A coevolution model of defensive medicine, litigation and medical malpractice insurance

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Abstract

We model the interactions between physicians and patients, subject to clinical and legal risks, by means of an evolutionary game. In each instant of time, there is a large number of random pairwise encounters between members of the two populations. In each encounter, a physician heals a patient. The outcome of the healing process is uncertain and may result in patient harm; if that happens, the patient may sue the physician for medical malpractice. Physicians have to choose between two alternative treatments, D and ND, with different levels of benefits and risks.The treatment D is less risky than the alternative treatment ND, but has the disadvantage of providing a lower expected benefit to the patient. Therefore its provision corresponds to practicing "negative" defensive medicine.

Physicians prevent, at least partially, negligence charges by buying medical malpractice insurance. This transfers the risk of litigation from the physician to the insurer.

The dynamics we analyze are determined by a three-dimensional discrete-time dynamic system, where the variables x and y are, respectively, the shares of defensive physicians and litigious patients, while the variable a represents the insurance premium.

In such a context we show that, depending on the policy's price calculation principle as well as on model's parameters related to the accuracy of the judicial system and legal reforms, the game's final outcome could be an appealing equilibrium point in which the defensive strategy of physicians and litigious behavior of patients vanish, an interior (Nash) equilibrium in which all strategies coexist, or even an oscillatory behavior arisen via a Neimart-Sacker bifurcation in which strategies coexist in a recurrent manner. Furthermore, we state a "no chaos" conjecture, supported by analytical, numerical and empirical arguments.

Keywords: Defensive medicine, Discrete-time evolutionary dynamics, Medical malpractice insurance

1. Introduction

Defensive medicine is a diversion from best medical practice that allows physicians to defend themselves against malpractice lawsuits (Ehrlich and Becker, 1972), putting patients at risk of damage from procedures that are unnecessary or unsuitable (Tancredi and Barondess, 1978). Defensive medicine may be either "positive" or "negative". The former consists of additional medical services of negligible or no medical value performed to deter patients from filing malpractice claims or to persuade the legal system that the standard of care was met (Antoci et al. 2016)[1]. The latter, on the other hand, takes the shape of avoidance behavior (Feess, 2012): physicians try to avoid a source of legal danger by adopting safer but less effective therapies. Both these diversions from best medical practices generate damages to patients, and additional costs for health care systems. In high-risk specialties, defensive medicine is a global problem. In the United States, 93 percent of respondents said they practiced it (Studdert et al., 2005), and similar percentages were found in Europe (Palagiano, 2013; Garcia-Retamero and Galesic, 2014; Ramella et al., 2015; Osti and Steyrer, 2016), China (He, 2014), and Japan (Hiyama et al., 2006). Throughout their careers, U.S. doctors will almost likely encounter a claim and will almost certainly pay an indemnity (Jena et al., 2011). When adverse events occur, clinical safety may increase the likelihood of being sued; for example, claims for anesthesia-related death scarcely decreased after a tenfold decrease in mortality rate (Eichhorn, 1989; Kohn et al., 2000).

The magnitude of the problem is enormous. The medical liability system, which includes defensive medicine, is expected to cost the US more than 55 billion of dollars a year, or 2.4 percent to 10% of total health care

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[1] Excessive usage of Caesarean sections to deliver babies (Dubay et al., 1999, 2001; Fees, 2012) and excessive exposure to radiation in diagnostics (Hendee, 2010) are two examples.

spending (Kessler and McClellan, 1996; Price Waterhouse Coopers, 2006; Mello et al., 2010). Defensive medicine costs the Italian public health care system more than 10 billion of euros per year, accounting for 10.5 percent of total costs (Palagiano, 2013). Only radiography, orthopedics, and trauma surgery cost the Austrian public health system roughly 420.8 million of euros per year, or 1.62 percent of total expenditures (Osti and Steyrer, 2016).

The adoption choices of defensive medicine practices may be conditioned by medical malpractice insurance, which is a sort of professional liability insurance that protects health care providers from medical malpractice lawsuits. Its market has gone through periods of turmoil (such as in the mid-1970s, mid-1980s, and early 2000s, see Baker (2005)), which could be attributed in part to a rise in lawsuits. In fact, malpractice claims grew at a rate of over 10% per year in the 1970s and 1980s (U.S. General Accounting Office, 1986; Danzon, 1991), but have remained relatively stable since then (Jena et al., 2011). Over the course of the underwriting cycle (Baker, 2005), the cost of medical malpractice insurance can change significantly (Rodwin et al., 2008), more so than other insurance markets. This cost is determined by the liability system (Danzon et al., 2004), but the impact of tort changes is still theoretically unclear (Avraham and Schanzenbach, 2010).

