. In this case, the stable manifold of Q^* separates the basins of attraction of $\tilde{Q}$ and the boundary stationary state $\hat{Q}$. Then it becomes interesting to know more about the shape of such two-dimensional surface. To this end we prove the following.

Theorem 3. *Assume system (9) has two stationary states in $\mathcal{B}$, a saddle $Q^* = (K^*, N^*, P^*)$ and a sink $\tilde{Q} = \left(\tilde{K}, \tilde{N}, \tilde{P}\right)$. Then in a neighborhood of Q^* the stable manifold of Q^* is given by the graph of a smooth function $P = \varphi(K, N)$, where (K, N) belongs to a region S, in the plane (K, N), containing (K^*, N^*). As a consequence, for each $(K_0, N_0) \in S$, there exists exactly one value $P_0^T = \varphi(K_0, N_0)$ such that: a trajectory starting from (K_0, N_0, P_0), with $P_0 < P_0^T$, tends to the boundary point $\hat{Q}$; a trajectory starting from (K_0, N_0, P_0), with $P_0 > P_0^T$, tends to the sink $\tilde{Q}$; the trajectory starting from $\left(K_0, N_0, P_0^T\right)$ tends to the saddle Q^*.*

Proof. See Appendix A.3 □

Fig. 4 shows trajectories starting from the same initial values of K and N, but different initial values of P, approaching either the boundary point $\hat{Q}$ or the sink $\tilde{Q}$.

According to Theorem 3, when two stationary states exist in $\mathcal{B}$, the initial pollution level $P(0)$ may play a crucial role, given the initial values $K(0)$ and $N(0)$ of the other two variables, in determining equilibrium selection. In fact, if $K(0)$ and $N(0)$ satisfy the conditions of the theorem, then there exists a threshold value P_0^T such that, starting from $(K(0), N(0), P(0))$, the economy converges to $\hat{Q}$ (where it becomes specialized in the F-sector) if $P(0) < P_0^T$, while it converges to $\tilde{Q}$ (where the economy gets industrialized) if $P(0) > P_0^T$. A higher initial level $P(0)$ of pollution implies lower labor productivity in the F-sector; therefore, in our model, low productivity of labor in the resource-dependent sector is the engine of the industrialization process. Such a feature is shared with the well-known theoretical literature on the "curse of natural resources", which has focused on various mechanisms through which the abundance of environmental resources may inhibit growth processes (for a review, see van der Ploeg [40]). Most current explanations for the curse of natural resources have a crowding-out logic (see Sachs and Warner [37]): natural resources crowd-out the activity of a sector X; the activity of sector X drives growth; therefore, natural resources harm growth. Sachs and Warner [35,36] identify sector X with traded-manufacturing activities; in Gylfason et al. [20] and Gylfason [19], sector X represents education, and so on. In all this literature, the expansion of sector X is always desirable. Indeed, given that it does not generate negative externalities, it always fuels economic growth and leads to an increase in the welfare of economic agents. On the contrary, in the model of the present paper, the development of sector X (the I-sector) may be welfare reducing.

Theorem 3 affirms that in a neighborhood of the saddle Q^* the stable manifold of Q^* can be interpreted as the graph of a function $P = \varphi(K, N)$. Moreover, it follows from the proof of the theorem in Appendix A.3 that such a manifold, in a neighborhood of Q^*, can also be seen as the graph of a function $N = \psi(K, P)$. Hence, if (K_0, P_0) is sufficiently close to (K^*, P^*),

