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DO WE NEED MORE TIME FOR LEISURE?

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Abstract

"We need more time: more time for leisure" Linton Kwesi Jonhson used to dub. Our study of an OLG economy with endogenous labor supply gives a rational to the dub poet's claims. In the model we present the golden rule is defined as the pair of capital-labour ratio and individual labour supply which maximises the steady state utility of each generation. When, other things equal, agents prefer to work more the higher (lower) the opportunity cost of leisure, individual labour supply as a function of capital per unit of labour reaches a minimum (maximum) at the golden age. Therefore, a reduction in the length of the working week could well be welfare improving whenever labour supply is increasing (decreasing) in the opportunity cost of leisure and the economy is getting closer to (farther from) the golden age.

JEL codes: J22, E21, O44

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1. Introduction

Dynamic efficiency has been a key issue in the growth literature since the seminal contributions by Phelps (1961,1962,and 1965). As shown by Samuelson (1958) and Diamond (1965), infinite-horizon competitive economies with finite-lived agents might reach dynamically inefficient steady states such that the marginal rate of return to physical capital would be lower than the rate of growth of population. Under these circumstances, Pareto optimality can be restored by imposing an intergenerational redistribution scheme from young to old individuals opportunely tuned so to achieve the Golden Rule level of physical capital accumulation, which is defined as the long run level of capital that maximises per-capita consumption of each generation.

The empirical evidence produced by Abel et al. (1989) indicates that economies such as the US and the OECD countries are indeed dynamically efficient (see also Feldstein adn Summers (1977)). Accordingly, various economists have argued in favour of policies aimed at increasing the propensity to save. In fact, as discussed by Pagano and Jappelli (1999), while forcing higher savings by repressing consumption credit in a dynamically efficient overlapping generation (OLG) economy is welfare-detrimental for the very first generation, it would make all subsequent generations better off. [1] However, other authors, like for instance Ibbotson (1987) and Mishkin (1984) find that, in the time interval which goes from the *20s* until the *80s*, both in the US and in various other countries the mean value of riskless interest rates was well below the economy's average growth rate, which would rather indicate a situation of dynamic inefficiency. If anything, in accordance to that, one should recommend an intergenerational redistribution aimed at slowing the accumulation of capital rather than at encouraging it.

Independently of the endless debate on whether economies are dynamically efficient or not,[2] the fact that market economies might fail to achieve the optimal steady state process of capital accumulation, which Phelps defines as Golden Age, provides a strong motivation for the large literature focusing on the welfare properties of long run growth paths and the related policy issues. While many contributions in that area rely on the hypothesis of inelastic labour supply, there

[1] Hence, their statement that "[...] the Pareto criterion is extremely demanding and need not forestall all policies intervention. Even if there is not obvious candidate for a social welfare function when households are not altruistically linked, any benevolent planner will try to trade-off the interests of current and future generations by weighting their utility appropriately [...]"(Jappelli and Pagano, 1999, p...).

[2] Blanchard and Fisher (1989) argue that at the time the debate was still open.

is evidence that aggregate labour supply moves over time both at high frequencies (see Lucas and Rapping (1969), Hansen (1985) and Rogerson (1988)), and low frequencies (see Maddison (1991), (1995) and Evans et al. (2001)). Such empirical regularities motivate the study of growth models with endogenous labour supply at least on two different grounds: *i.* what are the consequences of labour supply decisions for the steady state stock of accumulable inputs; *ii.* how labour supply changes with the steady state level of accumulable inputs.

There is a well established literature based on endogenous labour decisions that reconsiders the various benchmark results obtained assuming inelastic labour supply, like for instance the growth-effects of taxation (Stokey and Rebelo 1995)), the accumulation of human capital (Ortiguera 2000, Duranton, 2001), and the interplay between portfolio and retirement decisions (Kingston 2000). Some of these studies are however based on Ramsey-type models and have therefore little to say about the dynamic efficiency properties of the decentralised steady-state equilibria. Others, while based on the OLG framework, neither look at the golden value of individual labour supply, nor they provide an analysis of how labour supply differs from its golden value depending on the steady state level of capital accumulation.