In this paper, we model the interactions between physicians and patients, subject to clinical and legal risks, by means of an evolutionary game. Physicians have to choose between two alternative treatments, with different levels of benefits and risks, and can prevent, at least partially, negligence charges by buying medical malpractice insurance. Patients may sue the physicians for medical malpractice when adverse events occur. The adoption process of choices by patients and physicians is modeled by a three-dimensional discrete-time dynamic system based on the exponential replicator dynamics proposed in Cabrales and Sobel (1992), augmented by an equation describing the time evolution of the assurance premium. This way we describe bifurcations of the system, in particular Neimart -Sacker bifurcations, as well as is ruled out the period doubling bifurcation. Moreover, we investigate the possible convergence of trajectories to the boundary (precisely, to states where either all or no patients litigate). Finally, we are led to formulate, on the basis of analytic, numerical and statistic observations, a no-chaos conjecture, under specific assumptions on the system parameters.

Our paper builds on three previous works. In Antoci et al. (2016 and 2018) we analyzed the adoption processes of positive and negative defensive medicine, respectively, in a context in which physicians cannot self-protect themselves buying an insurance policy. In Antoci et al. (2019) we analyzed the coevolution of defensive medicine choices and the price of insurance policies in a context in which physicians practice "positive" defensive medicine.

The following is a breakdown of the article's structure. The model and its assumptions are introduced in the next section. The consequent evolutionary dynamics is analyzed in Section 3. Section 4 provides comments on mathematical results of Section 3. Section 5 shows some numerical simulations. Section 6 concludes. The proofs of Theorems 1 and 2 are postponed in the Appendix.

2. The game

We assume that, at each instant of time $t \in [0, +\infty)$, a large number of pairwise encounters (via random matching) occur between physicians and patients. In each encounter, the physician provides a *treatment* to the patient. This treatment has an uncertain outcome, and may cause harm to the patient. If this happens, the patient can sue and bring the physician to court, accusing him of medical malpractice. The outcome of the legal proceedings is uncertain. The physician may choose to reduce the risk of being sued (and possibly convicted) by providing a treatment D that has the advantage of being less risky than an alternative treatment *ND*, but also the disadvantage of providing a lower expected benefit to the patient with respect to *ND*. We will say that physicians who opt for choice D implement defensive medicine.

The benefits/costs of agents are expressed in monetary (economic) terms, and may also incorporate psychological and reputation factors. Treatments D and *ND* provide the patient with certain benefits equal to $B_D > 0$ and $B_{ND} > 0$, respectively, and uncertain harm $H > 0$, which can occur with (exogenous) probabilities q_D and q_{ND}. Thus, the patient's expected benefits are $\tilde{B}_D = B_D - q_D H$ and $\tilde{B}_{ND} = B_{ND} - q_{ND} H$. Similarly, treatments provide the physician with expected benefits $B_D^{PH} > 0$ and $B_{ND}^{PH} > 0$. We assume $B_{ND}^{PH} > B_D^{PH}$, $\tilde{B}_{ND} > \tilde{B}_D$ and $0 < q_D < q_{ND} < 1$; that is, despite the higher clinical risk, *ND* treatment can be considered the optimal treatment.

If the patient suffers the uncertain harm H, he may decide to initiate a malpractice lawsuit incurring an (ex ante) cost $C_L > 0$. If the patient wins the litigation, he will receive compensation amounting to $E > 0$.

Let us assume that the judge finds the physician guilty with (exogenous) probabilities p_D and p_{ND}, which may depend on the type of treatment. We will assume $0 \leq p_{ND},\ p_D \leq 1$, without any assumption about the relative values of p_D and p_{ND}.

The (one-shot) game works as follows. The physician can play two (pure) strategies, D or *ND*. Similarly, the patient can play two strategies: he can choose whether to sue the physician if he suffers the damage H from the treatment (strategy L) or not (strategy *NL*). Each player chooses one's own strategy ex ante, without knowing the other players' strategy.

