Review of Economic Dynamics 54 (2024) 101232

Contents lists available at ScienceDirect

Review of Economic Dynamics

journal homepage: www.elsevier.com/locate/red

Full Length Article

Public wages, public employment, and business cycle volatility: Evidence from U.S. metro areas ☆

Claire A. Boeing-Reicher, Vincenzo Caponi [a,b,c,*]

[a] University of Sassari, Sassari, Italy
[b] CRENOS, Cagliari-Sassari, Italy
[c] IZA, Bonn, Germany

ARTICLE INFO

JEL classification:
E6
E32
J64

Keywords:
Job search
Public employment
Public wages
Business cycle
Volatility

ABSTRACT

We revisit the question about whether a larger public sector stabilizes or destabilizes the economy. Based on results from two causal identification approaches, we show that a higher rate of public-sector employment reduces volatility in, i.e. stabilizes, private-sector employment growth, with at most a slight crowding-out of private employment. Public wages, meanwhile, increase private wages but appear not to be destabilizing. The stabilizing effect of public employment with limited crowding out is at odds with standard search and matching models that contain a public sector, which predict 1:1 crowding out and strong destabilization. To improve the performance of such models, we follow Gomes (2015) and add a product market that can replicate what Gomes calls the Business Cycle Wealth Effect. We also point out that the government procures output directly from the private sector. When the model has these two features, then it can generate stabilizing effects of public employment on private employment, with reduced crowding out.

1. Introduction

Standard search and matching models predict that the presence of a public sector should increase the volatility of public employment and output. To see how well these predictions line up with measurement, we set up a panel dataset at the U.S. MSA level with which we measure causal relationships among public wages, public employment, and business cycle volatility. We find results that are puzzling from the perspective of a standard search and matching model. Specifically, based on causal analyses using an SVAR and shift-share instruments, we find that an increase in long-run public-sector employment crowds out private-sector employment to a limited degree, reduces private wages, and results in less volatility in private employment over the business cycle. Meanwhile, higher public wages ‘crowd in’ private wages, with an ambiguous effect on the level of and volatility in private employment over the business cycle. The results on volatility are at odds with the implications of standard search and matching models augmented with public employment, as formulated by Quadrini and Trigari (2007) and subsequent researchers. These models predict a strong destabilizing effect of both public employment and public wages on private employment, because a higher rate of public employment improves the outside option of private-sector workers, and a higher outside option leads to higher volatility via mechanisms shown

☆ We wish to thank seminar participants in Kiel, Rennes, and the University of Canterbury. We also wish to thank Antonio Afónso, Pedro Gomes, and the participants at the UECE Conference on Economic and Financial Adjustments, the 2016 Kyoto meeting of the Econometric Society, and the 2016 Taipei International Conference on Growth, Trade and Dynamics. Also, this work has been financed in part by Fritz Thyssen Foundation grant Az.10.15.1.015IB (Boeing-Reicher).

* Corresponding author at: University of Sassari, Sassari, Italy.
E-mail address: vcaponi@uniss.it (V. Caponi).

https://doi.org/10.1016/j.red.2024.05.001
Received 12 September 2018; Received in revised form 11 April 2024

Available online 21 May 2024

by Hagedorn and Manovskii (2008). To solve this puzzle, we show that adding several product market features to the model helps it to generate a negative effect of public employment on volatility. These product market features consist of allowing for income effects in the manner of Baxter and King (1993) and Gomes (2015), and accounting for the fact that the public sector sources much of its consumption from the private sector, which results in a demand spillover between the public and private sector that further stabilizes the outside option.

The standard search model generates a counterfactually positive effect of public employment on volatility through the following mechanism. Under both random and directed search, the public sector acts as a competitor to the private sector in the labor market, so that higher public wages or a greater chance of finding a public-sector job cause the value of unemployment for a private-sector worker to rise. This should crowd out private employment and make private employment more responsive to productivity shocks (i.e. increase volatility). This occurs because higher public wages or public employment shift the wage curve outward, and higher public wages or public employment also steepen both the wage curve and job creation curve. A steeper wage curve and steeper job creation curve make equilibrium market tightness more sensitive to shocks. This is the same intuition behind the results of Hagedorn and Manovskii (2008), who show that an increase in the flow value of unemployment results in a stronger response of market tightness to shocks. These results are theoretically robust within a standard search and matching framework, but they are at odds with our empirical findings.

To resolve this puzzle, we extend the search and matching model in the following ways. First we follow Gomes (2015) and add a labor-leisure tradeoff, which Gomes calls Business Cycle Wealth Effect. This effect is the same mechanism introduced by Baxter and King (1993) into RBC models, whereby the flow value of unemployment and thus the worker's outside option are governed by an income effect. This results in an outside option that increases with private consumption and is thus procyclical. Since a larger public sector crowds out private consumption, a larger public sector results in a lower outside option, which has a stabilizing effect in line with Hagedorn and Manovskii (2008). However, this effect by itself cannot explain the negative effect of the public sector on volatility that we see in the data. In that light, we note that a significant portion of government consumption comes from intermediate purchases from the private sector. If we allow for some public consumption to come from the private sector, then a larger public sector crowds out private consumption to a larger degree than before, which amplifies the stabilizing effect of the public sector. These two features together are sufficient to generate a negative relationship between the size of the public sector and volatility. Based on these results, we argue that it is important to take the structure of government consumption and product markets into account when looking at the effects of public wages and public employment on the private sector.[1]

Our results add to a growing literature which analyzes the interactions among public wages, public employment, and their private counterparts. Most of this literature focuses primarily on understanding the direct effects of public employment and wages on labor markets. For instance, Quadrini and Trigari (2007) set up a search and matching model on which we base our approach. They find that high public wages or public employment should drive up volatility in private employment and in total employment. Taking a different approach, Burdett (2012) sets up a model similar to that of Burdett and Mortensen (1998) but with a public sector. Burdett's model predicts that public employment and public wages should crowd out private employment. More recently, Afonso and Gomes (2014) set up a similar model to that of Quadrini and Trigari (2007). They find that high public wages should crowd out private employment while raising private wages, while public employment should crowd out private employment (more than one-to-one) while also raising private wages. All of these results regarding levels of employment and wages are compatible with our search and matching model without product markets, based on the same intuition governing these models. Building on this intuition, Gomes (2015) examines the possibility of using procyclical public wages to stabilize business cycles, in a model that contains product markets. This result is driven by the search and matching model's prediction that public wages crowd out private employment. Furthermore, Bradley et al. (2015) set up a model with worker-level heterogeneity, which they estimate using British data. They go on to analyze a number of fiscal consolidation scenarios, with consolidation 'crowding in' private consumption. Meanwhile, Albrecht et al. (2015) use a similar model to discuss the cross-sectional behavior of wages in Colombian data, and Caponi (2017) analyzes public wage and employment policies in Italy, finding that differences in public wage and employment levels (vs. private productivity) can account for a large portion of the north-south unemployment differential.

While we focus on search and matching models, our empirical results also address an older DSGE literature on the effects of government policies on volatility. Galí (1994), Fatás and Mihov (2001), Andrés et al. (2008), Reicher (2014), and others find that government spending and some of its components are negatively correlated in the data with volatility in both private output and overall output, and these results echo our results on public employment. However, in standard theoretical models of the business cycle without public employment, it is difficult (but possible) to generate a stabilizing effect of government consumption on private output. For instance, Andrés et al. (2008) point out that a medium-scale DSGE model with sticky prices and capital adjustment costs can generate a stabilizing effect of government spending when rule-of-thumb consumers are inserted into that model, and when that model features a high degree of nominal and real rigidities. In light of this literature, with its emphasis on product markets, our results also show that a richer description of product markets can move theoretical models closer to the data.

2. A panel for US metro areas and our empirical approaches

The empirical questions we want to answer are the following, over the medium to long run:

[1] Both the random and directed search model give similar results to each other with respect to public employment. The one place where they differ is that the random search model generates a positive effect of public wages on private wages, while the directed search model generates an opposite effect.

Table 1
Summary statistics for observables, annual.

	Mean	Std.	Min.	Max.
Log private wage W_p	3.66	0.16	3.26	4.39
Private employment rate E_p	0.478	0.079	0.232	0.690
Log private prod. π	-2.74	0.20	-3.26	-2.03
Log public wage W_g	4.02	0.14	3.72	4.53
Public employment rate E_g	0.0896	0.0422	0.0365	0.3508
Unemployment rate U	0.0672	0.0193	0.0323	0.2108
Population (m)	0.621	1.243	0.060	12.765

This table features the cross-sectional sample mean, standard deviation, minimum, and maximum for the main observables. The dataset in levels spans the period 1969 through 2018, for 382 metro areas. Source: BEA and authors' calculations.

- What is the effect of an increase in public employment on the level of private employment, private wages, and volatility in private employment, holding public wages constant?
- What is the effect of an increase in public wages on the level of private employment, private wages, and volatility in private employment, holding public employment constant?

To answer these questions, we adopt two different causal identification approaches and compare results with each other. The first approach is a panel SVAR with which we measure the effect of a long-run increase in public employment or public wages on our outcome of interest. With that approach we see negative (but small) effects of public employment on private employment and on private wages, reflecting incomplete "crowding out". We also see a strong negative of public employment on volatility in private-sector employment growth. We also see a strong positive effect of public wages on private wages. The second approach is a panel two-stage least squares regression using shift-share (Bartik) instruments to instrument for public employment or public wages at the local level. That approach gives results that are consistent in direction and magnitude with the SVAR, although results from the shift-share model are much less precisely estimated than for the SVAR.

2.1. The research question and the SVAR dataset

We first run a panel structural vector autoregression over all U.S. MSAs using annual data from 1969 through 2018. Our VAR has the following five variables, with the first four variables taken as first differences.[2] Our variables are as follows:

- The year over year change in the public employment rate, which is federal, state, and local government employment divided by population.
- The year over year change in the log public wage, where the public wage is the total compensation of public employees per public employee deflated by the national PCE deflator.
- The year over year change in the private employment rate, which is private employment (employment that is not public) divided by population.
- The year over year change in the log private wage, which is the total compensation of private employees per private employee deflated by the national PCE deflator.
- The change in the square of the year over year change in the log private employment rate, whose long-run average is a measure of volatility in private employment growth.

We also include the following exogenous variables as nuisance variables in our SVAR estimation:

- Dummy variables to represent fixed effects for each MSA, to control for unobserved differences across MSAs.[3]
- Dummy variables to represent fixed effects for each time period to control for common shocks across time. We omit one fixed effect to avoid multicollinearity.

Table 1 shows summary statistics for our data taken in growth rates from 1970 through 2018. We have removed Carson City, Nevada, and Midland, Texas, because they contributed by far the two largest outliers to changes in log private wages. In particular, both MSAs have a large discontinuity in private wage data at the SIC/NAICS cutoff in 2001 and a high amount of volatility otherwise.

[2] Based on unit root tests, the presence or absence of unit roots here is ambiguous–when we do a series of univariate panel unit root tests following the methodology of Levin et al. (2002), our test statistic values cluster on both sides of the 5-percent critical value of 27 that is implied by observing changes in 382 MSAs over 48 years (1971-2018) after taking growth rates and changes in squared growth rates. Given this ambiguity and given that our annualized data appear to be highly persistent, we are running our VAR in first differences.

[3] Our results are robust to excluding MSA dummies; these results are available upon request. Also, given that we are taking first differences in variables that exhibit near-random-walk-like behavior, and given that our sample spans 47 years, we are not concerned with small-sample bias in coefficients on lagged variables.

Table 2
Correlations among variables in our panel SVAR dataset.

