

Estimating the macroscopic capillary length from Beerkan infiltration experiments and its impact on saturated soil hydraulic conductivity predictions

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*Original*

Estimating the macroscopic capillary length from Beerkan infiltration experiments and its impact on saturated soil hydraulic conductivity predictions / Di Prima, S.; Stewart, R. D.; Castellini, M.; Bagarello, V.; Abou Najm, M. R.; Pirastru, M.; Giadrossich, F.; Iovino, M.; Angulo-Jaramillo, R.; Lassabatere, L.. - In: JOURNAL OF HYDROLOGY. - ISSN 0022-1694. - 589:(2020), p. 125159. [10.1016/j.jhydrol.2020.125159]

*Availability:*

This version is available at: 11388/234779 since: 2020-06-22T23:24:46Z

*Publisher:*

*Published*

DOI:10.1016/j.jhydrol.2020.125159

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(Article begins on next page)

# 1 **Estimating the macroscopic capillary length from Beerkan infiltration experiments and its** 2 **impact on saturated soil hydraulic conductivity predictions**

3 Simone Di Prima<sup>a,b\*</sup>, Ryan D. Stewart<sup>c</sup>, Mirko Castellini<sup>d</sup>, Vincenzo Bagarello<sup>e</sup>, Majdi R. Abou Najm<sup>f</sup>,  
4 Mario Pirastru<sup>a</sup>, Filippo Giadrossich<sup>a</sup>, Massimo Iovino<sup>e</sup>, Rafael Angulo-Jaramillo<sup>b</sup> and Laurent  
5 Lassabatere<sup>b</sup>

6 <sup>a</sup> Department of Agricultural Sciences, University of Sassari, Viale Italia, 39, 07100 Sassari, Italy.

7 <sup>b</sup> Université de Lyon; UMR5023 Ecologie des Hydrosystèmes Naturels et Anthropisés, CNRS, ENTPE, Université Lyon 1, Vaulx-  
8 en-Velin, France.

9 <sup>c</sup> School of Plant and Environmental Sciences, Virginia Polytechnic Institute and State University, Blacksburg, VA, United State.

10 <sup>d</sup> Council for Agricultural Research and Economics-Agriculture and Environment Research Center (CREA-AA), Via Celso Ulpiani  
11 5, 70125 Bari, Italy.

12 <sup>e</sup> Department of Agricultural, Food and Forest Sciences, University of Palermo, Palermo, Italy.

13 <sup>f</sup> Department of Land, Air and Water Resources, University of California, Davis, CA 95616, United States.

14 \* Corresponding Author. E-mail: [sdiprima@uniss.it](mailto:sdiprima@uniss.it)

## 15 **Abstract**

16 The macroscopic capillary length,  $\lambda_c$ , is a fundamental soil parameter expressing the relative  
17 importance of the capillary over gravity forces during water movement in unsaturated soil. In this  
18 investigation, we propose a simple field method for estimating  $\lambda_c$  using only a single-ring  
19 infiltration experiment of the Beerkan type and measurements of initial and saturated soil water  
20 contents. We assumed that the intercept of the linear regression fitted to the steady-state portion of  
21 the experimental infiltration curve could be used as a reliable predictor of  $\lambda_c$ . This hypothesis was  
22 validated by assessing the proposed calculation approach using both analytical and field data. The  
23 analytical validation demonstrated that the proposed method was able to provide reliable  $\lambda_c$   
24 estimates over a wide range of soil textural characteristics and initial soil water contents. The field  
25 testing was performed on a large database including 433 Beerkan infiltration experiments, with the  
26 99% of the experiments yielding realistic  $\lambda_c$  values. The generated  $\lambda_c$  values were then used in  
27 conjunction with four different methods for estimating saturated soil hydraulic conductivity,  $K_s$ .  
28 Estimated  $K_s$  values were close to those generated by a reference method, with relative error < 25%  
29 in nearly all cases. By comparison, assuming constant or soil-dependent  $\lambda_c$  values caused relative  
30 errors in  $K_s$  of up to 600%. Altogether, the proposed method constitutes an easy solution for  
31 estimating  $\lambda_c$ , which can improve our ability to estimate  $K_s$  in the field.

32 **Keywords:** infiltration, macroscopic capillary length, Beerkan, ring infiltrometer, hydraulic  
33 conductivity.

#### 34 **Highlights**

- 35 • The macroscopic capillary length ( $\lambda_c$ ) is a key hydraulic parameter.
- 36 • We propose a simple field method to estimate  $\lambda_c$  from steady-state infiltration.
- 37 • We assessed the proposed calculation approach using both analytical and field data ( $n = 433$ ).
- 38 • The proposed method can improve our ability to estimate hydraulic conductivity in the field.

#### 39 **1. Introduction**

40 The macroscopic capillary length,  $\lambda_c$  (L), was first described by Bouwer (1964) and expresses  
41 the relative importance of capillary over gravity forces during water movement in unsaturated soil  
42 (Raats, 1976). More specifically, low  $\lambda_c$  values (e.g.,  $0 < \lambda_c \leq 10$  mm) indicate a dominance of  
43 gravity over capillarity, and are typically found in coarse-textured or highly structured porous media  
44 (Reynolds et al., 2002). Alternately, high  $\lambda_c$  values (i.e.,  $> 1000$  mm) indicate dominance of  
45 capillarity over gravity, as found in many fine-textured or poorly structured porous media (**Table**  
46 **1**).  $\lambda_c$  can also be used to estimate the macroscopic pore radius (i.e., the average radius of the pores  
47 that are active in flow) via the Young-Laplace equation (Kutilek and Nielsen, 1994), as suggested  
48 by Roulier et al. (2002). Because it indicates soil capillarity,  $\lambda_c$  is included in many infiltrometer  
49 methods for calculating  $K_s$  (e.g., Bagarello et al., 2017, 2014d, 2004; Elrick and Reynolds, 1992;  
50 Nimmo et al., 2009; Reynolds and Elrick, 1990; Stewart and Abou Najm, 2018a; Wu et al., 1999).  
51 It is thus important to accurately constrain  $\lambda_c$  under field conditions.

52 Previously developed methods to estimate  $\lambda_c$  have all presented some limitations. For instance,  
53 the two-ponding depths method by Reynolds and Elrick (1990) requires measuring steady state flow  
54 rates under two distinct water ponding conditions, thus inducing considerable effort and  
55 experimental complexity. Bagarello et al. (2013) proposed empirical equations to estimate  $\lambda_c$ ;

56 however, those results were site specific and therefore lacked generality. In addition, those authors  
57 used the cumulative linearization method (Smiles and Knight, 1976), which can fail in the presence  
58 of layered media, entrapped air, and vertical soil water content gradients (Vandervaere et al., 2000).  
59 Other methods are based on the analysis of transient state data, as for the case of the Method 1 by  
60 Wu et al. (1999) and Approach 2 by Stewart and Abou Najm (2018a). However, these approaches  
61 require accurate characterization of the transient state, which can be challenging under specific field  
62 conditions such as highly permeable, slightly sorptive and water-repellent soils (Di Prima et al.,  
63 2019). Therefore, alternative methods for estimating  $\lambda_c$  from simple and replicable infiltration  
64 experiments have the potential to substantially reduce the amount of work necessary to accurately  
65 estimate soil hydraulic properties.