Summarizing the conditions on the parameters, we have:

$$B_{ND}^{PH} > B_D^{PH}, \tilde{B}_{ND} > \tilde{B}_D, 0 < q_D < q_{ND} < 1, 0 \le p_{ND}, p_D \le 1, E > 0, C_L > 0$$

We denote by $x(t)$ the share of physicians who choose strategy D at time t, and by $1 - x(t)$ the share of physicians who choose strategy ND; $0 \le x(t) \le 1$.

Similarly, we denote by $y(t)$ the share of litigious patients at time t and by $1 - y(t)$ the share of non-litigious patients; $0 \le y(t) \le 1$.

We assume that insurance against malpractice litigation is mandatory for physicians. We denote by $a(t)$ the price of the insurance policy at time t. The insurance company pays compensation E to the litigious patient who won the malpractice lawsuit, and requires the physician to pay an extra price equal to $\varepsilon a(t)$, with $\varepsilon > 0$, in order to incentive best practice choices on the part of the physician. Finally, we assume that physician's payoffs are also negatively affected by the reputation damage $R > 0$ in case of conviction.

Physician's payoff matrix is:

$$\begin{array}{ccc} & L & NL \\ D & B_D^{PH} - a(t) - q_D\, p_D\, [R + \varepsilon a(t)] & B_D^{PH} - a(t) \\ ND & B_{ND}^{PH} - a(t) - q_{ND}\, p_{ND}\, [R + \varepsilon a(t)] & B_{ND}^{PH} - a(t) \end{array} \tag{1}$$

while patient's payoff matrix is:

$$\begin{array}{ccc} & D & ND \\ L & \tilde{B}_D - q_D\,(C_L - p_D E) & \tilde{B}_{ND} - q_{ND}\,(C_L - p_{ND} E) \\ NL & \tilde{B}_D & \tilde{B}_{ND} \end{array} \tag{2}$$

The price of insurance policy $a(t+1)$ at time $t+1$ that would allow for a balanced budget (in expected value), considering the values of the variables at time t as an estimate of the values at time $t+1$ (static expectations), must satisfy the following balanced budget condition for insurance (N denotes the size of the population of doctors):

$$-\left[q_D\, p_D E \cdot y(t)\right] \cdot x(t) N - \left[q_{ND}\, p_{ND} E \cdot y(t)\right] \cdot \left[1 - x(t)\right] N + a(t+1) \cdot N = 0 \tag{3}$$

where:

1. $q_D\, p_D E \cdot y(t)$ represents the expected loss for each physician adopting the strategy D ($y(t)$ is the probability of being matched with a litigious patient);
2. $x(t)N$ is the number of physicians adopting the strategy D;
3. $q_{ND}\, p_{ND} E \cdot y(t)$ is the expected loss for each physician adopting the strategy ND;
4. $[1 - x(t)]\, N$ is the number of physicians adopting the strategy ND.

Equation (3) can be written as follows:

$$a(t+1) = \{q_D\, p_D E \cdot x(t) + q_{ND}\, p_{ND} E \cdot [1 - x(t)]\} \cdot y(t) \tag{4}$$

This form of policy pricing implies that $a(t+1) = 0$ for $y(t) = 0$. We define the pricing dynamic more generally by adding to the value of (4) a constant mark-up $\overline{a} \ge 0$:

$$a(t+1) = [q_D\, p_D E \cdot x(t) + q_{ND}\, p_{ND} E \cdot [1 - x(t)]] \cdot y(t) + \overline{a} \tag{5}$$

We will analyze the exponential replicator dynamics used in Bischi and Merlone (2017) and in Bischi et al. (2018), based on the monotone selection dynamics proposed in Cabrales and Sobel (1992), augmented by equation (5). Replicator dynamics are driven by the expected payoffs of the four strategies D, ND, L, NL (here after, we drop the time t):

$$\begin{aligned} M_1(y,a) &= \left[B_D^{PH} - a - q_D\, p_D\,(R + \varepsilon a)\right] y + \left(B_D^{PH} - a\right)(1 - y) = \\ &= B_D^{PH} - a - q_D\, p_D\,(R + \varepsilon a)\, y \end{aligned}$$