Both these two aspects appear to be relevant from a welfare view-point: *a.* What is the golden rule level of labour supply? *b.* Is the level of labour supply greater or smaller than its golden rule level when the economy is dynamically efficient/inefficient and how does it change as the economy gets closer to its Golden Age?

This paper constitutes a prime attempt to explore these questions. We restrict our attention to a standard Diamond model in which production is characterised by decreasing returns in accumulable inputs. Agents are identical and live for two periods, have a linearly-separable utility function defined over consumption in both periods and leisure, and supply labour when young to finance life-time consumption. The paper is closely related to Nourry (2001) and Nourry and Venditti (2001) who provide a more general analysis of the dynamics of OLG models with endogenous labour supply. However, differently from these contributions, we focus our analysis on the welfare properties of the steady state achieved by the *laissez faire* economy.

The model we consider yields a steady state equilibrium such that the level of labour supply is a function of the steady state level of capital. The *Golden Age* is consequently defined as the pair of capital labour ratio and labour supply which maximises the steady state utility of the representative agent subject to the typical resource constraint faced by a benevolent dictator. Clearly enough, steady state labour supply being determined by the steady state level of capital implies that labour does not play any role as a policy to achieve the *Golden Age*. But, how much do agents work in the decentralised economy compared to the golden rule level of labour supply? And, how does individual labour supply change if the economy's steady state gets closer to the *Golden Age*? Our main finding is that, whenever agents tend to work more the higher is the opportunity cost of leisure, individual labor supply will be decreasing in the steady-state capital labor ratio as long as the economy is dinamically efficient, and increasing otherwise. These conclusions are reversed whenever agents tend to work less the higher the opportunity cost of leisure. In our framework, the opportunity cost of leisure is correctly defined as the utility loss from forgone consumption, taking into account the intertemporal allocation of income preferred by the individual. The intuition behind our results lies in the effects induced by a change in the steady state level of capital on the opportunity cost of leisure so defined. When the economy is dynamically efficient, the effect of an increase in the steady state value of capital per unit of labour on the level of wages and the future value of savings[3] is such that the opportunity cost of leisure is reduced. Therefore, agents driven by the substitution effect will work less, while those driven by the income effect will work more. These conclusions are reversed in the case of a dynamically inefficient economy. Agents driven by the substitution effect (income effect) in their labour-leisure decision would find it optimal to work more (less) as capital increases.

To put it in other words, if agents are driven by the substitution effect (income effect), individual labor supply considered as a function of the capital labour ratio, reaches a global minimum (maximum) at the golden age.

These results have some potential implications for the 35 hours-debate in Europe, which up until now has been questioned from an academic perspective mainly within standard labour economics literature such as in the case of Marimon and Zilibotti (1999). Kaldor's stylised fact that the capital-output ratio should be constant is widely accepted by economic theorist. The constancy of this ratio directly implies that, with constant returns of scale, also capital per unit of effective labour should be constant. However, empirical evidence based both on the World Penn Data and the Extended World Penn Data developed by Duncan Foley and Adalmir Marquetti shows that in countries such as the EU, the US, Japan, Canada and Australia, the capital-output ratio has been growing steadily in the period

[3] Note that the future value of savings will generally depend both on the wage and the interest rate.

1963-90, which leads to a legitimate presumption that also the long run level of capital per unit of effective labour has been indeed growing also. If so, our analysis suggests that, provided that these economies are dynamically efficient and agents are driven by the substitution effect in their labour-leisure decisions, it might indeed be optimal to reduce individual labour supply over time, which might offer a justification for the request of a reduction in the length of the working week, assuming this was originally set optimally by the law.

It is needless to say, that a proper assessment of the above implications would require a model which combines endogenous labour supply and long run growth, which is beyond the scope of this paper where we are only concerned with changes in the level of labour supply induced by changes in the steady state level of capital-labour ratio.

The paper is organised as follows. Section 2 presents the model. Section 3 describes the dynamics and the steady state properties. Section 4 carries out the welfare analysis. A final section concludes.