	(1)	(2)	(3)	(4)	(5)
(1) Public E rate change	1.00	-0.13	-0.01	-0.03	-0.06
(2) Public W growth	-0.13	1.00	-0.03	0.20	0.00
(3) Private E rate change	-0.01	-0.03	1.00	0.20	0.09
(4) Private W growth	-0.03	0.20	0.20	1.00	0.05
(5) Private E rate growth^2	-0.06	0.00	0.09	0.05	1.00

This table features contemporaneous cross-correlations for the main observables. The dataset in changes/growth rates spans the period 1970 through 2018, for 382 metro areas. Source: BEA and authors' calculations.

Table 3
Summary statistics for our panel SVAR dataset.

Variable	Mean	Std	Min	25 pctile	50 pctile	75 pctile	Max
Public E rate	0.0910	0.0455	0.0300	0.0617	0.0766	0.1046	0.4636
Log public W	-0.7150	0.2519	-1.4154	-0.9045	-0.7263	-0.5269	0.1868
Private E rate	0.4427	0.0899	0.1227	0.3827	0.4428	0.5024	0.7663
Log private W	-0.9605	0.2010	-1.5632	-1.0979	-0.9663	-0.8388	0.2079
Public E rate change	-0.0001	0.0035	-0.0716	-0.0011	-0.0000	0.0010	0.1436
Public W growth	0.0132	0.0256	-0.2240	-0.0009	0.0132	0.0283	0.1917
Private E rate change	0.0032	0.0111	-0.1007	-0.0023	0.0042	0.0097	0.1641
Private W growth	0.0077	0.0327	-0.3059	-0.0087	0.0079	0.0250	0.3411
Private E rate growth squared	0.0008	0.0022	0.0000	0.0001	0.0003	0.0008	0.1556

This table features summary statistics for the main observables. The dataset in changes/growth rates/changes in squared growth rates spans the period 1971 through 2018, for 382 metro areas. Source: BEA and authors' calculations.

That said, our results are not sensitive to the inclusion/exclusion of these two MSAs. Meanwhile, Table 2 shows a 5 by 5 contemporaneous correlation matrix for our analysis dataset. In these data, increases in public employment have a slight negative correlation with changes in private employment, private wages, and squared private employment growth. Meanwhile, increases in public wages are positively correlated with decreases in private employment, increases in private wages, and no change in squared employment rate growth. These correlations are exploratory and not causal, but they are in the same directions as our causal estimates. Table 3 shows the summary of the statistics of the panel data used.

2.2. The SVAR specification and identification

We denote the five endogenous variables of interest as a 5 by 1 column matrix $x_{i,t}$ for MSA i at time t, and the other nuisance variables as z_{it}. Given that information criteria favor one lag, our dynamic system has the following representation:

$$x_{i,t} = Ax_{i,t-1} + Bz_{i,t} + \epsilon_{i,t}, \tag{1}$$

where the reduced-form innovations $\epsilon_{i,t}$ are iid across time and have a covariance matrix such that $E(\epsilon_{i,t}\epsilon_{i,t}') = \Sigma$.

We can also represent our long-run innovations $\eta_{i,t}$ such that $\eta_{i,t} = \Lambda\epsilon_{i,t}$, where $\Lambda = (I - A)^{-1}$. If we split $\eta_{i,t}$ into its first two elements $\eta_{i,t}^{pub}$ corresponding with public employment and wages, and its three remaining elements $\eta_{i,t}^{priv}$ corresponding with private employment, wages, and employment volatility, then the causal effects of public variables on private variables are the coefficients in the following stacked regression:

$$\eta_{i,t}^{priv} = \alpha\eta_{i,t}^{pub} + \xi_{i,t}^{priv}, \tag{2}$$

where the coefficients α are a 3 by 2 matrix representing the effects of long-run public employment and wages on long-run private employment, wages, and employment volatility; and $\xi_{i,t}^{priv}$, represents a set of shocks to private employment, wages, and employment volatility that is orthogonal to $\eta_{i,t}^{pub}$. Since $\eta_{i,t}^{pub}$ is likely to respond to private employment variables in the long run, it does not suffice to estimate this regression by OLS–instead, we need some ancillary identifying assumptions.

Our identifying assumptions are as follows. Following Blanchard and Perotti (2002), we assume that the first two elements of $\epsilon_{i,t}$, which we denote as $\epsilon_{i,t}^{pub}$, are set by public authorities via a slow-moving budgeting and hiring process. This implies that public authorities set their employment and wage levels before they observe the realizations of the elements of $\epsilon_{i,t}$ corresponding with private-sector variables. For this assumption to be true in annual data, we must assume no year-ahead anticipation effects of the sort discussed by Ramey (2011, 2016).[4] If the Blanchard-Perotti assumptions are true, then we can also treat $\epsilon_{i,t}^{pub}$ as orthogonal to $\xi_{i,t}^{priv}$,

[4] Ramey shows that anticipation effects are important but mainly operate within a subannual horizon. Further, the main anticipation effect in Ramey's dataset pertains to the Korean War, which is not in our dataset.

in which case $\epsilon_{i,t}^{pub}$ is a valid instrument for $\eta_{i,t}^{pub}$. If that instrument is also relevant, then we can obtain a consistent estimate α by using a two-stage least squares (2SLS) regression.

We calculate our coefficients and standard errors using 'standard' first-stage and second-stage regression outputs using standard 2SLS formulas, though the nominal standard errors given by 2SLS are too small, because both our left-hand-side and right-hand-side variables are themselves estimated quantities. To correct for this downward bias in standard errors, we also present GMM standard errors based on the delta method following Lütkepohl (1990), with a variance-covariance matrix of our estimate $\hat{\alpha}$ given by:

$$Cov(vec(\hat{\alpha}')) = \delta Cov(h)\delta', \tag{3}$$

where the stacked coefficients $\hat{\beta} = [\hat{A}', \hat{B}']'$ are given by their usual OLS formula of $(X'X)^{-1}X'Y$ where X and Y are stacked data matrices; $\hat{\Sigma}$ is the maximum likelihood estimate of the covariance matrix to the innovations, which is given by the inner product $\hat{\epsilon}'\hat{\epsilon}/N$; $\hat{\epsilon}$ is the stacked matrix of residuals; the total number of observations is given by N; $h = [vec(\hat{\beta})', vech(\hat{\Sigma})']'$; and δ is a Jacobian matrix containing derivatives of each element $\hat{\alpha}_m$ with respect to each element of h. Since $\hat{\beta}$ and $\hat{\Sigma}$ are independent from each other, we parameterize $Cov(h)$ as block-diagonal, with the upper left block given by the usual formula $(X'X)^{-1} \otimes \hat{\Sigma}$. The lower right block is given by the formula for the covariance matrix of the stacked unique elements of the residual covariance matrix $\hat{\Sigma}$, which is given by $H(2\hat{\Sigma} \otimes \hat{\Sigma}/N)H'$, where H is a matrix that 'picks off' the unique elements of the vec operation to implement the $vech$ operation.

In calculating our Jacobian matrix, we find it useful to express our two-stage least squares estimator $\hat{\alpha}$ as a closed-form function of VAR estimates $h = [vec(\hat{\beta})', vech(\hat{\Sigma})']'$, where $\hat{A}$ is given by the first 5 by 5 submatrix of $\hat{\beta}'$, and $\hat{\Lambda} = (I - \hat{A})^{-1}$. If we denote the upper-left 2 by 2 submatrix of $\hat{\Sigma}$ corresponding with innovations to public employment and wages as $\hat{\Sigma}_{pub}$ and the first two columns of $\hat{\Sigma}$ corresponding with innovations to public employment and wages as $\hat{\Sigma}_{*,pub}$, then the estimated contemporaneous effect of a contemporary identity-matrix innovation to public employment and wages is given by $\hat{C} = \hat{\Sigma}_{pub}^{-1}\hat{\Sigma}_{*,pub}$. Meanwhile, the estimated long-run effect of an identity-matrix innovation to public employment and wages is given by $\hat{D} = \hat{\Lambda}\hat{C}$. If we denote the upper-left 2 by 2 submatrix of $\hat{D}$ corresponding with innovations to public employment and wages as $\hat{D}_{pub}$ and the first two columns of $\hat{D}$ corresponding with innovations to public employment and wages as $\hat{D}_{*,pub}$, then the long-run estimate of the treatment effect $\hat{\alpha}$ is equal to $\hat{\alpha} = \hat{D}_{pub}^{-1}\hat{D}_{*,pub}$, which is numerically identical to the coefficients given by the 2SLS regression using estimated quantities.

2.3. The SVAR estimates

Before we present 2SLS results, we first test for the relevance of our first-stage instruments. In a first stage we regress long-run innovations to public variables on short-run innovations to public employment and public wages as follows. That is, we estimate first stage regression:

$$\eta_{i,t}^{pub} = \sum_{i=1}^{2} \pi_i \epsilon_{i,t}^{pub} + \xi_{i,t}^{pub}, \tag{4}$$

where i stands for employment and wage.

The first panel of Table 4 shows the results from this exercise. When we regress the long-run innovation to public employment on short-run innovations to public employment and wages, we observe coefficients of 1.11 and 0.001, respectively with standard errors of 0.0005 and 0.0001. The two coefficients jointly have an F statistic of 2.2 million against a null hypothesis that they equal of zero. Meanwhile, when we regress the long-run innovation to public wages on the short-run innovations to public employment and public wages, we get coefficients of 0.026 and 1.07 with standard errors of 0.0041 and 0.0007, respectively. The two coefficients jointly have an F statistic of 1.2 million against a null hypothesis that they equal zero. These F statistics are well above any cutoffs recommended as a safeguard against weak instruments, and the coefficients near one and zero furthermore tell us that there is not a large discrepancy between short-run and long-run innovations to public employment or to public wages.

Since we are confident in the relevance of our first-stage instruments, we then present results for our 2SLS estimates in the second panel of Table 4, using both conventional heteroskedasticity-consistent (i.e. nominal) standard errors and delta-method standard errors that fully account for the fact that our regressors and outcome variables are derived from a VAR model. Altogether, there are $N = 18,336$ observations in our regression at an MSA-date grain, and there are 434 right-hand-side variables including fixed effects, in addition to 15 unique terms in the variance-covariance matrix.

From our estimates, the following patterns become clear. First of all, our data indicate that public employment weakly crowds out private employment at a rate of about 0.1 private workers per public worker, driving down private wages slightly in the process. This estimate is statistically distinguishable from both 0 and 1, indicating weak crowding out. Furthermore, a higher rate of public employment tends to stabilize fluctuations in private employment. On the other hand, public wages do not clearly cause private employment to rise or fall–our estimates for this effect are too imprecise to say anything clearly. Higher public wages do appear to cause private wages to rise, while having an unclear effect on the volatility of private employment. Furthermore, point estimates indicate a positive effect on volatility, but this effect is not statistically distinguishable from zero.[5]

[5] In a separate set of results available upon request, we have also implemented Blanchard-Perotti short-run identification, and all of our impulse responses are compatible with our long-run causal effects.

Table 4
SVAR model with long-run identification.