$$\begin{aligned} M_2(y,a) &= \left[B_{ND}^{PH} - a - q_{ND}\, p_{ND}\,(R + \varepsilon a)\right] y + \left(B_{ND}^{PH} - a\right)(1 - y) = \\ &= B_{ND}^{PH} - a - q_{ND}\, p_{ND}\,(R + \varepsilon a)\, y \end{aligned}$$

$$\begin{aligned} P_1(x) &= \left[\tilde{B}_D - q_D\,(C_L - p_D E)\right] x + \left[\tilde{B}_{ND} - q_{ND}\,(C_L - p_{ND} E)\right](1 - x) = \\ &= \tilde{B}_{ND} - q_{ND}\,(C_L - p_{ND} E) + \left[\tilde{B}_D - \tilde{B}_{ND} + q_{ND}\,(C_L - p_{ND} E) - q_D\,(C_L - p_D E)\right] x \end{aligned}$$

$$P_2(x) = \tilde{B}_D x + \tilde{B}_{ND}(1-x) = \\ = \tilde{B}_{ND} + \left(\tilde{B}_D - \tilde{B}_{ND}\right) x$$

The discrete dynamic system can be expressed as:

$$\begin{aligned} x(t+1) &= x(t) \tfrac{\exp kM_1(t)}{x(t)\exp kM_1(t)+(1-x(t))\exp kM_2(t)} \\ y(t+1) &= y(t) \tfrac{\exp kP_1(t)}{y(t)\exp kP_1(t)+(1-y(t))\exp kP_2(t)} \\ a(t+1) &= E\left\{q_D p_D x(t) + q_{ND} p_{ND}\left[1-x(t)\right]\right\} y(t) + \overline{a} \end{aligned} \tag{6}$$

where k is a positive parameter. In fact, k accounts for different choices of the numeraire and can be also interpreted as an adjustment speed. Moreover, we observe that the dynamics of the policy price is independent of k, since, when E and $\overline{a}$ are multiplied by k, so is $a(t+1)$. Hence, both members in the third equation are multiplied by k, which can be canceled. This implies that the policy price can remain expressed in the original currency (euros, dollars etc.), which is much more convenient.

3. Analysis of the dynamical system

Let $p_{ND}q_{ND} > p_D q_D$ (the case $p_{ND}q_{ND} < p_D q_D$ will be discussed later) and denote by Π the open parallelepiped $\{0 < x < 1, 0 < y < 1, \overline{a} < a < \widehat{a}\}$, where $\widehat{a} = \overline{a} + p_{ND}q_{ND}E$, while its closure will be denoted by $\overline{\Pi}$.

3.1. Boundary equilibria

Besides the four vertices:

$Q_1 = (0,0,a_1)$ (all physicians play *ND* and all patients play*NL*)
$Q_2 = (0,1,a_2)$ (all physicians play *ND* and all patients play*L*)
$Q_3 = (1,0,a_3)$ (all physicians play *D* and all patients play*NL*)
$Q_4 = (1,1,a_4)$ (all physicians play *D* and all patients play*L*)

where the a_i are easily computed, there may be a further equilibrium inside the plane $y = 1$, say $\widetilde{Q} = (\widetilde{x}, 1, \widetilde{a})$, $0 < \widetilde{x} < 1$, in the case $\frac{1}{\varepsilon}\left(\frac{B_{ND}^{PH} - B_D^{PH}}{q_{ND}p_{ND} - q_D p_D} - R\right) - \overline{a} - q_{ND}p_{ND}E$ has the same sign but is smaller in absolute value than $q_D p_D E - q_{ND}p_{ND}E$.

3.2. Interior equilibrium

An interior equilibrium can exist if $M_1(t) = M_2(t)$, $P_1(t) = P_2(t)$, $a(t+1) = a(t)$. Denoting the possible equilibrium as $Q_0 = (x_0, y_0, a_0)$, it is easily computed that $0 < x_0 < 1$ implies $(C_L - p_D E)(C_L - p_{ND}E) < 0$, while, posing $a(t) = F(x(t), y(t))$, one is led to the second degree equation in y:

$$(R + \varepsilon F(x,y))\, y\, (q_{ND}p_{ND} - q_D p_D) = B_{ND}^{PH} - B_D^{PH}$$

so that a necessary condition for $y_0 \in (0,1)$ is $\left(B_{ND}^{PH} - B_D^{PH}\right)(q_{ND}p_{ND} - q_D p_D) > 0$, i.e., $q_{ND}p_{ND} - q_D p_D > 0$. In such a case the second degree equation admits exactly one positive solution y_0, corresponding to an interior equilibrium if $0 < y_0 < 1$. Hence, the interior equilibrium, if it exists, is unique.