2. The Model

We consider a perfectly competitive one-good closed economy populated by a continuum of size 1 of infinitely-lived atomistic firms and overlapping generations of individuals. At each time t a new generation of N_t individuals is born. Each of these individuals lives for two periods and gives birth to $1 + n$ individuals in her/is second period of life. Hence, the generation-size evolves according to $N_{t+1} = (1 + n)N_t$. Individuals are born with no endowment but one unit of time which they allocate to labour and leisure in their first period of life. In the first period of their life they supply labour to firms, consume part of the resulting income and save the rest to finance second period consumption. Agents derive direct utility from consumption in the two period of their life and leisure (only in the first period of life). Their preferences are described by the following CES function

$$U_t = \frac{1}{1-\sigma}\left(c_{1t}^{1-\sigma} + \frac{1}{1+\rho}c_{2t+1}^{1-\sigma} + b(1-l_t)^{1-\sigma}\right), \tag{2.1}$$

where U_t is the individual utility of a member of generation t, c_{1t} and c_{2t+1} represent consumption in the first and the second period of life respectively, $l_t \in [0, 1]$ is labour supply, so that $1 - l_t$ is leisure, ρ is the subjective discount rate, b and σ are positive parameters.

Firms use a constant return to scale technology described by

$$Y_t = F(K_t.L_t) = K_t^{\alpha} L_t^{1-\alpha}, \tag{2.2}$$

where Y_t is production, K_t is physical capital, L_t is labour and $\alpha < 1$ is the product-share of capital. Labour and capital are paid their marginal productivity:

$$\begin{aligned} w_t &= (1-\alpha) K_t^{\alpha} L_t^{-\alpha} &(2.3)\\ r_t &= \alpha K_t^{\alpha-1} L_t^{1-\alpha} - 1, &(2.4) \end{aligned}$$

where, for simplicity, we assume full capital depreciation.

2.1. The individual problem

Each young individual maximises (2.1) subject to the standard constraint

$$c_{1t} + \frac{c_{2t+1}}{R_{t+1}} = w_t l_t, \tag{2.5}$$

where $R_{t+1} = 1 + r_{t+1}$ is the gross interest rate. Condition (2.5) states that the present value of consumption should be equal to the labour income $w_t l_t$. The maximisation problem faced by each individual is

$$\max_{\{l_t, c_{1t}, c_{2t+1}\}} L = \frac{1}{1-\sigma}\left(c_{1t}^{1-\sigma} + \frac{1}{1+\rho} c_{2t+1}^{1-\sigma} + b(1-l_t)^{1-\sigma}\right) + \lambda(w_t l_t - c_{1t} - \frac{c_{2t+1}}{R_{t+1}}), \tag{2.6}$$

where λ is the Lagrangean multiplier. The solution implies the following saving (S_t^*) and labour supply (l_t^*) choices

$$\begin{aligned} S_t^* &= \frac{1}{1+(1+\rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}}} w_t l_t^* = s(R_{t+1}) w_t l_t^* &(2.7)\\ l_t^* &= \frac{1+(1+\rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}}}{1+(1+\rho)^{\frac{1}{\sigma}} + (R_{t+1} w_t)^{\frac{\sigma-1}{\sigma}} (1+\rho)^{\frac{1}{\sigma}} b^{\frac{1}{\sigma}}} = l\left(R_{t+1}, w_t\right), &(2.8) \end{aligned}$$

where

$$s(R_{t+1}) = \frac{1}{1+(1+\rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}}} \tag{2.9}$$

is the propensity to save. As usual, savings are increasing in R_{t+1} for $\sigma < 1$ and decreasing otherwise.[4] Similarly, for the labour supply, $\sigma < 1$ implies $\partial l_t^*/\partial w_t > 0$ and $\partial l_t^*/\partial R_{t+1} > 0$, while the opposite is true for $\sigma > 1$.[5] Finally, S_t^* is increasing in w_t if $\sigma < 1$, while the impact of a change in w_t is ambiguous if $\sigma > 1$.[6] Summing up

$$\left.\begin{aligned} S_t^* &= \widehat{S}(\overset{+}{R}_{t+1}, \overset{+}{w}_t) \\ l_t^* &= l\left(\overset{+}{R}_{t+1}, \overset{+}{w}_t\right) \end{aligned}\right. \text{ if } \sigma < 1$$

and

$$\left.\begin{aligned} S_t^* &= \widehat{S}(\overset{-}{R}_{t+1}, \overset{?}{w}_t) \\ l_t^* &= l\left(\overset{-}{R}_{t+1}, \overset{-}{w}_t\right) \end{aligned}\right. \text{ if } \sigma > 1.$$