66 The first objective of this investigation was to validate a simple field method to estimate  $\lambda_c$  that  
67 requires only a single-ring infiltration experiment taken to steady-state conditions (Lassabatere et  
68 al., 2006) and estimates for initial and saturated soil water contents. To meet this objective, we first  
69 developed the theoretical analysis for the estimation of  $\lambda_c$  from a single Beerkan run. We then  
70 validated the proposed method using analytically generated data, involving soils with different  
71 texture and initial water contents, and an empirical infiltration database that included data from 433  
72 field measurements collected during previous investigations. The second objective was to evaluate  
73 how  $\lambda_c$  values generated by our approach affected predictions of  $K_s$  from infiltration experiments.  
74 Here we used four different models to estimate  $K_s$  from steady-state infiltration, and then compared  
75 those results with both reference values and with those estimated using constant and soil-dependent  
76  $\lambda_c$  values.

## 77 **2. Theory**

78 For water infiltrating into an unsaturated soil from a constant head source, the soil matric flux  
79 potential,  $\phi_m$  ( $L^2 T^{-1}$ ), is defined as (Gardner, 1958):

$$80 \quad \phi_m = \int_{h_i}^{h_0} K(h)dh \quad h_i \leq h \leq h_0 \quad (1a)$$

81 where  $K$  ( $L T^{-1}$ ) is hydraulic conductivity and  $h$  (L) is water pressure head, with an initial value  
 82  $h_i$  (L) and a source pressure head  $h_0$  (L). Eq. (1a) simplifies to Eq. (1b) when water infiltrates from a  
 83 ponded source:

$$84 \quad \phi_m = \int_{h_i}^0 K(h)dh \quad h_i \leq h \leq 0 \quad (1b)$$

85 The macroscopic capillary length,  $\lambda_c$  (L), is defined as (Philip, 1985; Smith et al., 2002):

$$86 \quad \lambda_c = \frac{\phi_m}{\Delta K} \quad (2)$$

87 where  $\Delta K$  represents the difference between the saturated soil hydraulic conductivity,  $K_s$  ( $L T^{-1}$ ),  
 88 and the initial soil hydraulic conductivity,  $K_i$  ( $L T^{-1}$ ), i.e.,  $\Delta K = K_s - K_i$ .

89 According to White and Sully (1987), Eq. (2) can be rewritten as:

$$90 \quad \lambda_c = \frac{b S^2}{\Delta\theta \Delta K} \quad (3)$$

91 where  $b$  is a dimensionless constant dependent on the shape of the soil water diffusivity function,  
 92  $S$  ( $L T^{-0.5}$ ) is the soil sorptivity (Philip, 1957),  $\Delta\theta$  stands for the difference between the saturated,  $\theta_s$   
 93 ( $L^3 L^{-3}$ ), and initial,  $\theta_i$  ( $L^3 L^{-3}$ ), volumetric soil water contents, i.e.,  $\Delta\theta = \theta_s - \theta_i$ . For field soils,  $b$  is  
 94 commonly set equal to 0.55 even though it can theoretically vary from 1/2 to  $\pi/4$  (White and Sully,  
 95 1987).  $K_i$  is often assumed negligible, such that  $\Delta K = K_s$  (White and Sully, 1992).

96 Estimating  $\lambda_c$  with Eq. (3) requires prior determination of sorptivity and hydraulic conductivity.  
 97 These quantities can be estimated thanks to water infiltration experiments and fitting to the quasi-  
 98 exact implicit (QEI) model developed by Haverkamp et al. (1994) or its related approximate  
 99 expansions (see Lassabatere et al., 2009 for more details). Haverkamp et al. (1994) proposed the  
 100 following approximate expansion for the description of the steady-state for three-dimensional (3D)  
 101 water infiltration from a disc source while maintaining a zero water pressure head at the soil  
 102 surface:

$$103 \quad I_{3D}^{+\infty}(t) = \left( K_s + \frac{\gamma S^2}{r \Delta\theta} \right) t + \frac{S^2}{2(1-\beta)\Delta K} \ln\left(\frac{1}{\beta}\right) \quad (4)$$

104 where  $r$  (L) is the radius of the source, and  $\gamma$  and  $\beta$  are two infiltration constants, often fixed at  $\gamma$   
 105 = 0.75 and  $\beta = 0.6$  (Haverkamp et al., 1994). Eq. (4) was later extended to include infiltration  
 106 experiments from cylindrical sources with a slightly ponded water source (Ross et al., 1996) with  
 107 negligible effect on results.

108 Eq. (4) is a linear equation of the form:

$$109 \quad I_{3D}^{+\infty}(t) = i_s t + b_s \quad (5)$$

110 with  $b_s$  (L) and  $i_s$  (L T<sup>-1</sup>) defined as functions of hydraulic conductivity and sorptivity as follows:

$$111 \quad i_s = K_s + \frac{\gamma S^2}{r \Delta \theta} \quad (6a)$$

$$112 \quad b_s = \frac{S^2}{2(1-\beta)\Delta K} \ln\left(\frac{1}{\beta}\right) \quad (6b)$$

113 In this study, we use Eq. (6b) to quantify the ratio between sorptivity and the difference in  
 114 hydraulic conductivity, as previously suggested by Castellini et al. (2018):

$$115 \quad \frac{S^2}{\Delta K} = \frac{b_s}{C} \quad (7a)$$

$$116 \quad C = \frac{1}{2(1-\beta)} \ln\left(\frac{1}{\beta}\right) \quad (7b)$$

117 Eqs. (3) and (7) can be combined to explicitly solve for  $\lambda_c$ :

$$118 \quad \lambda_c = \frac{b S^2}{\Delta \theta \Delta K} = \frac{b}{\Delta \theta} \frac{b_s}{C} \quad (8a)$$

$$119 \quad \lambda_c = \frac{b}{\frac{1}{2(1-\beta)} \ln\left(\frac{1}{\beta}\right) \Delta \theta} \frac{b_s}{C} \quad (8b)$$

120 Under the common assumptions that  $b = 0.55$  and  $\beta = 0.6$ , Eq. (8b) can be simplified as follows:

$$121 \quad \lambda_c = 0.861 \frac{b_s}{\Delta \theta} \quad (9)$$

122 Eq. (9) constitutes a considerable simplification, as  $\lambda_c$  can now be estimated by only using the  
 123 steady-state infiltration data (to determine  $b_s$ ) and a measurement of the initial and saturated soil  
 124 water contents,  $\theta_i$  and  $\theta_s$ . Indeed,  $b_s$  is calculated as the intercept of the linear regression fitted to the  
 125 steady-state portion of the experimental infiltration curve (Eq. 5), so  $b_s$  calculation does not require  
 126 the use of Eq. (6b). Note that the simplified proposed method combines equations related to two

127 approaches with distinct, but not necessarily incompatible, assumptions. The first approach by  
 128 White and Sully (1987) was originally developed assuming the Gardner (1958) model for the  
 129 hydraulic conductivity function. The second approach developed by Haverkamp et al. (1994) and  
 130 Smettem et al. (1994) does not expect any specific hydraulic functions, but requires that these  
 131 functions follow a specific equation defining the infiltration constant  $\beta$  (equation 6 in Haverkamp et  
 132 al., 1994).