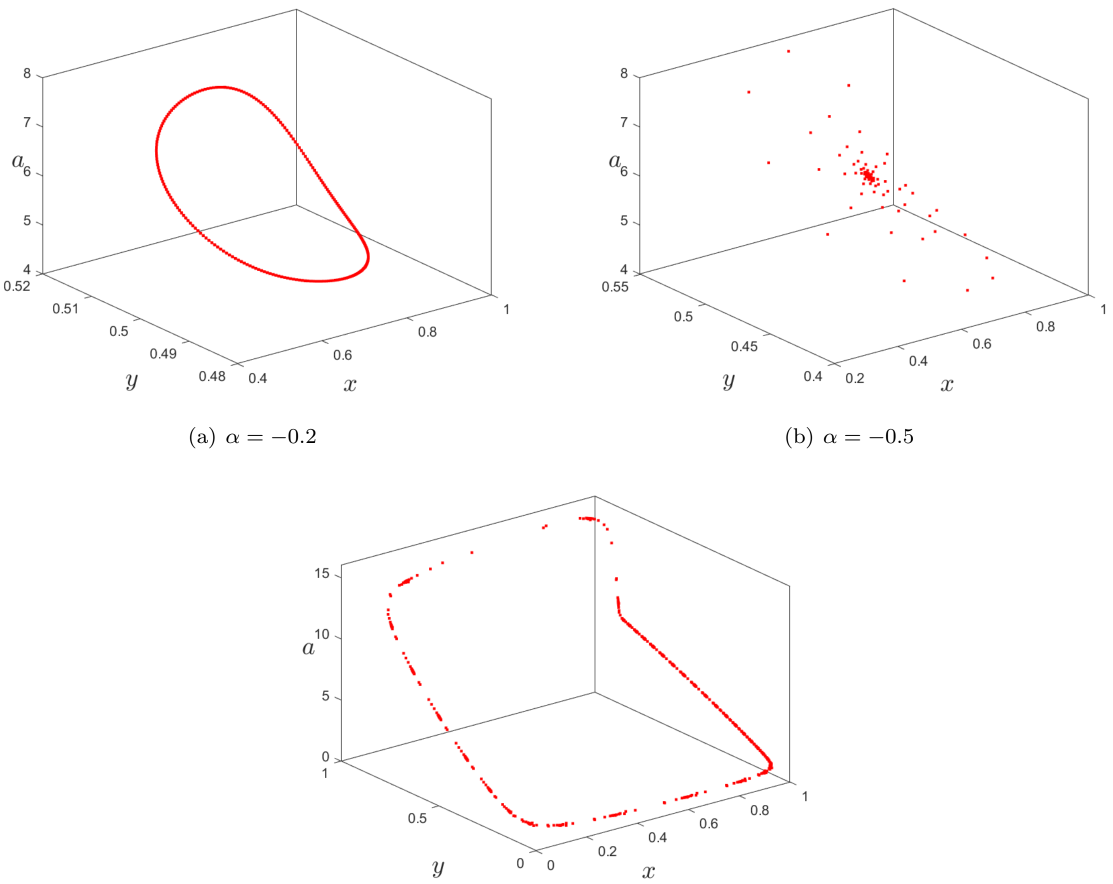
3.3. Stability of the interior equilibrium

Suppose the interior equilibrium $Q_0 = (x_0, y_0, a_0)$ exists. Calling J the corresponding Jacobian matrix, we have to compute the eigenvalues, i.e. the zeroes $\lambda_1, \lambda_2, \lambda_3$ of the equation

$$\det(J - \lambda I) = 0$$

If $|\lambda_1|, |\lambda_2|, |\lambda_3| < 1$, the equilibrium is attracting. Setting, by convenience of computation, $\lambda = 1 + \alpha$, we find that:

$$J - (1+\alpha) I = \begin{pmatrix} -\alpha & b & c \\ d & -\alpha & 0 \\ e & f & -1-\alpha \end{pmatrix}$$



(a) $\alpha = -0.2$

(b) $\alpha = -0.5$

(c) $\alpha = -0.85$

Figure 1: Parameter values are provided in Table 1.