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[4] Differentiation of S_t^* with respect to R_{t+1} yields

$$\frac{\partial S_t^*}{\partial R_{t+1}} = \frac{ds(R_{t+1})}{dR_{t+1}} w_t l_t^* + \frac{\partial l_t^*}{\partial R_{t+1}} s(R_{t+1}) w_t.$$

Since $s(R_{t+1})\ w_t$ and l_t^* are always nonnegative, we have that $\dfrac{\partial S_t^*}{\partial R_{t+1}}$ is positive (negative) if both $\dfrac{ds(R_{t+1})}{dR_{t+1}}$ and $\dfrac{\partial l_t^*}{\partial R_{t+1}}$ are positive (negative). Moreover, since, these derivatives have always a common sign, $\dfrac{\partial S_t^*}{\partial R_{t+1}} > 0$ follows if $\sigma < 1$.

[5] Obviously, if $\sigma = 1$, both labour supply and propensity to save are constant: $l_t^* = (2 + \rho)/(2 + \rho + b\,(1 + \rho))$, $s = 1/(2 + \rho)$.

[6] The derivative of S_t^* with respect to w_t is

$$\frac{\partial S_t^*}{\partial w_t} = l_t^* + w_t \frac{\partial l_t^*}{\partial w_t}.$$

Hence, $\partial S_t^*/\partial w_t$ is surely positive if $\sigma < 1$ so that $\partial l_t^*/\partial w_t > 0$ holds. However as long as $\sigma > 1$, $\partial l_t^*/\partial w_t < 0$ follows so that the sign of $\partial S_t^*/\partial w_t$ becomes ambiguous. In this case the sign will depend on the magnitude of $|\partial l_t^*/\partial w_t|$. In particular $\partial S_t^*/\partial w_t < (>) 0$, if $e_w^l < (>) -1$, where

$$e_w^l = \frac{\partial l_t^*}{\partial w_t} \frac{w_t}{l^*}$$

is the elasticity of the work effort with respect to wages. Notice that $\partial S_t^*/\partial w_t < 0$ only if $\sigma >> 1$.

3. Dynamics and Steady state

In equilibrium $L_t = N_t l_t$ holds[7], so that output per effective worker, $Y_t/N_t l_t$, is equal to $y_t = f(k_t) = k_t^{\alpha}$, where $k_t = K_t/N_t l_t$ is capital per effective worker. The equilibrium values of the interest and the wage rates are given by

$$\begin{aligned} r_t &= \alpha k_t^{\alpha-1} - 1 && (3.1)\\ w_t &= (1-\alpha)\, k_t^{\alpha}. && (3.2) \end{aligned}$$

Capital evolves over time according to

$$K_{t+1} = S_t N_t, \qquad (3.3)$$

which, given expression (2.7), implies

$$(1+n) k_{t+1} l_{t+1} = \frac{1}{1 + (1+\rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}}} w_t l_t. \qquad (3.4)$$

Substituting for the equilibrium values of w_t and r_{t+1} we obtain the following accumulation equation

$$(1+n) k_{t+1} l_{t+1} = \frac{(1-\alpha)\, k_t^{\alpha}}{1 + (1+\rho)^{\frac{1}{\sigma}} \left(\alpha k_{t+1}^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}} l_t. \qquad (3.5)$$

The equilibrium level of labour supply evolves according to equation (2.8). Hence, the dynamics of the economy is described by the following system

$$\begin{aligned} k_{t+1} &= \frac{l_t\,(1-\alpha)\, k_t^{\alpha}}{(1+n)\, l_{t+1} \left[1 + (1+\rho)^{\frac{1}{\sigma}} \left(\alpha k_{t+1}^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}\right]} && (3.6)\\ l_{t+1} &= \frac{1 + (1+\rho)^{\frac{1}{\sigma}} \left(\alpha k_{t+2}^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}}{1 + (1+\rho)^{\frac{1}{\sigma}} + \left[(1-\alpha)\, k_{t+1}^{\alpha} \left(\alpha k_{t+2}^{\alpha-1}\right)\right]^{\frac{\sigma-1}{\sigma}} \left[(1+\rho)\, b\right]^{\frac{1}{\sigma}}} && (3.7) \end{aligned}$$

[7] In all subsequent analysis we drop the "$*$" for simplicity.