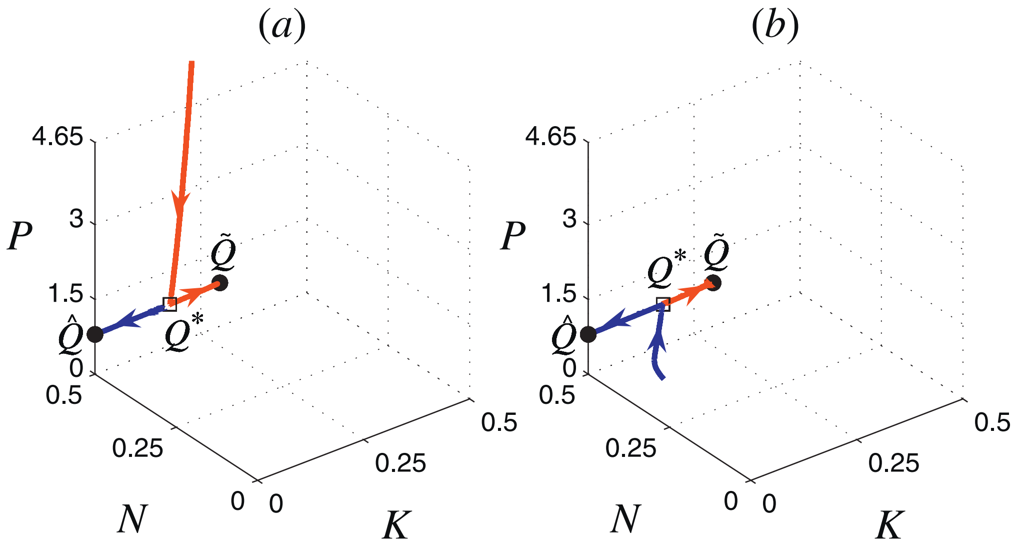

Fig. 4. Two examples illustrating Theorem 3: (a) the blue trajectory starts from $(K_0, N_0, P_0) = (K^* - 0.01, \bar{N} - K_0, N_0^{-1} - 1)$, whereas the red trajectory from $(K_0, N_0, 28)$; (b) the red trajectory starts from $\left(K_0', N_0', P_0'\right) = (K^* + 0.01, \bar{N} - K_0', N_0'^{-1} - 1)$, whereas the blue trajectory from $\left(K_0', N_0', 0\right)$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

there exist a $\delta > 0$ and a function $N_0^T = \psi(K_0, P_0)$ such that: a trajectory starting from (K_0, N_0, P_0), with $N_0^T < N_0 < N_0^T + \delta$, tends to the boundary point $\hat{Q}$; a trajectory starting from (K_0, N_0, P_0), with $N_0^T - \delta < N_0 < N_0^T$, tends to the sink $\tilde{Q}$; the trajectory starting from $\left(K_0, N_0^T, P_0\right)$ tends to the saddle Q^*. Consequently, also the initial value $N(0)$ of N may play a role in equilibrium selection. So, in the case in which the boundary point $\hat{Q}$ is attractive, the convergence to the interior sink $\tilde{Q}$ from a point close to the saddle Q^* can be considered as the consequence of a coordination failure of workers.

4. Concluding remarks

Our results on workers' wages are strictly linked to the assumption that the industrialization process generates negative environmental externalities, but not positive ones. In the case in which both types of externalities condition the dynamics of the economy, it may happen that negative externalities – through the mechanism analyzed in our paper – lead economic agents towards a better exploitation of positive externalities. Obviously, in such a context, the effect of positive externalities may counterbalance the effect of negative externalities.

Our model aims to highlight some undesirable scenarios that could be observed in an economy in which environmental regulation is not effective. Such a pessimistic scenario is often observed in developing countries, where ineffective environmental policies play a very relevant role in shaping economic dynamics. López [27] points out that indirect factors capable to prompt a welfare reducing structural change are inadequate policies aiming at fostering productivity in the modern sector in addition to a complete neglect of traditional subsistence sector of the rural poor. Although our conceptual framework is too simple fully to catch all dynamic aspects of the growth paths of developing countries, we believe that some of the latter are consistent with the narratives behind the model we have proposed and this encourages further research along the lines suggested in this paper.

Appendix A

A1. Local analysis

In order to prove Theorem 1, assume $Q_0 = (K_0, N_0, P_0) \in \mathcal{B}$ is a stationary state of system (9). Then it is easily computed that $\varphi(K_0) = 0$. Moreover, $\lim_{K \to \bar{N}} \varphi(K) = -\infty$ and $\varphi''(K) < 0$ as $K \in (0, \bar{N})$. It follows that the stationary states in $\mathcal{B}$ are at most two, according to the conditions stated in the theorem. Furthermore, the Jacobian matrix $J(Q_0)$ is given by

$$J(Q_0) = \begin{pmatrix} -l\delta & -l\delta & 0 \\ \dfrac{-m(1-\delta)}{K_0} & \dfrac{-m(1-\delta)}{K_0} - \dfrac{m(1-\beta)}{N_0} & \dfrac{-m\gamma}{1+P_0} \\ r(1-\delta) & -r\delta + q\beta & \dfrac{-q\gamma N_0}{1+P_0} - 1 \end{pmatrix}$$