where:

$$\begin{aligned}
b &= kx_0\left(1-x_0\right)\left(q_{ND}p_{ND}-q_Dp_D\right)\left(R+\varepsilon a_0\right)\\
c &= kx_0\left(1-x_0\right)\left(q_{ND}p_{ND}-q_Dp_D\right)\varepsilon y_0\\
d &= ky_0\left(1-y_0\right)\left(q_{ND}\left(C_L-p_{ND}E\right)-q_D\left(C_L-p_DE\right)\right)\\
e &= -Ey_0\left(q_{ND}p_{ND}-q_Dp_D\right)\\
f &= E(q_Dp_Dx_0+q_{ND}p_{ND}\left(1-x_0\right))
\end{aligned}$$

In fact, a necessary condition for Q_0 to be attracting is that, posed $H = J - I$, $\det H \leq 0$. We consider the generic case $\det H < 0$. In particular, assuming $q_{ND}p_{ND} - q_Dp_D > 0$, it follows from lengthy calculations that $b, c, f > 0$, while $d, e < 0$. Hence, the characteristic polynomial of H can be computed to be:

$$-\alpha^3 - \alpha^2 - p\alpha - q$$

with p and $q = -\det H$ positive. Therefore, if the three eigenvalues of H are real, they belong to the interval $(-1, 0)$ and consequently the eigenvalues of J belong to $(0, 1)$, implying the attractiveness of Q_0. However, when H (and thus J) has two complex conjugate eigenvalues, the real one, say α_0, lies, as we will see, in $(-2, 0)$, but the two complex conjugate eigenvalues of J may cross the circle of radius 1. Assume this occurs and pose $-\alpha_0 = \gamma$. If we consider H, whose trace is -1, the real part of the complex eigenvalues is equal to $(1+\gamma)/2$ and consequently those of J have real part $(1+\gamma)/2$. Thus, calling β the imaginary part, $((1+\gamma)/2)^2 + \beta^2 = 1$, that is $\beta^2 = 1 - ((1+\gamma)/2)^2$. Going back to H and considering the product of the eigenvalues, fairly long computations lead to the second degree equation:

$$-\gamma\left[\left(\frac{-1+\gamma}{2}\right)^2 + 1 - \left(\frac{1+\gamma}{2}\right)^2\right] = \det H = -q$$

Hence:

$$\gamma_{1,2} = \frac{1 \pm \sqrt{1-4q}}{2}$$

implying $q < \frac{1}{4}$.
Therefore, $-\gamma_i \in (-1, 0)$, $i = 1, 2$, must be an eigenvalue of H, i.e.:

$$-\gamma_i^3 + \gamma_i^2 + p\gamma_i + q = 0$$

which gives the two relations between p and q generating a Neimark-Sacker bifurcation.

3.4. Bifurcations and conjectures

When a Neimark-Sacker bifurcation occurs an invariant closed curve, say Γ, generically arises. In fact, naming $\delta = 1 - \gamma$ the real eigenvalue of J (the Jacobian matrix of the fixed point Q_0) corresponding to γ, it can be shown that, when $\delta \in (\delta_1, \delta_2) \subset (0, 1)$, Q_0 is an attractor, whereas for $\delta > \delta_2$ and $\delta < \delta_1$ an invariant closed curve Γ_i arises through the Neimark-Sacker bifurcation. In fact, when an interior equilibrium exists, $\det H < 0$ is checked to imply $p_{ND} > p_D$ and we will prove in Theorem1 that in such a case no trajectory starting in Π can tend to the boundary of $\overline{\Pi}$. Hence, some interior attractor exists and we can expect the curve generated by a Neimark-Sacker bifurcation to be attracting, as confirmed by numerical simulations. Moreover, by the Central Manifold Theorem, it lies on an invariant two-dimensional manifold. From then on other bifurcations occur. In order to investigate them, we can let δ, i.e. the real eigenvalue of J, vary, assuming the other two eigenvalues to be complex conjugate. Hence, consider first the case $\delta > \delta_2$. A bifurcation occurs when $\delta = 1$, corresponding to $\det H = 0$, so that $\det H > 0$ when $\delta > 1$. That can be seen as caused by p_{ND} becoming smaller than p_D (a verdict favorable to the patient is more probable if the physician adopts a defensive strategy), so that $C_L - p_D E < 0 < C_L - p_{ND} E$. When that occurs, we can conjecture that the invariant two-dimensional manifold, where Q_0 and Γ_2 lie, becomes a separatrix between the attracting basins of two boundary attractors (e.g., fixed points $(0, 0, a')$ and $(\widetilde{x}, 1, \widetilde{a})$. Consider, next, the opposite case $\delta < \delta_1$. We can let δ decrease until it crosses the value 0, after which we can expect that trajectories approaching the curve Γ_1 do so both rotating and flipping from one to the other side of the invariant surface where Γ_1 lies. Finally, a period-doubling bifurcation could occur when δ crosses the value -1, so that the new attractor would be constituted by a pair of invariant curves, between which the rotating trajectories oscillate. However, as we will see, the assumptions of our model rule out such a possibility.