3.1. Steady state: existence and stability

A *dynamic equilibrium* is a sequence $\{k_t, l_t\}_{t=0}^{\infty}$ that satisfies equations (3.6) and (3.7) with k_0 exogenously given. Given the accumulation equation (3.5), any steady state level of capital k satisfies

$$\phi(k) = (1+n)k - \frac{(1-\alpha)\,k^{\alpha}}{1+(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}} = 0. \tag{3.8}$$

In order to investigate the existence of non-trivial steady state values we focus on[8]

$$\psi(k) = \frac{\phi(k)}{k} = (1+n) - \frac{(1-\alpha)\,k^{\alpha-1}}{1+(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}}. \tag{3.9}$$

Proposition 1. *Our economy experiences a unique and saddle point stable steady state equilibrium $k^* : \psi(k^*) = 0$.*

Proof. $\psi(k)$ is a continuous function of k for $k \in (0, \infty)$. Moreover

$$\lim_{k\to\infty} \psi(k) = \lim_{k\to\infty} (1+n) - \frac{(1-\alpha)\,k^{\alpha-1}}{(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}+1} = (1+n) \tag{3.10}$$

$$\lim_{k\to 0} \psi(k) = \lim_{k\to 0} (1+n) - \frac{(1-\alpha)\,k^{\alpha-1}}{(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}+1} = -\infty, \tag{3.11}$$

where, in the case of $\sigma > 1$, (3.11) follows from the application of the Hôpital's rule. It then follows that there exist at least one value of k, call it k^*, such that $\psi(k^*) = 0$ holds, which means that our economy admits at least a steady state equilibrium k^*. As for uniqueness, we only need $\psi(k)$ to be monotonic in k. It can be easily verified that

$$\psi'(k) = \frac{(1-\alpha)^2 k^{\alpha-2}\left(1+\frac{1}{\sigma}(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}\right)}{\left[(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}+1\right]^2} > 0 \tag{3.12}$$

[8] Notice that in the standard Diamond model, $k = 0$ is a steady state equilibrium if $f(0) = 0$. But, as Nourry (2001) points out, with endogenous labor supply agents do not work if there is no production so that $f(k)$ is no longer defined for $k = 0$. Thus no trivial steady state exists ($k = 0$) and the dynamical system can be such that there is no steady-state equilibrium.

for all $k \in (0, \infty)$ which directly implies that k^* is unique. Therefore both the existence and the uniqueness of k^* are guaranteed for any value of $\sigma > 0$. As for stability, we can notice that since we have CES utility function and Cobb-Douglas production function, (which has a elasticity of capital labor substitution equal to 1), we can apply the sufficient condition stated by Nourry-Venditti (2001) according to which in a CES economy, if the elasticity of capital-labor substitution is greater than or equal to 1, the steady state is saddle-point stable.■

4. Welfare analysis

Since the seminal work by Diamond (1965) and Samuelson (1958) we are aware of the fact that OLG economies may well suffer of dynamic inefficiency, a situation which, applying the standard golden rule definition to our model, corresponds to a value of k such that

$$k > k^{GR} \equiv \left(\frac{\alpha}{1+n}\right)^{\frac{1}{1-\alpha}}, \tag{4.1}$$

where k^{GR} is the golden rule level of capital labour ratio, i.e. that particular level of k that maximises each generation per-capita consumption, a sufficient condition for individual welfare maximisation when utility depends only on consumption. However, in our model, individual utility depends both on consumption and labour supply. Therefore, labour supply should be taken into account when computing the golden rule for this economy. In particular, whenever all generations carry the same weight in the social welfare function, in order to maximise the steady state utility of each generation, the central planner will choose a pair $\{k^{GR}, l^{GR}\}$. In other words, the golden rule for our economy, g, is defined as $g = (k^{GR}, l^{GR})$. Formally, g solves the following problem