133 Eq. (9) may also simplify and improve estimates for  $K_s$ , as  $\lambda_c$  is an important and often unknown  
 134 parameter in many infiltration models. Four examples of methods that require  $\lambda_c$  to estimate  $K_s$  from  
 135 steady-state infiltration data include:

136 **i)** the One-Ponding Depth (**OPD**) method by Reynolds and Elrick (1990)

$$137 \quad K_s = \frac{\frac{i_s \pi r^2}{\lambda_c} (0.316 \frac{d}{r} + 0.184)}{r \left( \frac{H}{\lambda_c} + 1 \right) + (0.316 \frac{d}{r} + 0.184) \frac{\pi r^2}{\lambda_c}} \quad (10)$$

138 **ii)** Method 2 by Wu et al. (1999) (**WU2**)

$$139 \quad K_s = \frac{i_s}{0.9084 \left( \frac{H + \lambda_c}{G^*} + 1 \right)} \quad (11)$$

140 **iii)** the Steady version of the Simplified method based on a Beerkan Infiltration run (**SSBI**)  
 141 by Bagarello et al. (2017)

$$142 \quad K_s = \frac{i_s}{\frac{\gamma \gamma_w \lambda_c}{r} + 1} \quad (12)$$

143 **iv)** Approach 4 (**A4**) by Stewart and Abou Najm (2018b)

$$144 \quad K_s = \frac{i_s}{\left( \frac{H + \lambda_c}{G^*} + 1 \right)} \quad (13)$$

145 where  $d$  (L) is the ring insertion depth into the soil,  $r$  (L) is the ring radius,  $G^* = d + r/2$ ,  $H$  (L) is  
 146 the ponding depth of water, and  $\gamma_w$  is a dimensionless constant related to the shape of the wetting  
 147 front (White and Sully, 1987).  $\gamma$ , the infiltration constant defined above, was set equal to 0.75  
 148 (Smettem et al., 1994) and  $\gamma_w$  was set equal to 1.818, as suggested by Reynolds and Elrick (2002).

149 **3. Material and methods**

150 **3.1. Analytically generated data**

151 We assessed the accuracy of the proposed calculation approach for  $\lambda_c$  and  $K_s$  by using the same  
152 six soils considered by Hinnell et al. (2009) and Bagarello et al. (2017): sand, loamy sand, sandy  
153 loam, loam, silt loam, silty clay loam. These soils were chosen to cover a wide range of hydraulic  
154 responses. We modelled the infiltration experiments for these synthetic soils using the infiltration  
155 model proposed by Smettem et al. (1994):

156 
$$I(t) = I_{1D}(t) + \frac{\gamma S^2}{r_d \Delta \theta} \quad (14)$$

157 where  $I$  (L) is 3D cumulative infiltration and  $I_{1D}$  (L) is the 1D cumulative infiltration into an  
158 uniform, initially unsaturated soil profile, which can be modelled by the following implicit equation  
159 (Haverkamp et al., 1990):

160 
$$\frac{2\Delta K^2}{S^2} t = \frac{1}{1-\beta} \left[ \frac{2\Delta K}{S^2} (I_{1D}(t) - K_i t) - \ln \left( \frac{\exp\left(2\beta \frac{\Delta K}{S^2} (I_{1D}(t) - K_i t)\right) + \beta - 1}{\beta} \right) \right] \quad (15)$$

161 To also test the effect of the initial soil water content on parameters predictions, initial values of  
162  $Se$ , ranging from 0.1 to 0.8 were converted to equivalent  $\theta_i$  values for each soil using the  
163 relationship  $Se = (\theta_i - \theta_r)/(\theta_s - \theta_r)$ , with  $\theta_r$  ( $L^3 L^{-3}$ ) representing the residual water content. The  
164 sorptivity was then estimated as follows (Parlange, 1975):

165 
$$S = \sqrt{\int_{h(\theta_i)}^0 (\theta_s + \theta - 2\theta_i) K(h) dh} \quad (16)$$

166 The integrals in Eqs. (16) and (1) were computed using the `intg` function defined in Scilab  
167 (Campbell et al., 2010). The water retention curve and the hydraulic conductivity functions were  
168 calculated according to the van Genuchten–Mualem model (Mualem, 1976; van Genuchten, 1980):

169 
$$Se = \left[ \frac{1}{1 + (\alpha_{VG}|h|)^n} \right]^m \quad (17a)$$

170 
$$m = 1 - \frac{1}{n} \quad (17b)$$

171 
$$K(Se) = K_s Se^l \left[ 1 - (1 - Se^{1/m})^m \right]^2 \quad (17c)$$

172 where  $\alpha_{VG}$  ( $L^{-1}$ ) is an empirical parameter related to the water pressure head,  $n$  is the pore size  
 173 distribution index, and  $l$  is the pore connectivity parameter, which we assumed to be 0.5 following  
 174 Mualem (1976). Hydraulic parameters for the six synthetic soils were taken from Carsel and Parrish  
 175 (1988), with  $K_s$  values reported in that text used to represent the reference saturated hydraulic  
 176 conductivity (**Table 2**). Default values of  $\beta = 0.6$  and  $\gamma = 0.75$  were assumed, as commonly  
 177 suggested by many investigations (Angulo-Jaramillo et al., 2019).

178 To ensure steady-state conditions, each infiltration process was modelled for a period three times  
 179 longer than the maximum time for which the explicit short-term expansion of Eq.(15) (Haverkamp  
 180 et al., 1994) is considered valid, with  $t_{max}$  (T) calculated as follows (Lassabatere et al., 2006):

$$181 \quad t_{max} = \frac{1}{4(1-B)^2} \left( \frac{S}{K_s} \right)^2 \quad (18a)$$

$$182 \quad B = \frac{2-\beta}{3} \left( 1 - \frac{K_i}{K_s} \right) + \frac{K_i}{K_s} \quad (18b)$$

183 These analytical data were used to estimate the intercept,  $b_s$  (L), and the slope,  $i_s$  ( $L T^{-1}$ ), by  
 184 linear regression analysis of the last three data points of the cumulative infiltration time series.  
 185 Then, we defined the estimator for  $\lambda_c$ ,  $\hat{\lambda}_c$ , using Eq. (9) and the estimator for  $K_s$ ,  $\hat{K}_s$ , via the standard  
 186 predictive equations for  $K_s$  (Eqs. 10-13).

187 The reference macroscopic capillary length,  $\lambda_c$ , was calculated for each combination of soil type  
 188 and initial  $Se$  value using Eq. (2). Relative error,  $Er$ , was then calculated for each estimated value  
 189 for  $\hat{\lambda}_c$  and  $\hat{K}_s$  compared to the corresponding reference value (i.e.,  $\lambda_c$  and  $K_s$ ) as follows:

$$190 \quad Er(x) = 100 \times \frac{\hat{x}-x}{x} \quad (19)$$

191 where  $\hat{x}$  is the estimated value and  $x$  is the target, i.e., the reference value  $\lambda_c$  (Eq. 2) or  $K_s$  (**Table**  
 192 **2**). According to the accuracy criterion by Reynolds (2013), the estimates were deemed accurate  
 193 when they fell within the range  $0.75 \leq \hat{x}/x \leq 1.25$  (i.e.  $\leq 25\%$  error). This stringent criterion was  
 194 used because the parameters were estimated by analytically generated data, and therefore were free  
 195 of the perturbations embedded in field and laboratory measurements (e.g., measurement error,  
 196 random noise and natural variability).

### 3.2. The Beerkan infiltration database

In this investigation we also considered a large database of single ring (Beerkan) infiltration experiments carried out in four different countries, Italy, Burundi, France and Spain, during the period 2010-2017 (**Table 3**). Nearly half of the runs were carried out in Sicily, Italy (202 out of 433), and another  $\sim 1/3$  of the runs (152 out of 433) were carried out in Burundi in the African Great Lakes region. The tested soils covered a range of textures, from sandy to clayey (**Figure 1**).