It follows, by easy computations, that $sign[\det(J(Q_0))] = sign[\varphi'(K_0)]$ and $\mathrm{tr}(J(Q_0)) < 0$. Therefore, if $\varphi'(K_0) > 0$, Q_0 is a saddle with a two-dimensional stable manifold. If, instead, $\varphi'(K_0) < 0$, then the characteristic polynomial of $J(Q_0)$ is given by

$$-\lambda^3 + \mathrm{tr}(J)\lambda^2 - \sigma(J)\lambda + \det(J)$$

and it is easily calculated that

$$|\mathrm{tr}(J)| \cdot \sigma(J) > |\det(J)|$$

Therefore, the Routh–Hurwicz conditions yield that Q_0 is a sink. This completes the proof of the theorem.

A2. Global analysis

In order to illustrate the global dynamics generated by system (9) in the box $\mathcal{B}$, first of all we prove the following lemma.

Lemma 1. *Assume $\varphi(0) < 0$. Let $\mathcal{A} \subseteq \mathcal{B}$ be a positively invariant region without stationary states and suppose $\hat{Q} = (0, \bar{N}, \hat{P}) \in \partial A$,[6] where $\hat{P}$ is the solution of the equation $q\bar{N} - P(1+P)^{\gamma} = 0$. Then all the trajectories starting in $\mathcal{A}$ tend, as $t \to +\infty$, to $\hat{Q}$. Moreover, if $\mathcal{A} \subset \mathcal{B}$, then the boundary of $\mathcal{A}$ contains the two-dimensional stable manifold of the saddle Q^*.*

Proof. Given the assumptions of the lemma, if, by contradiction, a trajectory $\Gamma(t) = (K(t), N(t), P(t))$ starting in $\mathcal{A}$ does not converge to $\hat{Q}$, then it keeps oscillating. In particular $K(t)$ will reach a maximum, say, at t_1. Then $\ddot{K}(t_1) \le 0$ implies $\dot{N}(t_1) \ge 0$. In fact, since the existence of oscillating trajectories is an open condition, we can assume $\dot{N}(t_1) > 0$. Hence $N(t)$ would, in turn, reach a maximum before $K(t)$ reaches a minimum. Suppose that this occurs at $t_2 > t_1$. Then, as $\dot{K}(t_2) < 0$, $\ddot{N}(t_2) \le 0$ implies $\dot{P}(t_2) > 0$. Therefore we can set $t_2 = 0$, so that, in a right neighborhood of $t = 0$, $\dot{K}(t), \dot{N}(t) < 0$, $\dot{P}(t) > 0$. Now, consider, as above, the function $P(K, N)$ implicitly defined by $\dot{P} = f(K, N, P) = 0$ when $(K, N) \in (0, \bar{N})^2$, which can be continuously extended to the closed square $[0, \bar{N}]^2$. Then it is easily checked that, for any $K_0 \in (0, \bar{N})$, the graph of $P(K_0, N)$, $N \in [0, \bar{N}]$, has a parabolic shape, with $P(K_0, 0) = \bar{N}^{\delta} K_0^{1-\delta}$ and $P(K_0, \hat{N}) = \hat{P}$. The maximum value $P_{\mu}(K_0)$ is given by the solution of the system $f(K_0, N, P) = \frac{\partial f}{\partial N}(K_0, N, P) = 0$: hence it is easily checked that $P_{\mu}(K_0)$ is increasing with K_0 and $\lim_{K \to 0^+} P_{\mu}(K_0) = \hat{P}$. On the other hand, set $P_{\nu}(K_0) = (\bar{N} - K_0)^{\frac{\beta-1}{\gamma}} - 1$, $0 < K_0 < \bar{N}$, i.e., $P_{\nu}(K_0)$ is the P -coordinate of the intersection $\{\dot{K} = \dot{N} = 0, K = K_0\}$. Clearly $\frac{dP_{\nu}}{dK_0} > 0$. Since we assumed $\varphi(0) < 0$, it follows from straightforward computations that $\lim_{K \to 0^+} P_{\nu}(K_0) = P_{\nu}(0) = (\bar{N})^{\frac{\beta-1}{\gamma}} - 1 > \hat{P}$. Hence, let us go back to the trajectory $\Gamma(t) \subset \mathcal{A}$ for $t \ge 0$. Then, if K_0 is sufficiently small, $P_{\mu}(K_0) < P_{\nu}(0)$. It follows that, since $\dot{K}(t) < 0$ in a right neighborhood of $t = 0$, a possible maximal value of $P(t)$, say $P(t^*)$, $t^* > 0$, will satisfy $P(t^*) < P_{\nu}(0) < P_{\nu}(K^*)$. Therefore $K(t)$ keeps decreasing and in fact this implies $\lim_{t \to +\infty} K(t) = 0$.[7] Consequently $\lim_{t \to +\infty} N(t) = \bar{N}$ and $\lim_{t \to +\infty} \dot{N}(t) = 0$, so that, finally, $\lim_{t \to +\infty} P(t) = \hat{P}$. Therefore we have proven that, if $Q_0 = (K_0, N_0, P_0) \in \mathcal{A}$ and K_0 is small enough, then the trajectory through Q_0 converges, as $t \to +\infty$, to $\hat{Q}$. Now, suppose that the trajectory starting at some $Q_0 \in \mathcal{A}$, with, as above, $\dot{K}(Q_0) < 0$, $\dot{N}(Q_0) = 0$, $\dot{P}(Q_0) > 0$, reaches $\dot{K} = 0$ before $\dot{N} = 0$. Then, by the continuous dependence of the solutions from the initial conditions, there must be some Q_0 whose trajectory reaches a point $Q^* \in \{\dot{K} = 0\} \cap \{\dot{N} = 0\}$. Moreover, again by a continuity argument,[8] $\dot{P}(Q^*) \le 0$. But if $Q^* \in \mathcal{A}$, then it cannot be a stationary state. Hence, if the trajectory reaches Q^* in a finite time t^*, $\dot{P}(Q^*) < 0$, so that, as it is easily checked, in a right neighborhood of t^*, $K(t)$ keeps decreasing, and so on, implying that such a trajectory, and the nearby ones, converge to $\hat{Q}$. In fact, in order to have a trajectory not converging to $\hat{Q}$, the above Q^* had to be reached in infinite time. In other words, it had to be a stationary state and, precisely, a saddle. This completes the proof of the lemma.[9] □