$$\begin{aligned} \max_{c_1,c_2,l,k} U_t &= \frac{1}{1-\sigma}\left(c_1^{1-\sigma} + \frac{1}{1+\rho}c_2^{1-\sigma} + b(1-l)^{1-\sigma}\right) \qquad (4.2) \\ s.t.\ c_1 + \frac{c_2}{1+n} &= l(k^\alpha - k(1+n)) \end{aligned}$$

where the steady-state resource constraint has been derived by imposing the steady-state conditions on the following aggregate constraint

$$K_t^\alpha L_t^{1-\alpha} = K_{t+1} + c_{1t}N_t + c_{2t}N_{t-1}. \tag{4.3}$$

Accordingly, the centralised solution implies

$$
\begin{aligned}
k^{GR} &= \left(\frac{\alpha}{1+n}\right)^{\frac{1}{1-\alpha}} && (4.4)\\
l^{GR} &= \frac{(1+\rho)^{\frac{1}{\sigma}}+(1+n)^{\frac{1-\sigma}{\sigma}}}{(1+\rho)^{\frac{1}{\sigma}}+(1+n)^{\frac{1-\sigma}{\sigma}}+\left[(1-\alpha)\left(\frac{\alpha}{1+n}\right)^{\frac{\alpha}{1-\alpha}}\right]^{\frac{\sigma-1}{\sigma}}\left[b\left(1+\rho\right)\right]^{\frac{1}{\sigma}}} && (4.5)
\end{aligned}
$$

4.1. How does labour supply behave getting closer to the golden rule?

The steady state values for k and l in the decentralised economy are

$$
\begin{aligned}
k &= \frac{(1-\alpha)\,k^{\alpha}}{(1+n)\left[(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}+1\right]} && (4.6)\\
l &= \frac{1+(1+\rho)^{\frac{1}{\sigma}}\left(\alpha k^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}}{1+(1+\rho)^{\frac{1}{\sigma}}+\left[\alpha\,(1-\alpha)\,k^{2\alpha-1}\right]^{\frac{\sigma-1}{\sigma}}\left[(1+\rho)\,b\right]^{\frac{1}{\sigma}}} && (4.7)
\end{aligned}
$$

A comparison with the expressions (4.4) and (4.5) suggests that, as in the standard Diamond model, it is not guaranteed that the decentralised economy will spontaneously reach the *golden age* (Phelps, 1961). Also, in our model, for any $k \neq k^{GR}$,the agents will choose a level of labour supply l which is generally different from l^{GR}. However, l is a function of k such that if $k = k^{GR}$, then $l(k^{GR}) = l^{GR}$ follows. Therefore, setting k to its golden rule level is a *sufficient* condition for the economy to reach the *golden age:* $g = (k^{GR}, l^{GR})$.

Imagine a stationary economy which is not in its *golden age*, i.e. $(k, l) \neq g$. We then face the following question: under which conditions is the golden rule level of labour higher or lower than the steady state level reached by the decentralised economy?

The aim of this question is to investigate whether individuals work more or less than in the golden age, given a steady state level of capital k. In particular, how does the steady state level of labour supply l compare with the golden rule level l^{GR} assuming that k is respectively greater (dynamic inefficiency) or lower than k^{GR}? In other words, we are interested in distinguishing between the cases in which welfare maximisation is associated with a reduction of the level of labour supply relatively to that associated with the decentralised economy, as opposed to situations in which the reverse is true.

In steady state, $l(k) = l(R(k), w(k))$, (see equation (4.7)). Therefore, the labour supply l changes with k according to

$$l'(k) = l_w \frac{dw}{dk} + l_R \frac{dR}{dk}, \tag{4.8}$$

where $l_w(k)$ and $l_R(k)$ are the partial derivatives of l with respect to w and R respectively, evaluated at steady state. Given equation (4.7), we have