First Stage IV Long-run innovation	pub. emp. η_{Et}^{pub}		pub. wage η_{Wt}^{pub}	
	Coeff.	S.E	Coeff.	S.E.
Short-run innovations (IV)				
Public emp. rate ϵ_{Et}^{pub}	1.1097	0.0005	0.0262	0.0041
Log public wage rate ϵ_{Wt}^{pub}	0.0010	0.0001	1.0648	0.0007
F-Statistic	2.2e+06	Prob. 0.0000	1.2e+06	Prob. 0.0000

Long-run causal impacts of public emp. and log wages - IV2SLS

Dependent variable: Private empl. rate			
Explanatory variable	**Coeff. $\hat{\alpha}$**	**S.E. (nominal)**	**S.E. (δ method)**
Public empl. rate	-0.1000	0.0216	0.0311
Log public wage rate	0.0071	0.0038	0.0055
Dependent variable: Log private wage rate			
Explanatory variable	**Coeff. $\hat{\alpha}$**	**S.E. (nominal)**	**S.E. (δ method)**
Public empl. rate	-0.1758	0.0551	0.0780
Log public wage rate	0.0798	0.0098	0.0159
Dependent variable: Private empl. rate growth rate squared			
Explanatory variable	**Coeff. $\hat{\alpha}$**	**S.E. (nominal)**	**S.E. (δ method)**
Public empl. rate	-0.0664	0.0060	0.0096
Log public wage rate	0.0022	0.0011	0.0036

This table features long-run SVAR responses to a unit long-run shock to either the public employment rate or the log public wage, all else equal, for the main observables. This identification scheme assumes that policymakers set short-run public wages (at an annual frequency) before observing short-run and long-run shocks to private employment, wages, and volatility. The model includes 2-way (MSA and time) fixed effects. A model without MSA fixed effects is available upon request; it gives extremely similar results. The dataset spans the period 1970 through 2018, for 382 metro areas. Source: BEA and authors' calculations.

3. An alternative panel approach using shift-share instruments

To ensure that our empirical analysis is robust, we also present results from an alternative panel estimation methodology. In that methodology, we use a straightforward instrumental variables approach to estimate equations of the form:

$$x_{i,t}^{priv} = \beta x_{i,t}^{pub} + B z_{i,t} + \epsilon_{i,t}, \tag{5}$$

where $x_{i,t}^{priv}$ is the set of three private variables (the change in the private employment rate, the change in log private real wages, and the change in the variance of private employment growth) aggregated over 5-year intervals; $x_{i,t}^{pub}$ is our set of two public variables (the change in the public employment rate, and the change in log public real wages) aggregated over 5-year intervals indexed by t; and $z_{i,t}$ is our set of MSA and time dummies. We define public and private employment rates and real wages as in the previous section, while our measure of volatility is the within-MSA/time-interval variance of the 1-year change in log private employment. This panel dataset features 382 MSAs over 8 half-decades, for 3,056 observations.[6]

Since local public employment and log public wages are likely to respond to local economic conditions, we instrument changes in public employment and public wages using shift-share instruments originally developed by Bartik (1991). For the change in public employment rates from time $t-1$ to t, we take the initial public employment rate at time $t-1$ times the weighted average of (arithmetic) net employment growth rates in federal civilian, military, and state and local public employment (excluding the relevant MSA), weighted by MSA-specific employment shares at time $t-1$. Similarly, for the change in log public real wages at time t, we take the log of the weighted average of gross wage growth rates in federal civilian, military, and state and local public employment (excluding the relevant MSA) from time t to $t-1$, weighted by MSA-specific initial wage shares within the public sector at time $t-1$. By using both instruments, we are making the assumption that changes to trends in public employment and wages outside of a given MSA are uncorrelated with MSA-specific economic shocks. Additionally, we include our time and MSA fixed effects $z_{i,t}$ in our instrument list, omitting one fixed effect to avoid multicollinearity.

To ensure that our instruments are relevant, we first regress the change in public employment rates and in log public wage rates on the instrument set including fixed effects. Table 5 reports the results of the first stage regressions in the first panel. For the public employment regression, we test against the null hypothesis that the change in public employment rates is uncorrelated with the two shift-share instruments. This test gives us an F statistic of 4.48 (when we include MSA fixed effects) and 87.7 (when we exclude MSA fixed effects). Similarly, for the public wage regression, we get an F statistic of 9.85 (when we include MSA fixed effects) and

[6] We aggregate over 5-year intervals so that our results capture more of a "medium run" steady state than does an annual analysis. Also, this allows us to include the variance (and not just squared rates) in private employment growth as an outcome variable.

Table 5
Estimates of long-run causal impacts of public employment and log wages, Bartik identification.

First Stage IV	Public emp. x^{pub}_{Et}		Public wage x^{pub}_{Et}	
	Coeff.	S.E	Coeff.	S.E.
Instruments				
MSA fixed effects: Yes				
Public emp. rate z^{pub}_{Et}	0.6012	0.0023	1.2649	0.2234
Log public wage rate z^{pub}_{Wt}	0.0518	0.0373	0.8331	0.0418
F-Statistic	4.48	Prob. 0.0000	9.85	Prob. 0.0000
MSA fixed effects: No				
Public emp. rate z^{pub}_{Et}	0.6534	0.0275	0.8406	0.1528
Log public wage rate z^{pub}_{Wt}	0.0583	0.0074	0.8466	0.0411
F-Statistic	87.7	Prob. 0.0000	392	Prob. 0.0000

Second Stage IV Results				
Dependent variable: Private empl. rate				
Explanatory variable	**Coeff. est. $\hat{\beta}$**	**Std. err.**	**Coeff. est. $\hat{\beta}$**	**Std. err.**
Public empl. rate	-0.332	0.243	-0.211	0.145
Log public wage rate	0.112	0.035	0.102	0.031
MSA effects	Yes	Yes	No	No
F test for poolability	1.32			
(p-value)	0.000			
Dependent variable: Log private wage rate				
Explanatory variable	**Coeff. est. $\hat{\beta}$**	**Std. err.**	**Coeff. est. $\hat{\beta}$**	**Std. err.**
Public empl. rate	-0.959	0.554	-0.797	0.329
Log public wage rate	0.270	0.080	0.248	0.070
MSA effects	Yes	Yes	No	No
F test for poolability	1.17			
(p-value)	0.016			
Dependent variable: Private empl. growth volatility				
Explanatory variable	**Coeff. est. $\hat{\beta}$**	**Std. err.**	**Coeff. est. $\hat{\beta}$**	**Std. err.**
Public empl. rate	-0.061	0.016	-0.016	0.009
Log public wage rate	0.003	0.002	0.000	0.002
MSA effects	Yes	Yes	No	No
F test for poolability	0.18			
(p-value)	1.000			

This table shows responses to changes in the public employment rate or the log public wage, all else equal, over 5-year intervals, using shift-share (Bartik) instruments. The model includes 2-way (MSA and time) fixed effects or 1-way (time) fixed effects, respectively. The dataset spans a set of 5-year periods from 1975-79 through 2010-14, for 382 metro areas. Source: BEA, BLS, and authors' calculations.

392 (when we exclude MSA fixed effects). Given these F statistics, we have some concerns about weak instruments when we include MSA-specific fixed effects, but we are more comfortable with their relevance if we have a pooled regression.

Next, in the second panel of Table 5, we present the results from our panel estimates, with models that do and do not contain MSA fixed effects, respectively. The fixed effects regressions also contain results from an F test on poolability, i.e. that the MSA-specific fixed effects are zero. The columns report regression estimates alongside their heteroskedasticity-consistent standard errors. As we can see, the main results are directionally consistent with those from the panel SVAR, although the Bartik results are in general less precisely estimated. When we look at the effects of changes in public variables on changes in the private employment rate (top panel), we find some weak evidence of crowding out, on the order of 0.21 to 0.33 private workers per public worker, which is not statistically distinguishable from zero for either coefficient but is statistically distinguishable from one. These results are consistent with the results from the SVAR, which indicate weak crowding out. Meanwhile, we find some evidence of higher public wages 'crowding in' private employment, which is for both coefficients statistically distinguishable from zero. When we look at what affects changes in log private wages (center panel), we find a possible negative effect of public employment on private wages, which in the fixed effects model is not statistically distinguishable from zero but in the pooled model is statistically distinguishable from zero. Meanwhile, in both models, changes in log public wages pass through into log private wages at a rate of about 0.25 to 0.27, which is in both cases statistically distinguishable from zero. Finally, when we look at the effects of public employment and wages on private employment growth volatility, we find a negative effect of public employment of -0.061 under the fixed effects model which falls to -0.016 under the pooled model. The former value is statistically distinguishable from zero, while the latter is borderline at conventional levels (t = -1.889, p = 0.059). Meanwhile, we do not find any effects of public wages on volatility that are statistically distinguishable from zero.

4. A theoretical model

In order to understand our estimates, particularly on volatility, we construct a search and matching model in general equilibrium.

4.1. Household behavior

In our model, households, which are assumed to be large families that include all types of agents, trade on the goods market in order to maximize the present discounted value of utility H_t, subject to a sequence of budget constraints which apply at the household level. Households pool all of their consumption and income among their members.

The household's value function takes the form:

$$H_t = \widetilde{E_t} \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + v(G_{t+i}) - \overline{b}(E_{g,t+i} + E_{p,t+i}) \right], \tag{6}$$

where $\widetilde{E}$ denotes the expectation operator; β is a discount rate; C_{t+i} denotes private consumption with a preference parameter σ; $v(G_{t+i})$ is a monotonic, increasing function in real government consumption G_{t+i}, such that preferences are separable in government consumption and private consumption; and $E_{g,t+i}$ and $E_{p,t+i}$ denote public and private employment, with a marginal utility of unemployment given by $\overline{b}$. The budget constraints take the form:

$$C_{t+i} + T_{t+i} + B_{t+i} = (1 + r_{t+i})B_{t+i} + W_{g,t+i}E_{g,t+i} + b^u(1 - E_{g,t+i} + E_{p,t+i}) + W_{p,t+i}E_{p,t+i} + \Pi_{p,t+i}, \tag{7}$$

for consumption C_{t+i}, lump-sum taxes T_{t+i}, bondholdings B_{t+i} which yield a return r_{t+i}, public wages and employment $W_{g,t+i}$ and $E_{g,t+i}$, unemployment benefits per unemployed worker $b^u = h\bar{W}_p$, private wages and employment $W_{p,t+i}$ and $E_{p,t+i}$, and firm profits $\Pi_{p,t+i}$.

Maximization proceeds in a standard way. First, denoting the Lagrange multiplier on the budget constraint by Λ_{t+i}, the first-order conditions to the maximization problem imply that:

$$\Lambda_t = C_t^{-\sigma}. \tag{8}$$

Furthermore, the flow value of nonemployment or leisure in consumption units is denoted by b_t, which equals $h + \overline{b}/\Lambda_t$. In equilibrium, then, b is given by:

$$b_t = h + \overline{b}C_t^{\sigma}. \tag{9}$$

This setup endogenizes the flow value of nonemployment in a way that is compatible with the rest of the dynamic general equilibrium literature. As a special case, setting either $\overline{b}$ or σ to zero gives the "standard" search and matching calibration, which implies a risk-neutral, acyclical flow value of nonemployment. Meanwhile, setting σ to a larger value (such as one) gives an interpretation of this value as representing leisure in the presence of risk aversion, in line with the business cycle literature and with Gomes (2015). Furthermore, setting h to 0 gives an interpretation of the outside option as a value of leisure, following Baxter and King (1993).

4.2. The labor market (random search)

4.2.1. The environment

Production takes place in a search and matching economy, divided into the private and public sectors. We first present a random search model; later, we will present a directed search model. In the random search model, workers search in both sectors at once, and eventual placement in a sector occurs randomly. Apart from the inclusion of a public sector, this is a standard textbook search and matching economy. In this economy, future economic rents are discounted at a rate β; private matches are destroyed at a rate λ_p; public matches are destroyed at a rate λ_g; hires from unemployment into private employment occur at a rate p_p; hires from unemployment into public employment occur at a rate p_g; workers in private matches are paid a wage $W_{p,t}$; workers in public matches are paid a wage $W_{g,t}$; private matches produce output at a rate of productivity π; and unemployed receive unemployment benefits proportional to the steady state wage rate such that $b^u = h\bar{W}_p$ with an additional value of leisure, as discussed above. Firms and workers take the actions of the government as exogenous to their own decisions.

We note that like Gomes (2015), we do not have any on-the-job search since this greatly complicates our model (especially when including versions of on-the-job search with heterogeneity) and makes our theoretical results less interpretable. That said, we acknowledge that adding on-the-job search may change the dynamics of our model.