The Beerkan experiment is a variation of the single-ring infiltrometer technique, which consists of infiltrating water through a ring inserted shallowly (e.g., 1 cm) into the soil with a quasi-zero head of water imposed on the soil surface (Braud et al., 2005). All Beerkan experiments were carried out according to the methodology described by Lassabatere et al. (2006). First, a stainless steel ring was inserted shallowly into the soil ( $\sim 1$  cm). Then, water was poured on the confined soil surface in fixed volume increments ( $V$ ) to establish and maintain ponding conditions. The increments,  $V$ , ranged from 17 to 800 mL depending on ring diameter (**Table 3**). The energy of the falling water was dissipated with fingers to minimize the soil disturbance owing to water pouring, as commonly suggested (e.g., Di Prima et al., 2019). For each poured volume, the time needed for the water to infiltrate was recorded. The total number of poured volumes varied depending on time needed to reach steady state, as required by the Beerkan method (Angulo-Jaramillo et al., 2019).

We then estimated the intercept,  $b_s$  (mm), and the slope,  $i_s$  ( $\text{mm h}^{-1}$ ), of the regression line fitted to the cumulative infiltration time series. The final three data points were used, as those were assumed to represent steady state infiltration conditions. We estimated  $K_s$  by using Eqs. (10-13) and constraining  $\lambda_c$  through three different approaches:

- **Scenario 1:** determining  $\lambda_c$  through Eq. (9);
- **Scenario 2:** using  $\lambda_c = 83$  mm, taking into account that it represents the suggested first approximation value for most field soils (Elrick and Reynolds, 1992);

- **Scenario 3:** using a soil-dependent  $\lambda_c$  value according to **Table 1**. Specifically, we used  $\lambda_c = 250$  mm for soils with sand content  $< 20\%$ , 83 mm for sand contents between 20 and 70%, and 28 mm when the soil had  $> 70\%$  sand (Bagarello et al., 2017).

For the experimental dataset, the comparison of the estimator to the target is not possible. Indeed, we don't have any information on the real value of the macroscopic capillary length, given that previously developed methods to estimate  $\lambda_c$  have all presented some limitations, as discussed in the Introduction. Instead, we compared the estimates to representative values from the five soil capillarity categories suggested by Elrick and Reynolds (1992). Note that these categories were originally proposed to select a representative value for five soil texture-structure categories (**Table 1**) when calculating  $K_s$  by the OPD method (Angulo-Jaramillo et al., 2016). In this investigation, we also proposed range of values for each category as detailed in **Table 1**. The range values of the intermediate categories (strong, moderate and weak) were calculated as the mean of the representative values of two consecutive categories.

The same issue arises for the estimation of the saturated hydraulic conductivity. In this case, we chose to use the BEST-steady method proposed by Bagarello et al. (2014c) as a benchmark, as an independent  $K_s$  datum that can be used for assessing simplified procedures or validating newly developed methods. This method estimates  $K_s$  as follows:

$$K_{s,BEST} = \frac{C i_s}{A b_s + C} \quad (20a)$$

$$A = \frac{\gamma}{r(\theta_s - \theta_i)} \quad (20b)$$

Note that we also chose the Bagarello et al. (2014c) method because it requires the same experimental information as the  $\lambda_c$ -dependent methods considered in this investigation for estimating  $K_s$ , yet does not require an estimate of  $\lambda_c$ . We also avoided using laboratory measurements as benchmark, as they can induce experimental artifacts, such as soil compaction and samples biased by pores, that may limit their comparability with in-situ measurements (Haverkamp et al., 1999).

246 To compare  $K_s$  values estimated by Eqs. (10-13) with the reference  $K_{s,BEST}$  values obtained by  
247 Eq. (20), we again used the relative error metric (Eq. 19). We also calculated paired differences for  
248 each method, i.e.,  $K_{s,BEST} - K_s$  and checked them for normality using the Kolmogorov-Smirnov test.  
249 For non-normally distributed data we used the Wilcoxon signed rank test to evaluate the median  
250 difference between paired observations at the 95% confidence level. All statistical analyses were  
251 carried out using the Minitab© computer program (Minitab Inc., State College, PA, USA).

## 252 4. Results

### 253 4.1. Analytical validation

#### 254 4.1.1. Estimating $\lambda_c$ from analytically generated data

255 When applied to the six synthetic soils, Eq. (2) yielded the highest  $\lambda_c$  values for fine-textured  
256 soils (**Table 4**). This is logical, since for fine soils the capillary contribution to water flow was  
257 higher than for coarser soils. More specifically, high  $\lambda_c$  values were associated to initially flat  $K(h)$   
258 relationships, i.e., when a decrease in pressure head determined a moderate pore emptying (Angulo-  
259 Jaramillo et al., 2016; Reynolds, 1994). Moreover,  $\lambda_c$  values decreased for all soils as  $Se$  increased.

260 When cumulative infiltration was calculated for all synthetic soils (**Table 2**) using Eqs. (14) and  
261 (15), all curves exhibited a typical concave shape as a function of time (**Figure 2**). As the process  
262 approached steady state, cumulative infiltration curves became approximately linear with time. This  
263 behaviour shows how the influence of capillarity decreases as the wetting front moves away from  
264 the source and the hydraulic gradient decreases (Xu et al., 2012). Note that the duration of the  
265 infiltration process decreased for higher values of  $Se$ , as steady-state conditions were attained in less  
266 time. In these cases, capillary forces only influenced infiltration during the early stage of the  
267 process.

268 The value of the linear regression model intercept,  $b_s$ , estimated from each curve was used in  
269 conjunction with the known  $\theta_i$  and  $\theta_s$  values to calculate  $\lambda_c$  using Eq. (9). Both  $b_s$  and  $i_s$  decreased in  
270 all soils as  $Se$  increased.  $b_s$  ranged from 3.1 to 35.1 mm, with larger values corresponding to the

271 fine-textured silt loam and silty clay loam soils (**Table 4**). The slope of the linear regression model  
272  $i_s$  had values as low as 1.7 mm h<sup>-1</sup> (fine soil) and as high as 516.7 mm h<sup>-1</sup> (coarse soil)

273 The estimated  $\hat{\lambda}_c$  for the six synthetic soils ranged from 36.3 to 87.8 mm, and were classified  
274 only into weak or moderate capillary categories (Table 1), although those soils had textures which  
275 ranged from sand to silty clay loam. Relative error,  $Er(\lambda_c)$ , between estimated  $\hat{\lambda}_c$  and reference  $\lambda_c$   
276 values ranged from -7.9 to 23.3%, indicating that all  $\lambda_c$  values were accurate based on our stated  
277 criterion. The largest  $Er(\lambda_c)$  values were obtained for the coarse-textured sandy and loamy sand  
278 soils under initial wet conditions (**Figure 3**). Indeed, neglecting  $K_i$  is expected to introduce more  
279 uncertainty on  $\lambda_c$  estimations for higher  $Se$  values. Nevertheless,  $\lambda_c$  estimates were sufficiently  
280 accurate also in these cases, with error always < 25%.