The proof of the next lemma requires to *stretch* the box where system (9) is studied. Precisely, we consider the box $\mathcal{B}' = \{K, N \in (0, \bar{N}), P \in (0, +\infty)\}$. Clearly $\mathcal{B}' \supset \mathcal{B}$ and is positively invariant as well, with respect to (9). Hence we state

Lemma 2. *Consider system (9) defined in $\mathcal{B}'$. Then there exist trajectories lying in $\mathcal{B}'$ for all $t \le 0$ and tending, as $t \to -\infty$, to the boundary point $Q_{\infty} = (\bar{N}, 0, +\infty)$.*

We omit the rather technical proof, which can be found in Antoci et al. [1]. However, we observe that, although Lemma 2 is preparatory to proving next Lemma 3, it possesses an economical content as well. In fact, it affirms that we can always consider as a possible starting point for the dynamics of our economy a situation where the labor force employed in the traditional sector is (nearly) extinguished, the capital stock level is maximum and the pollution is indefinitely high.

[6] A positively invariant region is an open connected set such that all the trajectories starting in it remain there for all $t \ge 0$. By $\partial\mathcal{A}$ we denote the boundary of $\mathcal{A}$.

[7] It can be checked that $\lim_{t \to T^-} K(t) = 0$ implies $T = +\infty$.

[8] Q^* can be considered the limit of a sequence of points Q_n such that $\dot{K}(Q_n) > 0$, $\dot{N}(Q_n) = 0$, $\dot{P}(Q_n) < 0$.

[9] It can be shown that in case $\varphi(0) > 0$ (or $\varphi(0) = 0$ and $\varphi'(0) > 0$) no trajectory in $\mathcal{B}$ can tend, as $t \to +\infty$, to $K = 0$.