4.2.2. Flows and hiring costs

The matching technology is given by a Cobb-Douglas matching function over total vacancies V_t and searchers U_t, that is

$$M_t = AV_t^a U_t^{1-a}, \tag{10}$$

where M_t is the total number of matches in any sector and V_t is the total number of vacancies posted by the private and public sector, such that $V_t = V_t^G + V_t^P$. This implies that the probability for a private firm to hire an unemployed worker is given by:

$$\frac{M_t}{V_t} = AV_t^{a-1}U_t^{1-a} = A\theta_t^{a-1} = q(\theta_t) = q_t, \quad (11)$$

where $\theta = V/U$ is market tightness. Similarly, the probability for an unemployed worker to find a match is given by,

$$\frac{M_t}{U_t} = A\theta_t^a = p(\theta_t). \quad (12)$$

Since search is random, workers can end up being employed either in the public sector or in the private one depending on the relative number of the vacancies open. Therefore, $p(\theta_t)$ is the sum of the two probabilities. The probability for an unemployed worker to find a job in the private sector is given by:

$$p_{p,t} = \frac{M_t}{U_t}\frac{V_p}{V_t} = A\theta^a\frac{V_p}{V_t} = p(\theta_t)\theta_{p,t}, \quad (13)$$

where $\theta_p = V^P/U$. Similarly,

$$p_{g,t} = p(\theta_t)\theta_{g,t}, \quad (14)$$

with $\theta_g = V^G/U$, and $\theta_p + \theta_g = \theta$.

Employment flows are then given by:

$$E_{p,t+1} = E_{p,t} + p_{p,t}U_t - \lambda_p E_{p,t}, \quad (15)$$

and:

$$E_{g,t+1} = E_{g,t} + p_{g,t}U_t - \lambda_g E_{g,t}, \quad (16)$$

where $U_t = 1 - E_{p,t} - E_{g,t}$.

4.2.3. *Value functions and private wages*

The value functions for workers in public and private matches, measured in output units, are given by the two equations:

$$V^P_{E,t} = W_{p,t} - b_t + E_t m_{t+1}[(1-\lambda_p)V^P_{E,t+1} + \lambda_p V_{U,t+1}], \quad (17)$$

and:

$$V^G_{E,t} = W_{g,t} - b_t + E_t m_{t+1}[(1-\lambda_g)V^G_{E,t+1} + \lambda_g V_{U,t+1}]. \quad (18)$$

That is, both values include the wage paid by the job minus the flow value of unemployment, plus the discounted expectation of a change in status given by the probability of loosing the job represented by the separation rate λ. The stochastic discount factor is given by the discount factor β times the relative marginal of consumption in two different periods, such that:

$$m_t = \beta\frac{\Lambda_t}{\Lambda_{t-1}}. \quad (19)$$

Being unemployed has its flow value discussed previously. Unemployed workers have a chance to find a job in each of the two sectors, with a value given by:

$$V_{U,t} = b_t + E_t m_{t+1}[p^{p,t}V^P_{E,t+1} + p_{g,t}V^G_{E,t+1} + (1-p_{p,t}-p_{g,t})V_{U,t+1}]. \quad (20)$$

Turning to the values of posting a vacancy in the private sector $V^P_{V,t}$ and the value of a filled job $V_{J,t}$, we have,

$$V^P_{V,t} = -\kappa + E_t m_{t+1}[q_t V^P_{J,t+1} + (1-q_t)V^P_{V,t+1}], \quad (21)$$

and:

$$V^P_{J,t} = \pi_t - W_{p,t} + E_t m_{t+1}[(1-\lambda_p)V^P_{J,t+1} + \lambda_p V^P_{V,t+1}], \quad (22)$$

Free entry implies that the value of a filled job should equal the cost of filling that job, or, in other words, that $V^P_{V,t} = 0$ at any t. This gives:

$$\kappa = q_t E_t m_{t+1}V^P_{J,t+1} \Rightarrow E_t m_{t+1}V^P_{J,t+1} = \frac{\kappa}{q_t}, \quad (23)$$

Plugging this solution into Equation (22) gives:

$$V^P_{J,t} = \pi_t - W_{p,t} + (1-\lambda_p)c., \quad (24)$$

where $c = \frac{\kappa}{q_t}$ is the average cost of hiring a worker. Since these hiring costs create quasi-rents, employed workers and firms Nash bargain over these rents, in order to determine wages. In this bargaining process, workers have a bargaining weight β over the match surplus, while firms have a bargaining weight $1 - \beta$. This implies that the surplus is split according to the formula:

$$\eta V^P_{J,t} = (1 - \eta)(V^P_{E,t} - V_{u,t}). \tag{25}$$

4.3. Government production and fiscal policy

Following the setup of Quadrini and Trigari (2007), the government hires workers and pays them wages. Public employment and wages follow a rule-of-thumb fiscal reaction function. We assume that public employment E_g and the public wage W_g are set exogenously, according to political considerations, and that the government sets its policies according to the equations:

$$\ln E_{g,t} = \ln \bar{E}_g + \gamma_E (\ln E_{p,t} - \ln \bar{E}_p) + \epsilon_{E,t-1}, \tag{26}$$

and:

$$\ln W_{g,t} = \ln \bar{W}_g + \gamma_W (\ln W_{p,t} - \ln \bar{W}_p) + \epsilon_{W,t-1}. \tag{27}$$

with the shocks $\epsilon_{E,t-1}$ and $\epsilon_{W,t-1}$ being normally distributed around zero with variances σ^2_E and σ^2_W.

Since our empirical analysis involved estimating the causal effects of $\ln W_g$ and E_g holding the other constant, we start our simulations off with $\gamma_W = 0$ and $\gamma_E = 0$, and we also shut down public-sector shocks by setting these variances equal to zero. In results available upon request, we vary these coefficients to show the theoretical effects of various stabilization policies.

The government combines the output of its own workers with output that it purchases from the private sector, in order to produce a final government consumption good. We assume that government production is Cobb-Douglas where:

$$Y_{pub} = \Gamma E_g^{1-\zeta} (Y_g^{int})^{\zeta} - \kappa v_g, \tag{28}$$

where Y_{pub} is a government provided public good which is equal to the total output Y_g minus resources employed for recruiting (κv_g). Y_g^{int} is intermediate government purchases from the private sector. Given that public employment and wages are predetermined, the problem of the government is to set Y_g^{int} such that both inputs in production are used efficiently. That implies that W_g equals the marginal productivity of public labor, that is,

$$W_g = (1 - \zeta) P_g \Gamma E_g^{-\zeta} (Y_g^{int})^{\zeta}, \tag{29}$$

where P_g is a shadow price of government output. A similar first-order condition for Y_g^{int} gives:

$$1 = \zeta P_g \Gamma E_g^{1-\zeta} (Y_g^{int})^{\zeta - 1}. \tag{30}$$

Further algebra gives constant factor shares, such that:

$$Y_g^{int} = \zeta (Y_g^{int} + W_g E_g). \tag{31}$$

Alternately, we can write:

$$Y_g^{int} = \psi W_g E_g, \tag{32}$$

where the government purchases $\psi = \frac{\zeta}{1-\zeta}$ units from the private sector for every dollar in its real wage bill. We note that total value of the public output is given by $W_g E_g + Y^{int}$ in national accounts, since true public-sector productivity and hiring costs are unobservable.

The government also balances its budget, such that tax revenues equal the wage bill plus intermediate government consumption plus unemployment benefits. (The cost of keeping vacancies open come out of final output, such that vacancies are not purchased from the private sector.) This implies:

$$T = W_g E_g + \psi W_g E_g + h(1 - E_{g,t+i} + E_{p,t+i}). \tag{33}$$

To begin closing the model, substituting the budget constraint into household and firm constraints that private consumption is given by total private-sector production plus unemployment benefits minus the cost of keeping private vacancies open, plus earnings from working in the public sector minus taxes, that is:

$$C_t = \left[\pi_t E_{p,t} + h(1 - E_{g,t+i} + E_{p,t+i}) - \kappa V_t^P + W_{g,t} E_{g,t} - T_t = \pi_t E_{p,t} - \psi W_{g,t} E_{g,t} - \kappa V_t^P \right]. \tag{34}$$

To fully close the model, we need to specify law of motion for exogenous variables. To that end we assume that the process for the productivity is given by:

$$log(\pi_{t+1}) = \rho log(\pi_t) + \epsilon_{t+1} \tag{35}$$

where $\epsilon \sim N(0, \sigma^2_\epsilon)$.

Table 6
Deterministic Steady-State calibration, baseline model.

Target	Description	Value	Remarks
W_g/W_p	Gross pub. wage premium	1.0337	CPS data
$E_g/(E_g+E_p)$	Public employment share	0.1579	Cross-sectional mean
U	Unemployment rate	0.0673	Cross-sectional mean
λ_p	Private sep. rate	0.47	JOLTS nationwide mean
λ_g	Public sep. rate	0.0158	JOLTS nationwide mean
r	Discount rate	0.01	Target: 4 percent per year
β	Workers' bargaining power	0.5	Standard calibration
c/W_p	Hiring cost	0.14	
b/W_p	Outside option relative to wage	0.8596	
h/W_p	Replacement ratio of unemp. benefits	0.4	
$q(\theta)$	Hiring rate	0.7	from Quadrini and Trigari (2007)
ψ	Govt. demand parameter	1.421	NIPA, cross-sectional data
π	Productivity	1	Normalization
a	Match elasticity	0.37	Brügemann (2008)
σ	Consumption utility parameter	1	Log preferences
γ_W	Response of pub. to priv. wages	0	Want 'all else equal' effects
γ_E	Response of pub. to priv. empl.	0	Want 'all else equal' effects
ρ	Autoregressive parameter	0.95	from Quadrini and Trigari (2007)
σ_ϵ	St. dev. of productivity errors	0.007	from Quadrini and Trigari (2007).

This table lists a set of model calibrations for the baseline random search model calibrated to cross-sectional means (or aggregate means) for U.S. data, from 2001 to 2013. Source: BEA, BLS, and authors' calculations as discussed in main text.

5. Calibrating the model

5.1. Calibrating the main parameters

We calibrate our model to match aggregate data at a quarterly frequency. These calibration targets are shown in Table 6. We target public employment as a share of total employment of 15.79 percent, and we target an unemployment rate of 6.73 percent. Based on JOLTS data for layoffs, we target a rate of private separations λ_p of 4.7 percent per quarter, and we target a rate of public separations λ_g of 1.58 percent. We target these numbers because our model omits on-the-job search, which comprise the majority of quits. In the aggregate. Since the BLS does not publish stock-consistent flows broken out by public and private employers, and because these numbers are not corrected for misclassification, we view JOLTS layoffs as providing a better calibration target. In any case, our results are not sensitive to the exact choice of target.

For other parameters, we use standard values from the literature. For instance, we target a discount rate of four percent per year, or 1 percent per quarter. We normalize productivity π to one. Finally, we calibrate γ_W and γ_E to zero, as well as the standard deviations for the public variable shocks, for comparability with the causal effects estimated in the previous section. By doing this, we ensure that our theoretical exploration, like our causal estimation, answers the question, what is the effect of a unit change in public employment or the log public wage on private employment, log private wages, and private employment volatility, holding all other disturbances constant? In later sections, we vary these parameters in order to explore the theoretical implications of stabilization policies.

Next, we target a Nash bargaining weight η of 0.5, which is in keeping with the rest of the literature. Also, we set the matching function parameter a to 0.37, following Brügemann (2008), which is close to the value of 0.4 recommended by Elsby and Michaels (2013) and Petrongolo and Pissarides (2001). We also vary the consumption parameter σ between zero (the standard search and matching calibration) and one (a standard calibration from the business cycle literature). We also vary the outside option b (or equivalently the hiring cost c). We also assume that the unemployment benefit h equals 40 percent of steady-state private wages.