#### 281 **4.1.2. Estimating $K_s$ from analytically generated data**

282 The values of the slope,  $i_s$ , estimated from the analytically generated curves were used to  
283 calculate  $K_s$  by the four  $\lambda_c$ -dependent methods, i.e., OPD (Eq. 10), WU2 (Eq. 11), SSBI (Eq. 12),  
284 A4 (Eq. 13). Then, relative error,  $Er(K_s)$ , was calculated using Eq. (14) (**Figure 4**).  $Er(K_s)$  ranged  
285 from -9.5 to 3.1% for OPD, from -5.4 to 7.4% for WU2, from 2.2 to 24.7% for SSBI, and from -14  
286 to -2.4 % for A4. While we observe higher  $|Er(\lambda_c)|$  values for initial wet conditions (**Figure 3**), for  
287  $K_s$ , we observe similar trends between the four methods but also a consistent vertical shift of the  
288  $Er(K_s)$  values. For instance, for the SSBI method, lower errors corresponded to higher  $Se$  values.  
289 Conversely, for the A4 method, the errors always increased for increasing  $Se$  values, given that this  
290 method always underestimated  $K_s$ . We therefore argue that the discrepancies between the four  
291 methods were more relevant than the variations within a specific method due to different initial  
292 saturation degree. Nevertheless, the four methods always yielded  $K_s$  estimates close to the reference  
293 values, since  $|Er(K_s)|$  values were always < 25%. Mean  $|Er(K_s)|$  values were ordered as OPD < A4 <  
294 WU2 < SSBI, showing that the OPD method yielded the lower  $|Er(K_s)|$  values.

## 295 4.2. Field testing

### 296 4.2.1. Estimating $\lambda_c$ from the Beerkan infiltration database

297 Eq. (9) was also used to estimate  $\lambda_c$  from the field-based single ring (Beerkan) infiltration  
298 experiments. The procedure worked for nearly all Beerkan tests; however, six of the tests had  
299 infiltration rates that increased with time (i.e., the cumulative infiltration curves exhibited convex  
300 shapes). Fitting Eq. (5) to those data yielded negative value for the intercept,  $b_s$ , which led to  
301 negative values for  $\lambda_c$ , which is meaningless from a physical point of view. Those six cases – two at  
302 the Kinyami site, one at the Palermo – SAAF site, and three at the Crépieux-Charmy site – were  
303 excluded from subsequent analysis. The remaining 427 successful tests yielded  $\lambda_c$  values ranging  
304 from 1.5 to 737.7 mm (**Table 5**), thus covering the full range of soil capillarity categories suggested  
305 by Elrick and Reynolds (1992) (**Table 1**).

306 Across all soils, there was a consistent yet non-linear relationship between  $\lambda_c$  values and their  
307 corresponding intercepts  $b_s$  (**Figure 5a**). As shown by three different examples of  $\lambda_c$  estimation,  
308 cumulative infiltration shapes and times to steady-state conditions varied widely between soils with  
309 moderate (**Figure 5b**), strong (**Figure 5c**), and negligible capillarity (**Figure 5d**). In the first case  
310 (**Figure 5b**), cumulative infiltration exhibited the typical concave shape as a function of time. For  
311 this run, we estimated  $b_s$  value of 30.9 mm and a  $\lambda_c$  value of 49.2 mm (moderate capillarity). In the  
312 second case (**Figure 5c**), the cumulative infiltration curve exhibited a strong concave shape with a  
313  $b_s$  value of 209.6 mm, yielding a  $\lambda_c$  value of 598.7 mm (strong capillarity). This behavior is typical  
314 for very fine soils, with low permeability. For this run, capillary forces predominated for almost the  
315 entire duration of the experiment. In the third case (**Figure 5d**), the cumulative infiltration curve  
316 had an almost linear shape with a  $b_s$  value of 1.0 mm, which translated to a  $\lambda_c$  value of 1.5 mm, i.e.,  
317 lower than the considered threshold of 10 mm for negligible capillarity forces. This behaviour is  
318 typical for coarse-textured soils and occurs when the infiltration process is mainly driven by  
319 gravity. Altogether, 127 (29.7%) of the  $\lambda_c$  values represented strong capillarity conditions, 189

320 values (44.3%) represented moderate capillarity, 107 values (25.1%) represented weak capillary,  
321 and 4 values (0.9%) represented negligible capillarity.

#### 322 **4.2.2. Estimating $K_s$ from the Beerkan infiltration database**

323 The  $\lambda_c$  values determined through Eq. (9) were next used with four methods (i.e., OPD, WU2,  
324 SSBI, A4) to estimate  $K_s$  for the Beerkan dataset. The Wilcoxon signed rank test showed that all  
325 methods yielded  $K_s$  estimates significantly different from the BEST-steady values, and the  
326 differences between  $K_s$  and  $K_{s,BEST}$  were always non-normally distributed according to the  
327 Kolmogorov-Smirnov test. However, the discrepancies between methods were always  $\leq 25\%$ , with  
328 the exception of the SSBI method (**Figure 6a**). The WU2 method yielded the best overall fit with  
329 the BEST-steady values, with  $Er(K_s)$  values between -12.1 and 22.1%. The WU2 method yielded  
330 lower  $K_s$  estimates than the BEST-estimated values for 76% of the runs and higher  $K_s$  values for  
331 24% of runs, and the median  $K_s$  values for the two methods differed by only a factor of 1.002. The  
332 OPD and A4 methods also performed well, though those methods tended to under-predict  $K_s$  to a  
333 greater extent than WU2, with 82% of OPD runs and 91% of A4 runs under-predicting  $K_s$ . The  
334 SSBI method, by contrast, yielded  $K_s$  values that were higher than  $K_{s,BEST}$  with only a single  
335 exception. The  $Er(K_s)$  values ranged from -1.0 to 32.8%, with 21% of the runs (88 out of 427)  
336 yielding higher values than the considered threshold of 25%.

337 Using constant (Scenario 2) or a soil-dependent (Scenario 3)  $\lambda_c$  values resulted in greater  
338 difference between  $K_s$  and  $K_{s,BEST}$ . With the constant  $\lambda_c$  value (Scenario 2),  $Er(K_s)$  values ranged  
339 from -66.8 to 576.9% (**Figure 6b**), with 68.4% of OPD runs, 68.1% of WU2 runs, 55.7% of SSBI  
340 runs and 72.1% of A4 runs yielding higher values than the considered threshold of 25%. With the  
341 soil-dependent  $\lambda_c$  values (Scenario 3),  $Er(K_s)$  values ranged from -82.9 to 486.5% (**Figure 6c**), with  
342 70.5% of OPD runs, 69.3% of WU2 runs, 62.3% of SSBI runs and 74.0% of A4 runs yielding  
343 higher values than the considered threshold. These results also suggest that among the four  
344 considered methods, the SSBI was the least sensitive to the assumed  $\lambda_c$  value, with less runs (55.7

345 and 62.3% for scenario 2 and 3, respectively) yielding higher  $Er(K_s)$  values than the considered  
346 threshold of 25%.