Finally Lemma 3 states the following

Lemma 3. *Let $\mathcal{C} \subseteq \mathcal{B}$ be a positively invariant region containing exactly one stationary state, that is, the sink $\tilde{Q} = \left(\tilde{K}, \tilde{N}, \tilde{P}\right)$. Then all the trajectories starting in $\mathcal{C}$ tend, as $t \to +\infty$, to $\tilde{Q}$.*

Proof. The full proof of this lemma is given in Antoci et al. [1]. Here we just mention the main idea behind the proof. Assume, by contradiction, there exists in the region $\mathcal{C}$ described in the lemma's statement some ω-limit set Σ different from the sink $\tilde{Q}$. Then Σ is a compact set both positively and negatively invariant, whose trajectories are *oscillating.* Hence we reverse the time, i.e. we pose $\tau = -t$, and we show that we can choose $Q_0 = (K_0, N_0, P_0) \in \Sigma$ such that, deriving with respect to τ, $\dot{K}(Q_0) > 0$ and $\dot{N}(Q_0) < 0$. Hence the *negative* trajectory starting from Q_0, say $\alpha(\tau)$ with $\tau = -t$, should remain in $\mathcal{C}$ for all $\tau \in (0, +\infty)$. Vice-versa, we prove that, if $\alpha(\tau)$, with the above assumptions, remains in $\mathcal{B}' \supset \mathcal{C}$ for all $\tau \in (0, +\infty)$, then $\lim_{\tau \to +\infty} \alpha(\tau) = Q_\infty = \left(\bar{N}, 0, +\infty\right)$, which doesn't even belong to the closure of $\mathcal{C}$, thus reaching a contradiction.

Moreover, it can be seen that, if $\mathcal{C} \subset \mathcal{B}$, then the two-dimensional stable manifold of the saddle Q^* is part of the boundary of $\mathcal{C}$. □

A3. Stable manifold

In order to prove Theorem 3, let us consider the plane π of equation $N + K = \bar{N}$, intersecting the box $\mathcal{B}$ in a rectangle $\mathcal{R}$, where we choose P and N as coordinates, so that $\mathcal{R} = \left(0, \bar{P}\right) \times \left(0, \bar{N}\right)$. It follows from the proof of Lemma 1 that, when N is sufficiently high (hence K is sufficiently low), the trajectories from the corresponding strip of $\mathcal{R}$ tend to $\hat{Q}$. Hence the intersection of $\mathcal{R}$ with the stable manifold of Q^*, which separates the above two regimes, is a *curve* Γ contained in a strip $\left\{N_1' < N < N_2' / 0 < N_1' < N^* < N_2'\right\}$. Moreover, it follows again from the proof of Lemma 1 that, when $N_0 > N^*$, a trajectory from $\left(\bar{N} - N_0, N_0, P_0\right)$, with $P_0 \le N_0^{\frac{\beta-1}{\gamma}} - 1$, tends to $\hat{Q}$. On the other hand, it is easily computed that for $N_0 > N^*$, $K_0 = \bar{N} - N_0$, $P_0 > N_0^{\frac{\beta-1}{\gamma}} - 1$, $\dot{N}(K_0, N_0, P_0)$ and $\dot{P}(K_0, N_0, P_0)$ are < 0, while for $\tilde{N} < N_0 < N^*$, $K_0 = \bar{N} - N_0$, $P_0 < N_0^{\frac{\beta-1}{\gamma}} - 1$, $\dot{N}(K_0, N_0, P_0)$ and $\dot{P}(K_0, N_0, P_0)$ are > 0.