In our baseline setup, we target an overall replacement rate b/W_p of 0.8342, based on a value of c equivalent to 0.14 times the quarterly wage per worker. We choose this value to follow Elsby and Michaels (2013), Silva and Toledo (2009), and Hall and Milgrom (2008). This is a larger value than that used by Shimer (2005) but it is within the range used within the search and matching literature. Furthermore, this larger value is compatible with the estimates of Chodorow-Reich and Karabarbounis (2015), in which the outside option includes leisure and home production. Given the importance of this target, we also look at larger and smaller replacement rates.

5.2. Calibrating the public wage premium and government demand parameter ψ

To calibrate public wages, we need to ensure that public and private wages are comparable for a given worker. The problem is that our baseline dataset suggests that the public wage premium is unrealistically large when one takes worker composition into account. In fact, aggregate data suggest that public employees are compensated in the NIPA data at a rate of about 44 percent higher than private employees. However, most previous studies of the public wage or compensation premium, such as those of Katz and Krueger (1991), Borjas (2002), and Falk (2012a, 2012b) find a significantly smaller premium once they control for worker characteristics, most notably education. However, the estimates of Falk are only for the federal wage and compensation premiums, while we are

Table 7
Simulated moments from the random search model, varying a, σ, and ψ, baseline model.

Object	(1)	(2)	(3)	(4)
σ	0.00	1.00	1.00	1.00
a	0.37	0.37	0.37	0.50
ψ	1.42	0.00	1.42	1.42
Standard Deviations				
Log Private Employment	0.782	0.447	0.348	0.441
Log Private Wages	1.914	2.024	2.063	2.088
Log Productivity	1.807	1.809	1.810	1.809
Log GDP	2.447	2.178	2.097	2.173
Std as Share of GDP Std				
Log Private Employment	0.319	0.205	0.166	0.203
Log Private Wages	0.782	0.929	0.984	0.961
Log Productivity	0.739	0.831	0.863	0.832
log-Correlations with GDP				
Log Private Employment	0.979	0.982	0.982	0.984
Log Private Wages	0.998	1.000	1.000	1.000
Log Productivity	0.997	0.999	1.000	0.999

These tables list a set moments for the baseline random search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long and then discarding the first 16 years of observations. Standard deviations are reported in percents.

more interested in a premium that takes federal, state, and local employment into account. We therefore extend Falk's estimates to the overall public sector in Appendix A, and we arrive at a gross compensation premium of 1.0337, once worker characteristics are taken into account.

We also calculate the government's demand for output from the private sector, also adjusted for the mismeasurement of the public wage bill in private worker-productivity equivalents. To derive this, we note that, based on aggregate NIPA tables, the share of government consumption and gross investment that does not come from government value added, as a share of measured value added, equals 0.587. This gives a ratio of government intermediate consumption to the public wage bill of $\psi = 0.587/(1-0.587) = 1.421$. This represents the government's demand for the output of private workers as a ratio of the public wage bill, adjusted for the mismeasurement of public wages relative to worker productivity in aggregate data. Since this is an important parameter, we also vary it in our simulations. Finally, following Quadrini and Trigari (2007) we assume a AR(1) productivity process with unit mean, 0.95 autoregressive parameter, and $\epsilon_t \sim (0, 0.007)$.

6. Simulations based on the calibrated model

6.1. Simulating the effects of public wages and public employment

We simulate the model to calculate the following moments that are analogous to our empirical analysis: annualized log private employment, annualized log private wages, annualized log GDP, and annualized productivity. We calculate their standard deviations in levels and relative to log GDP. These statistics are summarized in Table 7 for the first four model variants, the first three of which reflect values of σ from 0 to 1, i.e. moving from an acyclical outside option to a more procyclical outside option. The fourth model variant involves taking ψ to zero, in line with standard search and matching models. Furthermore, in Table 8, we report numeric derivatives of steady-state exogenous variables E_g and $\ln W_g$, and $\ln \pi$ on steady-state private employment, log private wages, and the log standard deviation of private employment. To facilitate comparability with our empirical estimates, we show analogous empirical estimates on the right-hand side of Table 8.

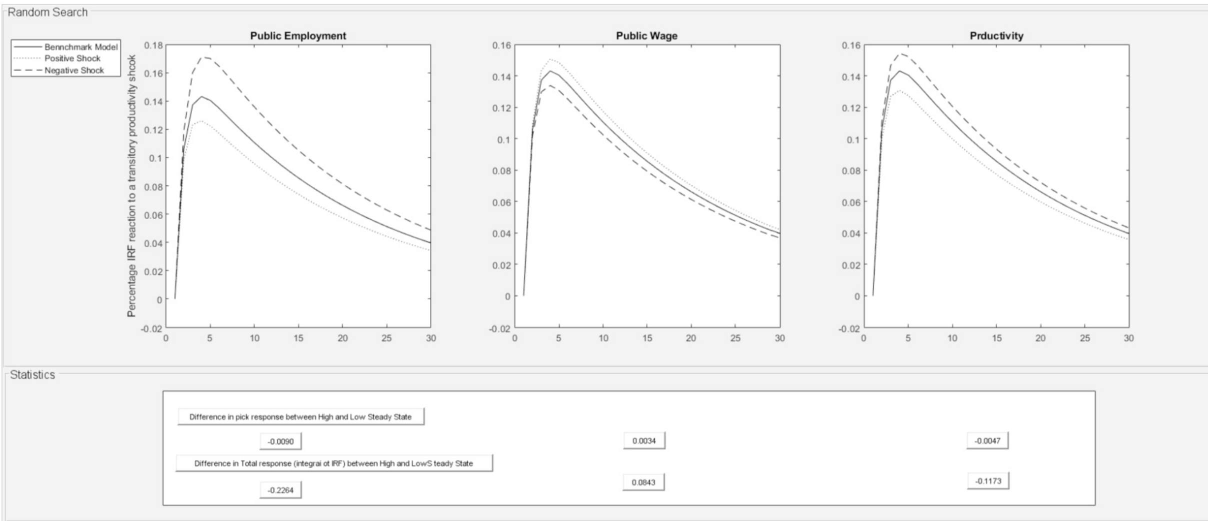
To calculate numeric derivatives, we shift each exogenous variable upwards and then downwards by a half percentage point; then we take the log differences of the long run simulated moments. We multiply that difference by 100, which gives the semi-elasticity of the change in the endogenous variables to changes in the exogenous ones. For the measure of volatility (log standard deviation), we ensure that our empirical results are comparable with our theoretical ones by taking the square root of the mean of the squared annualized growth.[7] When simulating the out-of-steady-state dynamics of the model, we can also look at impulse response functions, to gain more insights about the mechanisms at work that generates our results. Impulse responses for different steady-state variables are summarized in Fig. 1; for our shifts in parameters, responses are monotonic across time, which means that there are no important issues related to the timing of impulse responses in our current analysis.

[7] In Tables 9 and 10, we show results with HP-filtered series instead of annualized ones. We also try other measures of volatility such as raw standard deviations, the results are highly robust to the measure we choose. We compute all moments by simulating 300 independent series over 250 years and discarding the first 48 years.

Table 8
Implied empirical responses, varying a, σ, and ψ, baseline simulated model.

Object	(1)	(2)	(3)	(4)	SVAR	Bartik
σ	0.00	1.00	1.00	1.00		
a	0.37	0.37	0.37	0.50		
ψ	1.42	0.00	1.42	1.42		
Responses to log public wage shifter						
Volatility	8.787	4.951	2.337	0.825	*1.50*	*2.30*
Private wages	0.057	0.052	0.044	0.033	0.08	0.27
Private employment	-0.348	-0.308	-0.259	-0.327	*0.12*	*0.20*
Responses to public employment shifter						
Volatility	5.080	1.764	-6.066	-5.417	-25.20	-23.10
Private wages	0.016	-0.007	-0.059	-0.044	-0.10	-0.55
Private employment	-1.313	-1.172	-0.863	-0.773	-0.10	-0.33
Responses to log private productivity						
Volatility	-14.892	-6.058	-3.318	-1.766	-	-
Private wages	0.925	0.950	0.959	0.972	-	-
Private employment	0.510	0.341	0.290	0.365	-	-

This table lists a set (semi-)elasticities for the baseline random search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long, annualized, and then discarding the first 16 years of observations. Moments are the semi-elasticities, calculated as the log difference of the variable of interest resulting from incrementing and decrementing the control variable by a half percentage point. Empirical estimates are transformed to be comparable to their theoretical counterparts, with estimates in italics not statistically distinguishable from zero.



Calibrated IRF functions based on the benchmark calibrated model. Each panel represents the responses to the transitory shock in alternative scenarios.

Fig. 1. Impulse Response Functions to productivity shocks for different steady states - Benchmark Model.

6.2. Comparing models with and without product market features

As seen in Table 8, model variant (1) ($\sigma = 0; a = 0.37$) is a standard search and matching model without further product market features. Model variant (2) ($\sigma = 0.5; a = 0.37$) adds the income effect channel by letting b_t vary over time. Meanwhile, model (3) ($\sigma = 1; a = 0.37$) strengthens this effect by doubling the effect of consumption on the value of leisure. Model variant (4) is the same as model variant (3) but without the public consumption of intermediate goods ($\psi = 0$).

In model variant (1), an increase in public employment strongly crowds out private employment by more than one-to-one. This is because steady-state public wages are higher than steady-state private wages. Furthermore, an increase in public wages crowds out private employment. Moreover, an increase in public employment slightly increases private wages, since private workers now have the option of getting a higher wage in the public sector. Meanwhile, there is a strong positive effect of public wages on private wages, and public wages strongly crowd out private employment. As a result, the model predicts that public employment and public wages are both highly destabilizing, with a semi-elasticity of volatility with respect to public employment of +5, vs. -25 in the SVAR.

Table 9
HP-Filtered series: Simulated moments from the random search model, varying a, σ, and ψ, baseline model.

Object	(1)	(2)	(3)	(4)
σ	0.00	1.00	1.00	1.00
a	0.37	0.37	0.37	0.50
ψ	1.42	0.00	1.42	1.42
Standard Deviations				
Log Private Employment	0.296	0.169	0.132	0.168
Log Private Wages	0.782	0.821	0.839	0.843
Log Productivity	0.735	0.747	0.751	0.747
Log GDP	0.946	0.866	0.843	0.866
Std as Share of GDP Std				
Log Private Employment	0.313	0.196	0.156	0.194
Log Private Wages	0.827	0.947	0.995	0.973
Log Productivity	0.777	0.863	0.891	0.863
log-Correlations with GDP				
Log Private Employment	0.758	0.754	0.710	0.759
Log Private Wages	0.997	0.997	0.996	0.998
Log Productivity	0.985	0.984	0.981	0.986

These tables list a set moments from HP filtered series with $\lambda = 1600$ for the baseline random search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long and then discarding the first 16 years of observations. Standard deviations reported in percents.

Table 10
HP-Filtered series: Implied empirical responses, varying a, σ, and ψ, baseline simulated model.

Object	(1)	(2)	(3)	(4)	SVAR	Bartik
σ	0.00	1.00	1.00	1.00		
a	0.37	0.37	0.37	0.50		
ψ	1.42	0.00	1.42	1.42		
Responses to log public wage shifter						
Volatility	8.532	4.801	2.236	0.716	*1.50*	*2.30*
Private wages	-0.348	-0.308	-0.259	-0.327	0.08	0.27
Private employment	0.057	0.052	0.044	0.033	*0.12*	*0.20*
Responses to public employment shifter						
Volatility	4.991	1.814	-5.795	-5.092	-25.20	-23.10
Private wages	-1.313	-1.172	-0.863	-0.773	-0.10	-0.55
Private employment	0.016	-0.007	-0.059	-0.044	-0.10	-0.33
Responses to log private productivity						
Volatility	-14.440	-5.878	-3.190	-1.624	-	-
Private wages	0.510	0.341	0.290	0.365	-	-
Private employment	0.925	0.950	0.959	0.972	-	-

This table lists a set (semi-)elasticities for the baseline random search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long and then discarding the first 16 years of observations. Each series is also HP-filtered with $\lambda = 1600$. Moments are the semi-elasticities, calculated as the log difference of the variable of interest resulting from incrementing and decrementing the control variable by a half percentage point. Empirical estimates are transformed to be comparable to their theoretical counterparts, with estimates in italics not statistically distinguishable from zero.