## 347 **5. Discussion**

348 In this investigation, we developed a new procedure to estimate  $\lambda_c$  using simple Beerkan  
349 infiltration experiments and measurements of initial and saturated soil water contents (Eq. 9).  
350 Previous investigations also suggested that the measured infiltration curve contains the necessary  
351 information to estimate  $\lambda_c$  (Bagarello et al., 2014d, e.g., 2013; Stewart and Abou Najm, 2018a; Wu  
352 et al., 1999). However, those methods are based on the analysis of the transient infiltration process  
353 and can be subject to considerable error, particularly due to uncertainties with the duration of the  
354 transient phase (Vandervaere et al., 2000). In contrast, the proposed method uses measurements  
355 collected during the steady-state stage of the infiltration process, where the infiltration rate ( $i_s$ ) is  
356 assumed to be independent of the initial infiltration phase (Bagarello et al., 2013). Estimating  $\lambda_c$   
357 using the proposed method requires linear regression analysis of cumulative infiltration versus time  
358 to determine the intercept ( $b_s$ ). Because the magnitude of  $b_s$  depends on the entire cumulative  
359 infiltration curve (including the transient phase), that term is sensitive to the relative importance of  
360 capillary and gravity forces during ponded infiltration (Angulo-Jaramillo et al., 2019). Specifically,  
361 small  $b_s$  values indicate a linear infiltration curve, i.e., when gravity prevails over capillarity, which  
362 occurs primarily in coarse-textured and/or highly structured porous media. On the contrary, high  
363 intercept values indicate conditions when capillarity prevails over gravity, particularly in the  
364 transient infiltration phase, which occurs primarily in fine-textured soils. Therefore,  $b_s$  is expected  
365 to be a reliable predictor of the macroscopic capillary length, but one that necessitates collecting  
366 accurate data during the final stage of the infiltration process.

367 In this investigation, the proposed method (Eq. 9) was validated using both analytical and field  
368 data. The analytical verification demonstrated that Eq. (9) provided reliable  $\lambda_c$  estimates in nearly  
369 all conditions, including different soils and, for the same soils, under different initial soil water

370 contents. For the field data, verification was conducted using a set of 427 Beerkan infiltration  
371 experiments carried out on different soils having a range of textural characteristics, i.e., from sandy  
372 to clayey. That analysis showed that nearly all soils (i.e., 99.1% of the experiments) yielded  $\lambda_c$   
373 values falling within the realistic range  $10 \leq \lambda_c \leq 1000$  mm (Reynolds and Elrick, 2002); only four  
374 cases yielded  $\lambda_c$  values lower than 10 mm. Further, the proposed method predicted  $\hat{\lambda}_c$  values very  
375 close to the reference  $\lambda_c$  values, with all tests having relative errors between -23.2% and 7.9%. The  
376 consistency of Eq. (9) shows that it is a suitable method to constrain  $\lambda_c$ .

377 Many models to estimate  $K_s$  from field measurements (e.g., Eqs. 10-13), require knowledge of  
378  $\lambda_c$ , which is often estimated based on general descriptions of soil textural and structural  
379 characteristics (e.g., **Table 1**). Previous research has shown that choosing an incorrect capillarity  
380 category can lead to threefold or greater error in estimated  $K_s$  (Bagarello et al., 2014d), and that  $K_s$   
381 estimates are more sensitive to underpredictions of  $\lambda_c$  compared with overpredictions (Stewart and  
382 Abou Najm, 2018b). In this investigation we demonstrated that using constant or a soil-dependent  
383  $\lambda_c$  value may result in considerably greater relative error when predicting  $K_s$ . Specifically, using Eq.  
384 (9) resulted in relative errors  $< 25\%$  for all Beerkan tests when analyzed with three of the four  
385 methods, and  $< 30\%$  when the fourth method was used. By comparison, assuming  $\lambda_c = 83$  mm  
386 resulted in relative error up to 600%, and using a soil-dependent  $\lambda_c$  value caused relative errors of  
387 close to 500%.

388 Beyond its use in estimating  $K_s$ ,  $\lambda_c$  can provide information on soil pore structure and water  
389 retention (Stewart and Abou Najm, 2018b), making it important to have a simple method for use in  
390 the field. For example, the proposed method for constraining  $\lambda_c$  may also facilitate the estimation of  
391 dynamic indicators, such as the flow-weighted mean pore radius and the number of hydraulically  
392 active pores per unit area previously proposed by Warrick and Broadbridge (1992) and Watson and  
393 Luxmoore (1986). The flow-weighted mean pore radius represents the size of pores that are actively  
394 conducting and it expresses the ability of a soil to transmit water (Reynolds and Elrick, 2005).  
395 These indicators, quantitatively linked to  $\lambda_c$  and  $K_s$ , are useful to understand the effects of land use

396 and management on soil physical quality (Bouarafa et al., 2019; Castellini et al., 2019; Iovino et al.,  
397 2016). Therefore, accurate estimation of  $\lambda_c$  through Eq. (9) could also facilitate determination of  
398 dynamic indicators from infiltration experiments and improve soil quality assessment.

## 399 **6. Summary and conclusions**

400 In this investigation, we assessed a simple field method for estimating the macroscopic capillary  
401 length,  $\lambda_c$ , by only using a single-ring infiltration experiment of the Beerkan type and a  
402 measurement of the initial and saturated soil water contents. We validated the proposed method  
403 using both analytically generated data and a large database of 433 Beerkan infiltration experiments  
404 carried out in four countries (Italy, Burundi, France and Spain) over the period 2010-2017. The  
405 analytical validation supported our hypothesis that the intercept,  $b_s$ , is a reliable predictor of the  
406 macroscopic capillary length, while the testing carried out using the Beerkan database increased our  
407 confidence that the approach performs well under field conditions. Therefore, we conclude that the  
408 method proposed here constitutes an easy and effective solution for constraining  $\lambda_c$ , which at the  
409 same time can help users to better estimate  $K_s$  from field infiltration measurements. The proposed  
410 procedure may also avoid uncertainty due to an imprecise description of the transient state of  
411 infiltration, and any subjectivity caused by the selection of a representative  $\lambda_c$  value based solely on  
412 textural or structural characteristics.

413 **Funding:** This work was supported through the INFILTRON Project (ANR-17-CE04-0010,  
414 Package for assessing infiltration & filtration functions of urban soils in stormwater management;  
415 <https://infiltron.org/>) funded by the French National Research Agency (ANR), and the “Programma  
416 Operativo Nazionale (PON) Ricerca e Innovazione 2014-2020 (Linea 1 - Mobilità dei ricercatori,  
417 AIM1853149, CUP: J54I18000120001) funded by the European Regional Development Fund  
418 (ERDF) and the Italian Ministry of Education, University and Research (MIUR).

419 **Author Contributions:** SDP outlined the investigation and wrote the first draft of the manuscript.  
420 SDP, MI, VB, LL and RDS developed the theory. SDP, MAN, RAJ, FG, MP and MC planned the  
421 analytical validation and analyzed the field data. All authors contributed to discussing the results  
422 and developing the final version of the manuscript.

423 **Conflicts of Interest:** The authors declare no conflict of interest.

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589 **Table 1.** Soil capillarity categories suggested by Elrick and Reynolds (1992), and representative  $\lambda_c$  (mm)  
590 values. The suggested range values are also reported.

Soil capillarity category	Representative $\lambda_c$ (mm)	$\lambda_c$ range values
Very strong	$\geq 1000$	
Strong	250	$125 < \lambda_c < 1000$
Moderate	83	$42 \leq \lambda_c \leq 125$
Weak	28	$10 < \lambda_c < 42$
Negligible	$\leq 10$	

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593 **Table 2.** Soil hydraulic parameters for the five studied soils used to model the infiltration experiments,  
594 originally from Carsel and Parrish (1988).