Suppose, now, that, near Q^*, Γ is the graph of a decreasing function $N(P)$, so that $\frac{dN}{dP} < 0$ as P lies in a left neighborhood of P^*. Consider a closed *tract* Γ' of Γ where that occurs, $Q^* \notin \Gamma'$. Pose $H = N + K$ and take $Q_0 = (H_0, N_0, P_0) \in \Gamma'$ (hence $H_0 = \bar{N}$). To fix the ideas, we can assume Q_0 is the end-point of Γ' with the higher N. Recalling that on π $\dot{K} = 0$ and therefore $\dot{N} < 0$ implies $\dot{H} < 0$, consider a sufficiently small box $\mathcal{B}' = [H_0 - a, H_0] \times [N_0 - b, N_0] \times [P_0, P_0 + c]$, $a, b, c > 0$. The intersection of the side $\{P = P_0\}$ of $\mathcal{B}'$ with the stable manifold of Q^*, $\mathcal{T}$, is a curve α which can be parametrized by $N \in [N_0 - b, N_0]$. In fact, being T invariant (i.e., constituted by trajectories), along such a curve α $\frac{dH}{dN} = \frac{\dot{H}(H,N,P_0)}{\dot{N}(H,N,P_0)}$. Moreover, if $\mathcal{B}'$ is small enough, there exist $r_1 > r_2 > 0$ such that, in $\mathcal{B}'$, $-r_1 \le \dot{N}$, $\dot{P} \le -r_2$. Take now $Q_0' = \left(H_0, N_0, P_0'\right)$, with $P_0' = P_0 + \delta$, $\delta > 0$ being sufficiently small (so that Q_0' belongs to the basin of $\tilde{Q}$). Then the trajectory from Q_0' stays for a certain time in B', and in fact we can parametrize it too by N, so that it is represented by a curve $\gamma(N) = (H(N), N, P(N))$, $N_0 - b \le N \le N_0$. Then, along $\gamma(N)$, $\frac{dP}{dN} \ge \frac{r_2}{r_1} > 0$. Moreover, being $\dot{N} < 0$, it is easily computed that, for given H and N, $\frac{dH}{dN} = 1 + \frac{\dot{K}}{\dot{N}}$ is higher for a higher P . So, if δ is sufficiently small, the trajectory γ will reach the side $\{P = P_0\}$ of $\mathcal{B}'$ for some pair $(H_1, N_1) \in (H_0 - a, H_0) \times (N_0 - b, N_0)$. Should the trajectory remain "to the right" of T, then, for what we have noticed, the corresponding pair $(H_1, \hat{N}_1)$ on $\alpha = T \cap \{P = P_0\}$ would satisfy $\hat{N}_1 > N_1$. It follows that, for our choice of a sufficiently small $\mathcal{B}'$, we would have a point (H_1, N_1, P_1) on T with $P_1 > P_0$, leading to a contradiction, since we have supposed that γ lies "to the right" of T. Hence γ reaches the side $\{P = P_0\}$ of $\mathcal{B}'$ by intersecting the invariant manifold T, which again leads to a contradiction.

It follows that, for P lying in a right neighborhood of P^*, the points in $\mathcal{R}$ of the curve $C = \{N = (1+P)^{\frac{\gamma}{\beta-1}}, N \ge \tilde{N}\}$ (which is the graph of a decreasing function) lie to the right of Γ, and thus belong to the basin of attraction of $\tilde{Q}$. If it happened that Γ crossed again the curve C for some $P > P^*$, then there should exist, as it is easily observed, a *tract* of Γ where $\frac{dN}{dP} < 0$ with $P > P^*$. Again, by an argument analogous to the previous one, we are led to a contradiction, considering the trajectory from a point Q_0' sufficiently close to this *tract* and lying to the left of Γ (so that $\dot{N}\left(Q_0'\right)$, $\dot{P}\left(Q_0'\right) > 0$). Hence the curve C, for $P > P^*$, belongs to the basin of $\tilde{Q}$.

Suppose, now, that there exists a *tract* Γ' of Γ such that, for $P \in [P_1, P_2]$, $P_1 > P^*$, Γ' is the graph of a function $N(P)$ satisfying $\frac{dN}{dP} < 0$.[10] Hence, along Γ', $\dot{N}$, $\dot{P} < 0$ and, as above, $-r_1 \le \dot{N}$, $\dot{P} \le -r_2$ for suitable $r_1 > r_2 > 0$. Then we can consider a point Q_0' sufficiently close to Γ' lying to the right of Γ (i.e., for our assumption, in the basin of attraction of $\hat{Q}$). Again, it follows that the trajectory from Q_0' would *reach* the stable manifold of Q^* within a finite time, leading to a contradiction.

The same argument can be applied to rule out the existence of a *tract* Γ' of Γ where $\frac{dN}{dP} < 0$ as $P < P^*$ and $\tilde{N} < N < N^*$ (in this case Q_0' can be chosen as lying to the left of Γ', so that $\dot{N}\left(Q_0'\right)$, $\dot{P}\left(Q_0'\right) > 0$).

[10] Observe that, since $q < r$ (as the existence of two stationary states implies), the curve $\{\dot{P} = 0\} \cap \mathcal{R}$ is represented, near Q^*, by the graph of a function $N(P)$ such that $\frac{dN}{dP}(P^*) < 0$.

In conclusion, Γ is the graph of an increasing function $N(P)$, hence $P(N)$, in a strip $(N_1, N_2) \subset \mathcal{R}$, where $N_1 \leq \tilde{N} < N^* < N_2$, which implies that, for (K, N) belonging to a suitable region $\mathcal{S}$, the stable manifold of Q^* can be represented as the graph of a function $P(K, N)$.

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