$$\begin{aligned} l_w &= \frac{\left(\frac{1-\sigma}{\sigma}\right) R(k)^{\frac{\sigma-1}{\sigma}} w(k)^{-\frac{1}{\sigma}} \left[b(1+\rho)\right]^{\frac{1}{\sigma}} \left[1+(1+\rho)^{\frac{1}{\sigma}}\right]}{\left[1+(1+\rho)^{\frac{1}{\sigma}} + \left[R(k)w(k)\right]^{\frac{\sigma-1}{\sigma}} \left[b(1+\rho)\right]^{\frac{1}{\sigma}}\right]^2} \\ l_R &= \frac{\left(\frac{1-\sigma}{\sigma}\right) R(k)^{-1} w(k)^{\frac{\sigma-1}{\sigma}} \left[b(1+\rho)\right]^{\frac{1}{\sigma}}}{\left[1+(1+\rho)^{\frac{1}{\sigma}} + \left[R(k)w(k)\right]^{\frac{\sigma-1}{\sigma}} \left[b(1+\rho)\right]^{\frac{1}{\sigma}}\right]^2}. \end{aligned}$$

By substituting for l_w and l_R we obtain the following

$$l'(k) \gtreqless 0 \Leftrightarrow \begin{cases} \varepsilon_{w,k} \gtreqless \varepsilon_{R,k} s(R(k)) \text{ if } \sigma < 1 \\ \varepsilon_{w,k} \lesseqgtr \varepsilon_{R,k} s(R(k)) \text{ if } \sigma > 1 \end{cases} \tag{4.9}$$

where

$$\begin{aligned} \varepsilon_{w,k} &= \left| \frac{dw}{dk} \cdot \frac{k}{w(k)} \right| \\ \varepsilon_{R,k} &= \left| \frac{dR}{dk} \cdot \frac{k}{R(k)} \right| \\ s(R(k)) &= \frac{1}{1+(1+\rho)^{\frac{1}{\sigma}} R(k)^{\frac{\sigma-1}{\sigma}}}. \end{aligned}$$

Note that $\varepsilon_{w,k}$ and $\varepsilon_{R,k}$ measure the elasticities of wages and gross interest rate with respect to k. Hence, condition (4.9) can be interpreted as follows. Let us assume a marginal positive change in k. If $\varepsilon_{w,k} - \varepsilon_{R,k} s(R(k)) > 0$, the relative change in labour income, net of the negative relative change in the value of savings per unit of income, is positive. Accordingly, agents driven by the substitution effect in their labour-leisure decisions (i.e. individuals characterized by a $\sigma < 1$) should work more since leisure has become more expensive, i.e. its opportunity cost has increased. By contrast, for the very same reason, agents driven by the income effect (i.e. individuals with $\sigma > 1$) will find it optimal to work less. These conclusions are reversed as long as $\varepsilon_{w,k} - \varepsilon_{R,k} s(R(k)) < 0$. The crucial task becomes to evaluate condition (4.9) at the steady state value of k. In this respect, we can state the following result:

Proposition 2. *In steady state and for $\sigma < 1$, $l'(k) > (<)0$ for every $k > (<)k^{GR}$, while for $\sigma > 1$, $l'(k) > (<)0$ for every $k < (>)k^{GR}$.*

Proof. Substituting for the equilibrium values of w and R, condition (4.9), considered only with the "<" sign, can be re-written as

$$l'(k) < (>)0 \Longleftrightarrow \begin{cases} s(k) > (<)\dfrac{\alpha}{1-\alpha} & \text{if } \sigma < 1 \\ s(k) < (>)\dfrac{\alpha}{1-\alpha} & \text{if } \sigma > 1 \end{cases}. \tag{4.10}$$

In steady state, given equaton (3.8),

$$k = \frac{s(k)w(k)}{1+n} = \frac{s(k)(1-\alpha)k^{\alpha}}{(1+n)}$$

so that

$$s(k) = \frac{(1+n)}{(1-\alpha)k^{\alpha-1}}. \tag{4.11}$$

Substituting (4.11) into (4.10) , we obtain

$$l'(k) < (>)0 \Longleftrightarrow \begin{cases} (1+n) > (<)\alpha k^{\alpha-1} \text{ if } \sigma < 1 \\ (1+n) < (>)\alpha k^{\alpha-1} \text{ if } \sigma > 1 \end{cases}.$$

Which directly lead us to the following

$$l'(k) < (>)0 \Longleftrightarrow \begin{cases} k < (>)k^{GR} \text{ if } \sigma < 1 \\ k > (<)k^{GR} \text{ if } \sigma > 1 \end{cases}.$$

■

The above proposition describes how labour supply changes optimally as the economy's steady state moves either toward to or away from the golden age conditional on whether the economy is dynamic efficient or not and individual preferences.