Following Quadrini and Trigari (2007) and others, we find that the search and matching model predicts that public employment crowds out private employment to a high degree and is also destabilizing for the private economy.

Next we investigate model variant (2), which represents a baseline model where the spillovers from public activity, represented by ψ, into demand are turned off. This economy features a strong destabilizing effect of public wages and also of public employment, which is in line with a standard search and matching model but not with an economy that has intermediate purchases. Furthermore, this economy has a stronger crowding out of private employment by the public sector. This tells us that an income effect by itself is not sufficient to generate a stabilizing effect of public employment.

Table 11
Simulated Moments from the random search model, varying calibration targets.

Object	(1)	(2)	(3)	(4)
$c(\theta)/W_p$	0.140	0.140	0.280	0.070
W_g/W_p	1.000	1.067	1.034	1.034
Standard Deviations				
Log Private Employment	0.386	0.318	0.243	0.486
Log Private Wages	2.057	2.069	2.062	2.072
Log Productivity	1.821	1.799	1.813	1.808
Log GDP	2.139	2.061	2.013	2.210
Std as Share of GDP Std				
Log Private Employment	0.181	0.154	0.121	0.220
Log Private Wages	0.962	1.004	1.024	0.938
Log Productivity	0.851	0.873	0.901	0.818
log-Correlations with GDP				
Log Private Employment	0.982	0.982	0.979	0.985
Log Private Wages	1.000	1.000	1.000	1.000
Log Productivity	0.999	1.000	1.000	0.999

These tables list a set moments for the baseline random search model simulated changing some of our calibration targets. All moments are calculated running the model generating 300 quarterly independent series 250 years long and then discarding the first 16 years of observations. Standard deviations are reported in percent.

Next, we take the model with intermediate consumption but turn on the income effect channel ($\sigma = 0.5$ and $\sigma = 1$), as in model variants (3) and (4). Model (4) gives us the fully calibrated model according to Table 6, which we refer to as our 'baseline' model. Now, higher public employment crowds out private employment by somewhat less, while higher public employment also now decreases private wages and decreases volatility. Public wages, meanwhile, still crowd in private wages. These effects now all point in the same direction as their corresponding SVAR results, for those results that are statistically distinguishable from zero. Importantly, an increase in public employment decreases private wages and helps to stabilize private employment over the business cycle.

As a reminder, the stabilizing effect of public employment in models (3) and (34) come from two channels. The first channel is what Gomes (2015) refers to as the Business Cycle Wealth Effect. Under this channel, an increase in public employment or public wages cause private employment E_p and private consumption C to fall, which in turn decreases b. This undoes some of the destabilizing effects of public employment and public wages. The second channel comes from the additional term $\psi W_g E_g$ governing public consumption, which amplifies the income effect in response to public employment. This channel occurs because higher public employment and public wages motivate governments to increase intermediate consumption, whereby, for instance, public employees require pens, paper, and coffee to produce final output. The combination of these two channels is strong enough to generate a realistic stabilizing effect of public employment, without generating a stabilizing effect of public wages.

Moving beyond simulated moments, Fig. 1 shows the impulse response functions to a productivity shock equal to $2\sigma_\epsilon$. Each of the three graphs has three IRFs, a continous line for the benchmark model, dotted for the model with the exogenous variable increased by 0.5% and dashed for the model with the exogenous variable decreased by the same amount. The first graph reports the IRFs in the case we change public employment, the second public wages and the third long-run productivity. As we can see visually, the blue line is between the yellow (top) and red (bottom) in the first and third graph, which means that the reaction to a productivity shock is stronger when public employment and productivity have steady state lower levels, that is, those variables stabilize the cycle. The opposite is true for public wage. From the graph we can also appreciate that the strength of the response varies in intensity (pink) and duration in the same way.

6.3. Effects of the public wage premium target W_g/W_p and outside option target b

Next, we investigate the role of the steady state public wage premium W_g/W_p. While our estimates point toward the 'true' premium being relatively small, the role of W_g in the analytical results suggests that the size of this wage premium may have some effect on business cycle volatility. To investigate robustness, Tables 11 and 12 display simulated moments and derivatives for different steady state calibrations. Here, model variant (1) features no wage premium, such that $W_g/W_p = 1$, and model variant (2) features a net wage premium doubled relative to its baseline value, such that $W_g/W_p = 1.0674$. These two counterfactuals indicate that the baseline results on volatility and responses to public employment and wages are not sensitive to the exact level of the wage premium, although as intuition may suggest, a higher steady state wage premium dampens the stabilizing effects of higher public employment somewhat.

The other major parameter which might be expected to affect our results is the outside option b, which in our calibration strategy is a function of c. To examine the effects of this calibration choice, model variants (3) and (4) depict situations in which c/W_p is set to either half or twice its calibrated value. The model with a lower value of c as in (3) now features amplified volatility, while the

Table 12
Sensitivity to steady states, varying calibration targets.

Object	(1)	(2)	(3)	(4)	SVAR	Bartik
$c(\theta)/W_p$	0.140	0.140	0.280	0.070		
W_g/W_p	1.000	1.067	1.034	1.034		
Responses to log public wage shifter						
Volatility	2.815	1.991	1.822	1.892	*1.50*	*2.30*
Private wages	0.049	0.041	0.053	0.034	0.08	0.27
Private employment	-0.287	-0.238	-0.156	-0.396	*0.12*	*0.20*
Responses to public employment shifter						
Volatility	-9.133	-3.793	-1.455	-10.113	-25.20	-23.10
Private wages	-0.085	-0.037	-0.049	-0.052	-0.10	-0.55
Private employment	-0.707	-0.991	-1.068	-0.589	-0.10	-0.33
Responses to log private productivity						
Volatility	-3.713	-3.035	-2.891	-2.781	-	-
Private wages	0.955	0.962	0.952	0.968	-	-
Private employment	0.311	0.274	0.187	0.427	-	-

This table lists a set of model calibrations and implied (semi-)elasticities for the random search model changing some of our calibration targets.

opposite is true in model variant (4). This is a well-known result in the literature dating back to Hagedorn and Manovskii (2008). However, the effects of public employment and wages are still in the same directions as in our baseline analysis, which indicates that our results on stabilization are robust to changes in these parameters.

7. Conclusion

By looking at a panel dataset of U.S. metro areas, we have provided some empirical findings on the effects of public employment and public wages. Our findings suggest that a high rate of public employment does not cause much (if any) crowding out in private employment, while a high rate of public employment reduces both private wages and business cycle volatility. These facts are incompatible with the predictions of standard search and matching models augmented with a public sector, but we show that such models perform much better once they also allow for government policies to affect product markets directly.

Our results suggest several avenues for future work. First of all, on the empirical side, there remains much to be done in estimating the government wage or compensation premium purged of composition effects in light of current data limitations, particularly for state and local workers, and in linking these estimates with geography (e.g. MSAs) in a way that cross-sectional variation in the public wage premium could be examined. If this were to happen, then our current measures of public wage growth, which are hampered by sparse CPS data at a local level, could be made more accurate. Similarly, better localized data on intermediate public consumption could help improve how we understand this important channel through which government policies affect macroeconomic outcomes.

Another avenue for extension would be adding on-the-job search and more features of larger-scale macroeconomic models, particularly open-economy models. Our empirical analysis is of the effects of public spending shocks on what amount to small open economies, because those were the effects that we could econometrically identify, while our model has been one of a closed economy. It is possible that if public employment and wages are financed by transfers from other regions, then policies that are stabilizing at a regional open economy level may have quantitatively different effects at a national level. Such an analysis fruitfully teases apart open-economy from closed-economy effects and would require more data and analysis on fiscal transfers across MSAs or, more realistically, states and counties. There is also room to discuss the role for optimal government employment and wage policies, which would require further modeling efforts in light of the lack of empirical evidence on the 'crowding out' effect of public wages. Such an analysis would also require taking a stand on welfare, which is beyond the scope of our paper.

Data availability

All data and code is available as an on-line appendix

Appendix A. Estimating the government compensation premium

To estimate the government compensation premium, we follow Falk (2012a) in estimating the following regression using earnings per hour from 2001 to 2013 as reported in the following year's March Current Population Survey (CPS):

$$\ln W_{i,t} = \gamma_{G,t} \mathbb{1}_{G,i,t} + b'_G X_{i,t} + \varepsilon_{i,t}, \tag{A.1}$$

where $W_{i,t}$ is earnings per hour; $\mathbb{1}_{G,i,t,t}$ is an indicator variable equal to one if a worker worked in the government sector in time t and zero otherwise; $\gamma_{G,t}$ is the effect of public employment on log wages in time t; $X_{i,t}$ is a vector of control variables including time dummies, seven categories for educational attainment (unknown, through grade 6, through grade 11, a high school diploma

Table A.1
Gross public wage premium based on CPS data.

Year	Gross wage prem. (Geometric)	Gross wage prem. (Arithmetic)	Gross comp. prem. (Arithmetic)
2001	0.925	0.873	
2002	0.951	0.902	
2003	0.957	0.933	
2004	0.978	0.933	
2005	0.976	0.926	
2006	0.986	0.953	
2007	0.968	0.926	
2008	0.968	0.921	
2009	0.984	0.941	
2010	0.989	0.954	
2011	0.901	0.937	
2012	0.997	0.956	
2013	0.946	0.909	
Mean	0.973	0.931	1.034

This time series displays the estimated gross public wage premium W_t^{pr} for the years 2001 through 2013, based on CPS data and the authors' calculations. The time-series mean is calculated according to the timing conventions used elsewhere in the paper. The compensation premium is equal to mean wages adjusted using the ratios of federal and private compensation to wages for workers who worked full time, provided by Falk (2012b).

or equivalent, an associate's degree, a bachelor's degree, or anything beyond a bachelor's degree), five racial categories (unknown, white, black, native American, and anything else), state dummies, coded sex, and a third-degree polynomial in age; b_G is a vector of coefficients; and $\varepsilon_{i,t}$ is a white noise error term. In estimating this regression, we restrict our sample to native-born US citizens who lived in one of the fifty states or DC, reported usual hours of work strictly greater than 34 per week, worked strictly more than 51 weeks, were strictly over 15 years in age, were wage and salary workers, and reported wage and salary income for that year. Unfortunately, we are not able to adequately control for occupation (which is endogenous to the choice of sector) or metro area, since these things are badly measured in our dataset, and our sample is too small to adequately control for (or group by) metro area. This is particularly unfortunate since this hinders a proper cross-sectional comparison between the BEA and the CPS datasets, or the use of a CPS-based public wage premium in our panel analyses. We also do not control for firm/establishment size, since this is also endogenous to the choice of sectors.

In order to arrive at a public wage premium based on these regressions, we then calculate an arithmetic public wage premium for each year t based on the sample analogue to following formula:

$$W_t^{pr} = \frac{\mathbb{E}\left(\exp\left(\gamma_{G,t} + \varepsilon_{i,t}\right) | \mathbb{1}_{G,i,t} = 1\right)}{\mathbb{E}\left(\exp(\varepsilon_{i,t}) | \mathbb{1}_{G,i,t} = 0\right)}. \tag{A.2}$$

This estimate of the arithmetic wage premium explicitly includes exponentiated error terms $\varepsilon_{i,t}$ which have a smaller variance for government workers than for private workers, because of the way that wages are bargained. This is important since Jensen's inequality implies that this wage compression should push the arithmetic government wage premium down below the geometric wage premium given by $\exp(\gamma_{G,t})$.