Soil texture	Sand	Loamy Sand	Sandy Loam	Loam	Silt Loam	Silty Clay Loam
$\theta_r$	0.045	0.057	0.065	0.078	0.067	0.089
$\theta_s$	0.43	0.41	0.41	0.43	0.45	0.43
$\alpha_{VG}$ (mm <sup>-1</sup> )	0.0145	0.0124	0.0075	0.0036	0.002	0.001
$n$	2.68	2.28	1.89	1.56	1.41	1.23
$K_s$ (mm h <sup>-1</sup> )	297.0	145.9	44.2	10.44	4.5	0.7
$l$	0.5	0.5	0.5	0.5	0.5	0.5

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597 **Table 3.** Summary of the Beerkan infiltration database. Total number of Beerkan infiltration  
 598 experiments ( $N_{tot}$ ) = 433.

Country	Site	$N$	$D$ (cm)	$V$ (mL)	Coordinates	Reference
Burundi	Nyamutobo (Ruyigi)	77	15	150	3°27'50" S, 30°15'40" E	Bagarello et al. (2011)
Burundi	Kinyami (Ngozi)	20	15	150	2°54'30" S, 29°49'06" E	
Italy	Giampileri	11	15	150	38°4'8" N, 15°28'26" E	Bagarello et al. (2013)
Italy	Palermo - SAAF† (Sicily)	8	30	800	38°6'25" N, 13°21'6" E	Bagarello et al. (2014b)
Italy	Caccamo (Sicily)	4	30	800	37°52'34" N, 13°38'43" E	
Italy	Corleone (Sicily)	20	30	800	37°48'35" N, 13°17'49" E	
Italy	Sparacia (Sicily)	8	30	800	37°38'11" N, 13°45'50" E	
Italy	Palermo - SAAF (Sicily)	10	8.5	64	38°6'25" N, 13°21'6" E	Bagarello et al. (2014a)
Italy	Sparacia (Sicily)	10	8.5	64	37°38'10" N, 13°45'59" E	
Italy	Palermo - Parco d'Orleans (Sicily)	10	8.5	64	38°6'26" N, 13°20'59" E	
Italy	Villabate (Sicily)	10	8.5	64	38°4'53" N, 13°25'7" E	
Italy	Palermo - SAAF (Sicily)	12	15	200	38°6'25" N, 13°21'6" E	Bagarello et al. (2014c)
Italy	Palermo - SAAF (Sicily)	4	30	800	38°6'25" N, 13°21'6" E	
Italy	Pietranera (Sicily)	4	15	200	37°32'25" N, 13°30'44" E	
Italy	Pietranera (Sicily)	4	30	800	37°32'25" N, 13°30'44" E	
Italy	Caccamo (Sicily)	4	15	200	37°52'34" N, 13°38'43" E	
Italy	Corleone (Sicily)	20	15	200	37°48'35" N, 13°17'49" E	
Italy	Sparacia (Sicily)	8	15	200	37°38'11" N, 13°45'50" E	
Burundi	Nyamutobo (Ruyigi)	75	15	150	3°27'50" S, 30°15'40" E	
Italy	Palermo - Parco d'Orleans (Sicily)	10	15	150	38°6'26" N, 13°20'59" E	Alagna et al. (2016)
Italy	Palermo - SAAF (Sicily)	10	15	150	38°6'25" N, 13°21'6" E	Di Prima et al. (2016)
Italy	Palermo - Parco d'Orleans (Sicily)	10	15	150	38°6'26" N, 13°20'59" E	
Italy	Sparacia (Sicily)	10	15	150	37°38'10" N, 13°45'59" E	
France	Crépieux-Charmy (Lyon)	9	15	150	45°47'42" N, 4°53'19" E	
Spain	Les Alcusses de Moixent (Valencia)	10	8.5	48	38°48'33" N, 0°49'3" O	Di Prima et al. (2017)
Italy	Palermo - SAAF (Sicily)	5	5	17	38°6'25" N, 13°21'6" E	Di Prima et al. (2018a)
Italy	Palermo - Parco d'Orleans (Sicily)	5	5	17	38°6'26" N, 13°20'59" E	
Italy	Sparacia (Sicily)	5	5	17	37°38'10" N, 13°45'59" E	
Italy	Baratz Lake watershed (Sardinia)	40	8	43	40°41'53" N, 8°14'4" E	Di Prima et al. (2018b)

599 † Department of Agricultural, Food and Forest Sciences (SAAF = Scienze Agrarie, Alimentari e Forestali).  
 600  $N$  = Number of Beerkan infiltration experiments;  $D$  (cm) = ring diameter;  $V$  (mL) = water volume applied with each pouring.  
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**Table 4.** Summary of the soil hydraulic properties for the six synthetic soils.