4.2. Implications

Kaldor's stylised facts that the rate of return to capital and the labour share of income are roughly constant over time are well established and widely accepted among economic theorists. These two facts imply that the capital-output ratio, K/Y, should also be constant. Yet, the empirical evidence one could draw both from the original World Penn Data (wp) and the Extended World Penn Data (ewp)[9], which includes the original World Penn Data version 6 plus other aggregates' estimates by Duncan Foley and Adalmir Marquetti challenges such implication. For instance, in the period 1963-1990, the evidence for both EU countries, US, Canada and Australia is of a generalised and steady increase in K/Y as shown in Figure 1. The average growth rate of K/Y over the all period considered is either 1.46 (ewp) or 1.62 (wp) for Canada, 1.15 (ewp) or 1.34 (wp) for the USA, 3.8 (ewp) or 4.94 (wp) for Japan, and 1.77 (ewp) 2.3 (wp) for the UE.[10]

A feature of the above evidence which is of interest in the context of our analysis is that it implies a positive trend in the long run level of capital per unit of effective labour for the period considered[11]. Normalising the growth rate to zero, this implies an increase in the steady state level of the capital-labour ratio. The model then suggests that, conditional on dynamic efficiency and whenever agents are driven by the substitution effect in their labour-leisure decisions, in most of the countries considered the optimal level of individual labour supply has been decreasing in the period 1963-90. Then, assuming the length of the working week was originally set optimally by the laws still in place, one should conclude that there is a case for a reduction in the working week length. This conjecture could be of some interest for the working week reduction debate and it seems potentially worth a deeper investigation. However, it is our view that, a comprehensive assessment of the relevance of this argument, should be based

[9] Version 6 of the World Penn Data as well as its extended version by Duncan Foley and Adalmir Marquetti can be downloaded at http://homepage.newschool.edu/~foleyd/epwt/.

[10] See also Foley and Michl (1999).

[11] The existence of a positive trend in the capital per unit of effective labour is supported by Wolff (1991) who shows that in various industrialised countries capital per effective labour has been varying over long periods, quite differently from Kaldor's stylised evidence. For instance, over the period 1950-79, Italy (0.06%), Germany (0.15%), UK (0.71%), US (0.14%), Canada (0.38%) all experienced positive growth rates of capital per unit of effective labour. Moreover, over the same period, the average growth rate for the all sample of the G7 countries has been 0.25%. The only exception to this tendency is France whose growth rate of capital per unit of effective labour has been -0.47%.

on model which combines endogenous labour supply and long-run growth.[12] Such investigation goes beyond the scope of the present work whose main focus is on the effects of changes in the steady state level of capital-labour ratio on endogenous labour supply within the context of a standard overlapping generation model à la Diamond.

5. Conclusion

This paper considers the relationship between endogenous labour supply and the long-run level of physical capital per unit of labour in a standard overlapping generation model à la Diamond augmented to include leisure in the utility function of the individuals. Our main finding is that if individuals are driven by the substitution effect in their labour-leisure decisions the optimal level of individual labour supply reaches its minimum whenever the equilibrium value of the capital labour ratio is equal to its golden rule level. On the contrary, if agents are driven by the income effect, then the level of individual labour supply is maximum when the capital labour ratio is at its golden rule level. Consequently, in the case of a dynamic efficient economy, whenever agents work less the higher the opportunity cost of labour, they will optimally decide to work less following an (exogenous) increase in the long run level of capital per unit of effective labour. The opposite would be true if the economy were dynamically inefficient.

[12] Note that it would not be of much use to simply augment our model to allow for exogenous growth. In fact, because of the assumptions about individual preferences we adopt, with positive growth labour supply converges either to zero or to 1, which is clearly unsatisfactory in the light of empirical evidence.

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