A time series of geometric and arithmetic means for the estimated object W_t^{pr} can be found in Table A.1. This table shows that, over the course of our sample, the average government worker earns a wage 0.931 times as large as the that of the average comparable private worker, and that this ratio is somewhat smaller than that estimated using a geometric mean. Furthermore, this table shows that this ratio has trended slightly upward over time, particularly during the Great Recession. This is in line with the idea that public wages only partially adjust in response to movements in private wages, since private wages were sluggish during that time. Furthermore, some additional checks reveal that our low estimates for the public wage ratio are caused by the presence of state and local workers in our sample. In fact, running the wage regression but separately controlling for federal or state and local workers yields an arithmetic mean federal wage premium of 1.150 and an arithmetic mean state and local wage premium of 0.879 (not shown). These results are much more in line with those of Falk (2012a).

However, it is important to point out that this measured wage premium excludes supplements to wages and salaries, particularly health care and pension benefits. This is an important omission, because as Falk (2012b) points out, a larger share of federal compensation than private compensation comes in the form of these supplements. In fact, using data provided in Table 1 of Falk (2012b), total compensation for full time federal workers, excluding paid leave, exceeds wages by a ratio of 1.461, while a similar ratio for private workers is given by 1.316. When we apply these ratios to the wage premium for the total government, thereby implicitly assuming that the government sector as a whole behaves like the federal sector in this respect, we arrive at a government compensation premium of 1.034 (or 1.0337). This suggests that the average government worker likely does earn a modest premium over a comparable private worker, though the exact size of this premium depends on the systematic ways in which the total government sector might differ from the federal sector with respect to benefits.

Table B.1
Simulated Moments from the directed search model, varying a and σ, baseline model.

Object	(1)	(2)	(3)	(4)
σ	0.00	1.00	1.00	1.00
a	0.37	0.37	0.37	0.50
ψ	1.42	0.00	1.42	1.42
Standard Deviations				
Log Private Employment	1.365	0.801	0.652	0.669
Log Private Wages	2.037	2.125	2.157	2.160
Log Productivity	1.804	1.808	1.809	1.808
Log GDP	2.930	2.475	2.351	2.365
Std as Share of GDP Std				
Log Private Employment	0.466	0.324	0.277	0.283
Log Private Wages	0.695	0.859	0.917	0.913
Log Productivity	0.616	0.730	0.769	0.765
log-Correlations with GDP				
Log Private Employment	0.988	0.992	0.991	0.991
Log Private Wages	0.995	1.000	1.000	1.000
Log Productivity	0.995	0.999	0.999	0.999

These tables list a set moments for the directed search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long and then discarding the first 16 years of observations. Standard deviations are reported in percent.

Appendix B. The directed search model

The main difference between the directed search model and our random search model is that workers are self-selected into a pool of searchers depending which job they seek to find. This requires modifying the labor market block of the model, while leaving technology, goods markets, and government untouched, see Table B.1.

Random search implies that the unemployed pool is divided into two pools–those searching for a public job (s_t^G) and those looking for a private job (s_t^P). These add up to the total number of searchers, such that:

$$u_t = s_t^P + s_t^G. \tag{B.1}$$

Each sector has its own vacancy and unemployment pool, implying that there are distinct labor market tightnesses for the private sector and the public sector, such that $\theta_p = \frac{v^P}{s^P}$ and $\theta_g = \frac{v^G}{s^G}$. As in Quadrini and Trigari we assume that public vacancies are equal to the amount of job hires in each period, or, in other words, that the matching function for the public sector is given by $\min v^G, s^G$, assuming, realistically, that for any meaningful parametrization of the model $s^G > v^G$. Changing the matching function from this setup would not change the equilibrium of our model but would complicate it.

The rates at which jobs are found by workers are then p_p ad p_g, which only depend on the respective market tightness. Similarly, the rate at which employers find workers also differs; we denote these rates as q_p and q_g. These depend only on market tightness in a given sector.

Now, the value functions for an unemployed worker that searches for a private job are:

$$V_{U,t}^P = h + E_t m_{t+1}[p_{p,t} V_{E,t+1}^P + (1 - p_{p,t}) V_{U,t+1}], \tag{B.2}$$

for one that searches for a public job has the value function:

$$V_{U,t}^G = h + E_t m_{t+1}[p_t^G V_{E,t+1}^G + (1 - p_{g,t}) V_{U,t+1}], \tag{B.3}$$

where V_E^P and V_E^G are the values of being employed in the private and government sectors, respectively. The objects m_t and β have the same interpretation as for the random search model. Our value function for unemployment does not have a sectoral subscript because, while we have two distinct value functions for unemployment, in equilibrium the choice of workers to enter in one pool or the other will establish an equal value of the two, such that:

$$V_{U,t} = V_{U,t}^G = V_{U,t}^P. \tag{B.4}$$

The values of being employed in the private and government sectors are:

$$V_{E,t}^P = W_t^P - b_t + E_t m_{t+1}[(1 - \lambda_p) V_{E,t+1}^P + \lambda_p V_{U,t+1}], \tag{B.5}$$

and:

$$V^G_{E,t} = W^G_t - b_t + E_t m_{t+1}[(1-\lambda_g)V^G_{E,t+1} + \lambda_g V_{U,t+1}], \tag{B.6}$$

where λ_p and λ_g have the same interpretation as before.

Subtracting the value of unemployment in the private sector from the value of employment gives the following 'worker surplus' equation:

$$V^P_{E,t} - V^P_{U,t} = W^P_t - b_t + (1-\lambda_p)E_t m_{t+1}[V^P_{E,t+1} - V_{U,t+1}], \tag{B.7}$$

where b_t has the same interpretation as before.

Turning to the values of posting a vacancy in the private sector $V^P_{V,t}$ and the value of a filled job $V_{J,t}$, we obtain the laws of motion:

$$V^P_{V,t} = -\kappa + E_t m_{t+1}[q_{p,t}V^P_{J,t+1} + (1-q^{p,t})V^P_{V,t+1}], \tag{B.8}$$

and:

$$V^P_{J,t} = \pi_t - W_{p,t} + E_t m_{t+1}[(1-\lambda_p)V^P_{J,t+1} + \lambda_p V^P_{V,t+1}], \tag{B.9}$$

where π_t is gross worker productivity as before.

Free entry implies that $V^P_{V,t} = 0$, which allows us to simplify our job posting equation such that:

$$\kappa = \beta E_t m_{t+1} q_{p,t} V^P_{J,t+1}, \tag{B.10}$$

and:

$$V^P_{J,t} = \pi_t - W_{p,t} + (1-\lambda_p)E_t m_{t+1} V^P_{J,t+1}, \tag{B.11}$$

which in turn gives us the equilibrium value of a filled job:

$$V^P_{J,t} = \pi_t - W^P_t + (1-\lambda_p)\frac{\kappa}{q_{p,t}}. \tag{B.12}$$

Nash bargaining with a worker's bargaining weight η implies:

$$\eta V^P_{J,t} = (1-\eta)(V^P_{E,t} - V_{U,t}), \tag{B.13}$$

which leads, differently from the case of the random search model, to a closed-form solution for the wage rate at time t, such that:

$$W_{p,t} = \eta\pi_t + (1-\eta)b_t + \eta\kappa\theta_p, \tag{B.14}$$

which together with the low of motion given by equations (B.10) and (B.12) can be combined and rewritten as:

$$\frac{\kappa}{q_{p,t}} = E_t m_{t+1}\left(\pi_{t+1} - W_{p,t+1} + (1-\lambda_p)\frac{\kappa}{q_{p,t+1}}\right). \tag{B.15}$$

This equation defines the dynamic equilibrium of the system.

To close the model, the number of unemployed workers is given as one minus the number of employed workers in each sector, such that:

$$u_t = 1 - n^G_t - n^P_t. \tag{B.16}$$

The law of motion for private employment is given by:

$$n^P_t = (1-\lambda_p)(n^g_{t-1} + p_{p,t-1}s^P_{t-1}), \tag{B.17}$$

and the law of motion for public employment (which implicitly gives the number of matches necessary) is:

$$n^P_t = (1-\lambda_g)(n^G_{t-1} + p_{g,t-1}s^G_{t-1}). \tag{B.18}$$

We simulate this model and report results the same way that we do for the random search model.

B.1. Results from the directed search model

We discuss one more issue related to specification, and that is the specification of random search versus directed search. To that end, we respecify our model as a directed search model (see Appendix B for model equations) and calibrate that model to the same targets as our baseline model. We find that the effects of private employment under directed search strongly resemble those effects in our random search model. Put another way, our results on the stabilizing effects of private employment are robust to model specification. The one difference between the model variants is that under directed search, higher public wages lead to slightly lower private wages, using directed search, and then we present a set of computational results from that model.

Table B.2
Implied empirical responses, varying a and σ, directed search model.

Object	(1)	(2)	(3)	(4)	SVAR	Bartik
σ	0.00	1.00	1.00	1.00		
a	0.37	0.37	0.37	0.50		
ψ	1.42	0.00	1.42	1.42		
Responses to log public wage shifter						
Volatility	10.370	1.624	-2.539	-2.438	*1.50*	*2.30*
Private wages	0.000	-0.012	-0.021	-0.016	0.08	0.27
Private employment	-0.850	-0.724	-0.645	-0.617	*0.12*	*0.20*
Responses to public employment shifter						
Volatility	6.738	2.497	-4.681	-4.484	-25.20	-23.10
Private wages	-0.000	-0.021	-0.064	-0.053	-0.10	-0.55
Private employment	-1.459	-1.265	-0.891	-0.809	-0.10	-0.33
Responses to log private productivity						
Volatility	-16.890	-2.660	1.653	1.551	-	-
Private wages	0.986	1.016	1.025	1.021	-	-
Private employment	1.057	0.765	0.682	0.659	-	-

This table lists a set (semi-)elasticities for the directed search model simulated at our baseline calibration. All moments are calculated running the model generating 300 quarterly independent series 250 years long, annualized, and then discarding the first 16 years of observations. Moments are the semi-elasticities, calculated as the log difference of the variable of interest resulting from incrementing and decrementing the control variable by a half percentage point. Empirical estimates are transformed to be comparable to their theoretical counterparts, with estimates in italics not statistically distinguishable from zero.

The key moments and derivatives from this model are shown in Table B.2. Within Table B.2, model variants (1) through (4) correspond with directed-search equivalents of the random-search model variants (1) through (4). Comparing both of these tables, model variant (1) ($\sigma = 0; a = 0.37$) gives the same qualitative results as under random search. This equilibrium features a strong degree of crowding out in response to both public employment and public wages. Furthermore, these results suggest that adding directed search to a model without product market effects will make it difficult to reconcile a negative effect of public employment on private wages with a positive effect of public wages on private wages. A recent paper by Chassamboulli and Gomes (2018) reports similar results. Model variant (2), furthermore, is not sufficient to generate a stabilizing effect of public employment, just like with the random search model.

Model variants (3) and (4) still feature a strong income effect channel. This channel implies that anything that reduces private employment will also reduce private wages, by way of a lower flow value of unemployment. Because of this effect, this model variant still cannot explain the positive relationship between public wages and private wages that we see in the data, although it generates a stabilizing effect for public employment in a manner similar to the random search model. Altogether, these results imply that a directed search model can match most of (but not all of) our main empirical facts, particularly when that model features a richer product markets that feed into the flow value of unemployment.