Soil	$Se$	$h_i$ (mm)	$\theta_i$ ( $m^3m^{-3}$ )	$S$ ( $mm\ h^{-0.5}$ ) (Eq. 16)	$\phi$ ( $mm^2\ h^{-1}$ ) (Eq. 1b)	$\lambda_c$ (mm) (Eq. 2)	$\hat{\lambda}_c$ (mm) (Eq. 9)	$i_s$ ( $mm\ h^{-1}$ )	$b_s$ (mm)	$R_s$ ( $mm\ h^{-1}$ )			
										OPD (Eq. 10)	WU2 (Eq. 11)	SSBI (Eq. 12)	A4 (Eq. 13)
Sand	0.1	269.0	0.083	86.5	11308.6	38.1	37.2	516.7	15.0	304.3	319.1	341.3	289.8
	0.2	174.5	0.122	81.3	11298.6	38.0	37.0	515.3	13.2	304.2	319.0	341.0	289.7
	0.3	133.1	0.160	75.7	11268.5	37.9	36.8	513.3	11.5	303.7	318.5	340.3	289.3
	0.4	107.8	0.199	69.6	11202.4	37.7	36.7	510.6	9.8	302.4	317.2	338.8	288.1
	0.5	89.7	0.237	62.9	11076.7	37.3	36.7	506.5	8.2	299.8	314.4	335.9	285.6
	0.6	75.2	0.276	55.5	10854.0	36.5	37.1	500.4	6.6	294.9	309.2	330.7	280.9
	0.7	62.5	0.314	46.9	10467.3	35.2	38.2	491.1	5.1	286.1	299.8	321.6	272.3
	0.8	50.2	0.353	36.8	9776.1	32.9	40.5	476.0	3.6	270.3	282.7	305.4	256.8
Loamy Sand	0.1	483.8	0.092	58.2	5602.2	38.4	37.4	254.5	13.8	149.5	156.7	167.8	142.4
	0.2	276.4	0.128	54.7	5598.0	38.4	37.2	253.8	12.2	149.5	156.7	167.6	142.4
	0.3	195.6	0.163	50.9	5584.2	38.3	36.9	252.8	10.6	149.3	156.6	167.4	142.3
	0.4	150.0	0.198	46.8	5552.5	38.1	36.6	251.4	9.0	149.0	156.2	166.9	141.9
	0.5	119.2	0.233	42.3	5490.2	37.6	36.4	249.3	7.5	148.1	155.4	165.8	141.2
	0.6	95.9	0.269	37.3	5376.9	36.9	36.3	246.1	6.0	146.4	153.6	163.9	139.5
	0.7	76.5	0.304	31.5	5176.0	35.5	36.5	241.3	4.5	143.2	150.2	160.4	136.5
	0.8	58.9	0.339	24.6	4811.2	33.0	37.4	233.4	3.1	137.2	143.8	153.9	130.6
Sandy Loam	0.1	1765.2	0.100	36.0	2145.5	49.7	48.3	86.5	17.4	45.4	47.2	52.1	42.9
	0.2	799.2	0.134	33.8	2145.5	49.7	48.0	86.3	15.4	45.4	47.2	52.0	42.9
	0.3	494.2	0.169	31.5	2140.3	49.5	47.6	85.9	13.4	45.4	47.2	52.0	42.9
	0.4	344.1	0.203	29.0	2129.8	49.3	47.1	85.3	11.3	45.3	47.2	51.8	42.9
	0.5	253.1	0.238	26.2	2107.9	48.8	46.5	84.5	9.3	45.2	47.0	51.6	42.7
	0.6	190.3	0.272	23.1	2066.0	47.8	45.7	83.3	7.3	44.8	46.7	51.2	42.5
	0.7	142.4	0.307	19.5	1988.8	46.0	44.9	81.4	5.4	44.2	46.1	50.3	41.8
	0.8	102.3	0.341	15.2	1843.8	42.7	44.0	78.1	3.5	42.8	44.6	48.7	40.6
Loam	0.1	16941.8	0.113	20.9	719.7	69.2	67.6	24.3	24.9	10.7	11.0	12.6	10.0
	0.2	4883.1	0.148	19.6	719.6	69.2	67.2	24.2	22.0	10.7	11.0	12.6	10.0
	0.3	2330.8	0.184	18.3	718.8	69.1	66.6	24.1	19.1	10.7	11.0	12.6	10.0
	0.4	1354.4	0.219	16.8	716.5	68.9	65.9	23.9	16.2	10.7	11.0	12.6	10.0
	0.5	866.2	0.254	15.2	711.0	68.4	64.8	23.7	13.2	10.7	11.0	12.6	10.0
	0.6	579.6	0.289	13.4	699.3	67.2	63.2	23.3	10.3	10.7	11.0	12.5	10.0
	0.7	390.5	0.324	11.3	676.0	65.0	61.0	22.7	7.5	10.6	10.9	12.4	9.9
	0.8	252.5	0.360	8.8	629.2	60.5	57.7	21.6	4.7	10.4	10.8	12.1	9.8
Silt Loam	0.1	$1.4 \times 10^5$	0.105	16.3	402.8	89.5	87.8	12.3	35.1	4.6	4.7	5.6	4.3
	0.2	25267.3	0.144	15.3	402.8	89.5	87.3	12.2	31.1	4.6	4.7	5.6	4.3
	0.3	9318.5	0.182	14.3	402.6	89.5	86.6	12.2	27.0	4.6	4.7	5.6	4.3
	0.4	4529.8	0.220	13.2	401.8	89.3	85.7	12.1	22.9	4.6	4.7	5.6	4.3
	0.5	2531.6	0.258	11.9	399.6	88.8	84.4	12.0	18.8	4.6	4.7	5.6	4.3
	0.6	1519.5	0.297	10.5	394.4	87.6	82.4	11.8	14.7	4.6	4.7	5.6	4.3
	0.7	933.0	0.335	8.9	383.2	85.2	79.3	11.5	10.6	4.6	4.7	5.5	4.3
	0.8	553.5	0.373	7.0	359.1	79.8	74.2	10.9	6.6	4.6	4.7	5.4	4.3
Silty Clay Loam	0.1	$2.2 \times 10^7$	0.123	6.0	60.0	85.7	84.9	1.9	30.3	0.7	0.7	0.9	0.7
	0.2	$1.1 \times 10^6$	0.157	5.6	60.0	85.7	84.5	1.9	26.8	0.7	0.7	0.9	0.7
	0.3	$1.9 \times 10^5$	0.191	5.2	60.0	85.7	84.1	1.9	23.3	0.7	0.7	0.9	0.7
	0.4	53399.3	0.225	4.8	60.0	85.7	83.4	1.9	19.8	0.7	0.7	0.9	0.7
	0.5	19954.8	0.259	4.4	59.9	85.5	82.5	1.8	16.3	0.7	0.7	0.9	0.7
	0.6	8725.5	0.294	3.9	59.5	85.0	81.0	1.8	12.8	0.7	0.7	0.9	0.7
	0.7	4137.5	0.328	3.3	58.5	83.6	78.5	1.8	9.3	0.7	0.7	0.9	0.7
	0.8	1966.9	0.362	2.6	55.9	79.9	73.6	1.7	5.8	0.7	0.7	0.9	0.7

606 **Table 5.** Summary of the soil hydraulic properties estimated from the Beerkan database.

Statistic	$\lambda_c$	$K_s$ (mm h <sup>-1</sup> )				
	(mm) (Eq. 9)	BEST-steady (Eq. 20)	OPD (Eq. 10)	WU2 (Eq. 11)	SSBI (Eq. 12)	A4 (Eq. 13)
<i>N</i>	427	427	427	427	427	427
min	1.5	1.3	1.4	1.4	1.6	1.2
max	737.7	3550.9	3294.1	3493.8	3758.7	3173.7
mean	112.6	270.7	258.5	269.3	305.2	244.6
median	68.0	156.8	153.3	156.4	182.4	142.0
CV	101.3	142.0	140.1	141.3	137.6	141.3

607

608 **Figure 1.** Textural classification of the soils included in the Beerkan infiltration database.

609 **Figure 2.** Cumulative infiltration curves for different soils and initial effective saturation degrees,  $S_e$ . The  
610 curves were generated analytically using Eqs. (14) and (15) and the parameters listed in **Table 2**. The  
611 labels in ordinate and abscissa report respectively the total infiltrated water and the duration of the  
612 infiltration process. Note that the duration was fixed at three times the maximum time ( $t_{max}$ ) for which the  
613 explicit transient infiltration model proposed by Haverkamp et al. (19994) is valid.

614 **Figure 3.** Relative error,  $Er(\lambda_c)$ , of the estimated macroscopic capillary length compared to reference values  
615 for six synthetic soils listed in **Table 2**.

616 **Figure 4.** Relative error of the estimated values for saturated soil hydraulic conductivity,  $Er(K_s)$ , for six  
617 synthetic soils that were analyzed using four different methods. OPD = one-ponding depth (Eq. 10);  
618 WU2 = Method 2 (Eq. 11); SSBI = Steady version of the Simplified method based on a Beerkan  
619 Infiltration run (Eq. 12); A4 = Approach 4 (Eq. 13).

620 **Figure 5. (a)** Comparison between the estimated macroscopic capillary length,  $\lambda_c$ , and the intercept,  $b_s$ , of the  
621 regression line between cumulative infiltration and time under steady-state conditions. The soil  
622 capillarity categories are indicated by dashed horizontal lines. Also shown are examples of  $\lambda_c$  estimation  
623 for **(b)** moderate, **(c)** strong, and **(d)** negligible capillarity conditions.

624 **Figure 6.** Cumulative empirical frequency distribution of relative error in estimated saturated hydraulic  
625 conductivity,  $Er(K_s)$ , when 427 experiments in the Beerkan database were analyzed with the four  
626 considered methods (i.e., OPD, WU2, SSBI, A4) and  $\lambda_c$  was constrained **(a)** using Eq. (9), **(b)** assuming  
627  $\lambda_c = 83$  mm, **(c)** using a soil-dependent  $\lambda_c$  value. OPD = one-ponding depth (Eq. 10); WU2 = Method 2  
628 (Eq. 11); SSBI = Steady version of the Simplified method based on a Beerkan Infiltration run (Eq. 12);  
629 A4 = Approach 4 (Eq. 13).