Appendix C. Proofs of Propositions 1 to 4

C.1. Proposition 1

To understand the first-order effects of E_g and W_g, we first look at the special case of a symmetric calibration, where $\lambda_g = \lambda_p$ and where $W_g = W_p$. Looking at the effects of public employment, differentiating the composite labor market equilibrium condition with respect to E_g implies:

$$\frac{dE_p}{dE_g} = \frac{\alpha_g}{\alpha_p} + \varpi \frac{dc}{dE_g}, \tag{C.1}$$

for some constant ϖ. However, since under the symmetric calibration, $\alpha_g = -\alpha_p$, then the derivative of E_p with respect to E_g is given by:

$$\frac{dE_p}{dE_g} = -1 + \varpi \frac{dc}{dE_g}. \tag{C.2}$$

Next, we need to look at $\frac{dc}{dE_g}$. To do this, first, we rearrange equation the average hiring cost function to express c as a function of E_p and E_g, such that:

$$c = \kappa \left(\frac{\lambda_g E_g + \bar{\lambda}_p E_p}{1 - E_p - E_g} \right)^{\frac{1-a}{a}}. \tag{C.3}$$

Differentiating this expression with respect to E_g implies that:

$$\frac{dc}{dE_g} = \xi \left[1 + \frac{dE_p}{dE_g}\right], \tag{C.4}$$

for some coefficient ξ. Combining this equation with equation (C.2) implies:

$$\frac{dE_p}{dE_g} = -1 + \varpi\xi \left[1 + \frac{dE_p}{dE_g}\right]. \tag{C.5}$$

Solving this expression for $\frac{dE_p}{dE_g}$ implies that $\frac{dE_p}{dE_g} = -1$, and that $\frac{dc}{dE_g} = 0$. This is a situation of perfect crowding out. Furthermore, differentiating θ implies that $\frac{d\theta}{dE_g} = 0$, and differentiating the vacancy posting equation (C.7) in Online Appendix implies that $\frac{dW_p}{dE_g} = 0$. These results imply that public employment should crowd out private employment by one-to-one without having an effect on private wages.

C.2. Proposition 2

Next, we turn to the effects of public wages. Implicitly differentiating the vacancy posting condition (C.7) in Online Appendix implies that the derivative of W_p with respect to W_g is given by:

$$\frac{dW_p}{dW_g} = -(r + \bar{\lambda}_p)\frac{dc}{dW_g}. \tag{C.6}$$

To investigate this derivative, it is necessary to look at $\frac{dc}{dW_g}$. To do this, differentiating the expression (C.3) with respect to W_g implies that:

$$\frac{dc}{dW_g} = \xi\frac{dE_p}{dW_g}, \tag{C.7}$$

for the same positive value of ξ from above. Substituting this into the expression (C.6) implies:

$$\frac{dW_p}{dW_g} = -(r + \bar{\lambda}_p)\xi\frac{dE_p}{dW_g}. \tag{C.8}$$

Turning to the calculation of this derivative, differentiating equation (C.11) in Online Appendix with respect to W_g, after some algebra, implies:

$$\alpha_p\frac{dE_p}{dW_g} + \left[E_p(r + \bar{\lambda}_g)\eta\bar{\lambda}_p + (1 - E_p)(r + \bar{\lambda}_g)(r + \bar{\lambda}_p) - E_g r(r + \bar{\lambda}_p)\right]\xi\frac{dE_p}{dW_g} = 0. \tag{C.9}$$

This expression implies that, for meaningful calibrations, $\frac{dE_p}{dW_g} < 0$, which in turn given the previous equation implies that $\frac{dW_p}{dW_g} > 0$.[8]

C.3. Propositions 3 and 4

Taken together, these results show that an increase in E_g or W_g should be expected to increase the outside option of employed workers, in turn putting downward pressure on private employment. In addition, this increase in the outside option can be expected to increase the amount of volatility faced by the economy, where we use as our proxy for volatility the sensitivity of log private employment to productivity in the steady state given by $d\ln(E_p)/d\ln(\pi)$, which for a value of π equal to one, is equal to $d\ln(E_p)/d\pi$.[9] This derivative is given by:

$$\frac{d\ln(E_p)}{d\pi} = \frac{(r + \bar{\lambda}_g)(1 - \eta)(1 - E_p) - r(1 - \eta)E_g}{E_p\alpha_p}, \tag{C.10}$$

which for meaningful calibrations, is positive.

After taking derivatives of this derivative, the effect of public employment on business cycle volatility is given by:

$$\frac{d}{dE_g}\left[\frac{d\ln(E_p)}{d\pi}\right] = -\frac{r(1 - \eta)}{E_p\alpha_p} - \left[\frac{(r + \bar{\lambda}_g)(1 - \eta) - r(1 - \eta)E_g}{E_p^2\alpha_p}\right]\frac{dE_p}{dE_g}$$

[8] By meaningful calibrations, we refer to calibrations for which the model has an interior solution.

[9] Differentiating log volatility in this manner allows us to avoid having to calibrate the level of volatility.

$$- \left[\frac{(r + \bar{\lambda}_g)(1-\eta) - r(1-\eta)E_g}{E_p \alpha_p^2} \right] (r + \bar{\lambda}_g)(\eta \bar{\lambda}_p - (r + \bar{\lambda}_g)) \frac{dc}{dE_g}, \quad \text{(C.11)}$$

which in principle is ambiguous. However, as a special case, focusing on a symmetric calibration gives:

$$\frac{d}{dE_g} \left[\frac{d \ln(E_p)}{d\pi} \right] = \frac{(r + \bar{\lambda}_g)(1-\eta) - r(1-\eta)(E_g + E_p)}{E_p^2 \alpha_p} > 0, \quad \text{(C.12)}$$

which implies that higher public employment should result in higher private sector business cycle volatility, which is at odds with our empirical estimates.

Next, we look at the effect of public wages on business cycle volatility. This is given by:

$$\frac{d}{dW_g} \left[\frac{d \ln(E_p)}{d\pi} \right] = - \left[\frac{(r + \bar{\lambda}_g)(1-\beta) - r(1-\beta)E_g}{E_p^2 \alpha_p} \right] \frac{dE_p}{dW_g}$$
$$- \left[\frac{(r + \bar{\lambda}_g)(1-\eta) - r(1-\eta)E_g}{E_p \alpha_p^2} \right] (r + \bar{\lambda}_g)(\eta \bar{\lambda}_p - (r + \bar{\lambda}_g)) \frac{dc}{dW_g}, \quad \text{(C.13)}$$

which is in principle ambiguous, even under the symmetric calibration. However, under the special case when $a = 1$ and hence $\frac{dc}{dW_g} = 0$, the fact that $\frac{dE_p}{dW_g} < 0$ implies that $\frac{d}{dW_g} \left[\frac{d \ln(E_p)}{d\pi} \right]$ is positive, for meaningful calibrations. This implies that higher public wages could destabilize private employment, as we show later on in our simulations. On this matter, the empirical estimates are ambiguous.

Appendix D. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.red.2024.05.001.

References

Afonso, António, Gomes, Pedro, 2014. Interactions between private and public sector wages. Journal of Macroeconomics 39A, 97–112.

Albrecht, James, Robayo-Abril, Monica, Vroman, Susan, 2015. Public Sector Employment in an Equilibrium Search and Matching Model. Mimeo.

Andrés, Javier, Doménech, Rafael, Fatás, Antonio, 2008. The stabilizing role of government size. Journal of Economic Dynamics and Control 32 (2), 571–593.

Bartik, Timothy, 1991. Who Benefits from State and Local Economic Development Policies? W.E. Upjohn Institute.

Blanchard, Olivier, Perotti, Roberto, 2002. An empirical characterization of the dynamic effects of changes in government spending and taxes on output. The Quarterly Journal of Economics 117 (4), 1329–1368. Oxford University Press.

Baxter, Marianne, King, Robert G., 1993. Fiscal policy in general equilibrium. The American Economic Review 83 (3), 315–334.

Borjas, George J., 2002. The wage structure and the sorting of workers into the public sector. NBER Working Paper 9313.

Bradley, Jake, Postel-Vinay, Fabien, Turon, Hélène, 2015. Public Sector Wage Policy and Labor Market Equilibrium: a Structural Model. Mimeo.

Brügemann, Björn, 2008. What Elasticity of the Matching Function Is Consistent with U.S. Aggregate Labor Market Data? Vrije Universiteit Amsterdam. Mimeo.

Burdett, Ken, 2012. Towards a theory of the labor market with a public sector. Labour Economics 19 (1), 68–75.

Burdett, Ken, Mortensen, Dale, 1998. Wage differentials, employer size and unemployment. International Economic Review 39 (2), 257–273.

Caponi, Vincenzo, 2017. Public employment policies and regional unemployment differences. Regional Science and Urban Economics 63 (C), 1–12.

Chassamboulli, A., Gomes, P., 2018. Meritocracy, public-sector pay and human capital accumulation. Mimeo.

Chodorow-Reich, Gabriel, Karabarbounis, Loukas, 2015. The Cyclicality of the Opportunity Cost of Employment. Mimeo.

Elsby, Michael W.L., Michaels, Ryan, 2013. Marginal jobs and heterogeneous firms. American Economic Journal: Macroeconomics 5 (1), 1–48.

Falk, Justin, 2012a. Comparing Wages in the Federal Government and the Private Sector. CBO Working Paper 2012-3, January 2012.

Falk, Justin, 2012b. Comparing Benefits and Total Compensation in the Federal Government and the Private Sector. CBO Working Paper 2012-4, January 2012.

Fatás, Antonio, Mihov, Ilian, 2001. Government size and automatic stabilizers: international and intranational evidence. Journal of International Economics 55 (1), 3–28.

Galí, Jordi, 1994. Government size and macroeconomic stability. European Economic Review 38 (1), 117–132.

Gomes, Pedro, 2015. Optimal public sector wages. The Economic Journal 125 (587), 1425–1451.

Hagedorn, Marcus, Manovskii, Iourii, 2008. The cyclical behavior of equilibrium unemployment and vacancies revisited. The American Economic Review 98 (4), 1692–1706.

Hall, Robert E., Milgrom, Paul R., 2008. The limited influence of unemployment on the wage bargain. American Economic Review 98 (4), 1653–1674.

Katz, Lawrence F., Krueger, Alan B., 1991. Changes in the structure of wages in the public and private sectors. Research in Labor Economics 12, 137–172.

Levin, Andrew, Lin, Chien-Fu, Chun, Chia-Shang James, 2002. Unit root tests in panel data: asymptotic and finite-sample properties. Journal of Econometrics 108 (1), 1–24.

Lütkepohl, Helmut, 1990. Asymptotic distributions of impulse response functions and forecast error variance decompositions of vector autoregressive models. The Review of Economics and Statistics 72 (1), 116–125.

Quadrini, Vincenzo, Trigari, Antonella, 2007. Public employment and the business cycle. Scandinavian Journal of Economics 109 (4), 723–742.

Petrongolo, Barbara, Pissarides, Christopher A., 2001. Looking into the black box: a survey of the matching function. Journal of Economic Literature 39 (2), 390–431.

Ramey, Valerie A., 2011. Identifying Government Spending Shocks: It's all in the Timing The Quartely Journal of Economics 126 (1), 1–50.

Ramey, Valerie A., 2016. Macroeconomic Shocks and Their Propagation. In: Taylor, J.B., Uhlig, Harald (Eds.), Handbook of Macroeconomics, vol. 2. Elsevier, pp. 71–162.

Reicher, Claire A., 2014. A set of estimated fiscal rules for a cross-section of countries: stabilization and consolidation through which instruments? Journal of Macroeconomics 42, 184–198.

Shimer, Robert, 2005. The cyclical behavior of equilibrium unemployment and vacancies. The American Economic Review 95 (1), 25–49.

Silva, Jose I., Toledo, Manuel, 2009. Labor turnover costs and the cyclical behavior of vacancies and unemployment. Macroeconomic Dynamics 13 (S1), 76–96.