3.5. Behavior at the boundary

Assume the existence of an interior equilibrium, which implies, in particular, $(C_L - p_D E)(C_L - p_{ND} E) < 0$ and $\left(B_{ND}^{PH} - B_D^{PH}\right)(q_{ND}p_{ND} - q_D p_D) > 0$. Then, the main difference is constituted by the sign of $p_{ND} - p_D$. Precisely, if $p_{ND} - p_D > 0$, the real eigenvalue at the equilibrium Q_0, which we have chosen as a bifurcation parameter, is smaller than 1, whereas $p_{ND} - p_D < 0$ implies the above eigenvalue to be greater than 1. Consider, then, the former case. The dynamic system is defined in a closed parallelepiped:

$$\overline{\Pi} = \{0 \leq x \leq 1, 0 \leq y \leq 1, \overline{a} \leq a \leq \widetilde{a}\}$$

where the value of $\widetilde{a}$ depends on the sign of $q_{ND}p_{ND} - q_D p_D$. So, the first step is to study the local stability of the four equilibria corresponding to pairs of pure strategies adopted by physicians and patients, namely $Q_1 = (0, 0, a_1)$, $Q_2 = (0, 1, a_2)$, $Q_3 = (1, 0, a_3)$, $Q_4 = (1, 1, a_1)$. In fact, straightforward computations show that all the respective Jacobian matrixes J_i have one real eigenvalue larger than 1 and one real eigenvalue smaller than 1, corresponding to the first two diagonal entries. Hence, such equilibria are all saddles. Moreover, passing from Q_1 to Q_2, from Q_3 to Q_4, from Q_1 to Q_3 and from Q_2 to Q_4 the positions of eigenvalues larger and smaller than 1 in the Jacobian matrixes are inverted, due, precisely, to the existence of the interior equilibrium. Finally, one further equilibrium may exist on the plane $y = 1$, say $\widetilde{Q} = (\widetilde{x}, 1, \widetilde{a})$, where $\widetilde{x} > x_0$ and $\widetilde{a} < a_0$, as it is easily calculated. That implies $\widetilde{Q}$ not to be attracting either. What we expect, then, is that no open region exists in Π whose trajectories converge to the boundary.

Consider, now, the case $p_{ND} - p_D < 0$, corresponding to the above real eigenvalue of Q_0 larger than 1. Continuing to assume $B_{ND}^{PH} > B_D^{PH}$ and therefore, for the existence of the interior equilibrium, $q_{ND}p_{ND} > q_D p_D$, we can consider again the local stability of Q_1, Q_2, Q_3, Q_4. It turns out that Q_1 has two eigenvalues $\in (0, 1)$ and one eigenvalue equal to 0 and, in fact, it is attracting for trajectories starting sufficiently near it. On the other hand, Q_2 and Q_3 are, as above, saddles. As to Q_4, it is also attracting if no interior equilibrium $\widetilde{\mathbf{Q}}$ exists within the side $y = 1$. Otherwise, $\widetilde{Q}$ or a closed curve surrounding it (after a Neimark-Sacker bifurcation) may be attracting.

All that is consistent with the above conjectures. Precisely, we can conjecture that, when an interior equilibrium exists, if $p_{ND} - p_D > 0$, generic trajectories converge to an interior attractor; whereas, when $p_{ND} - p_D < 0$, trajectories converge, generically, either to Q_1 or to an attractor lying on $y = 1$, and the two basins of attraction are separated by an invariant surface through Q_0 .

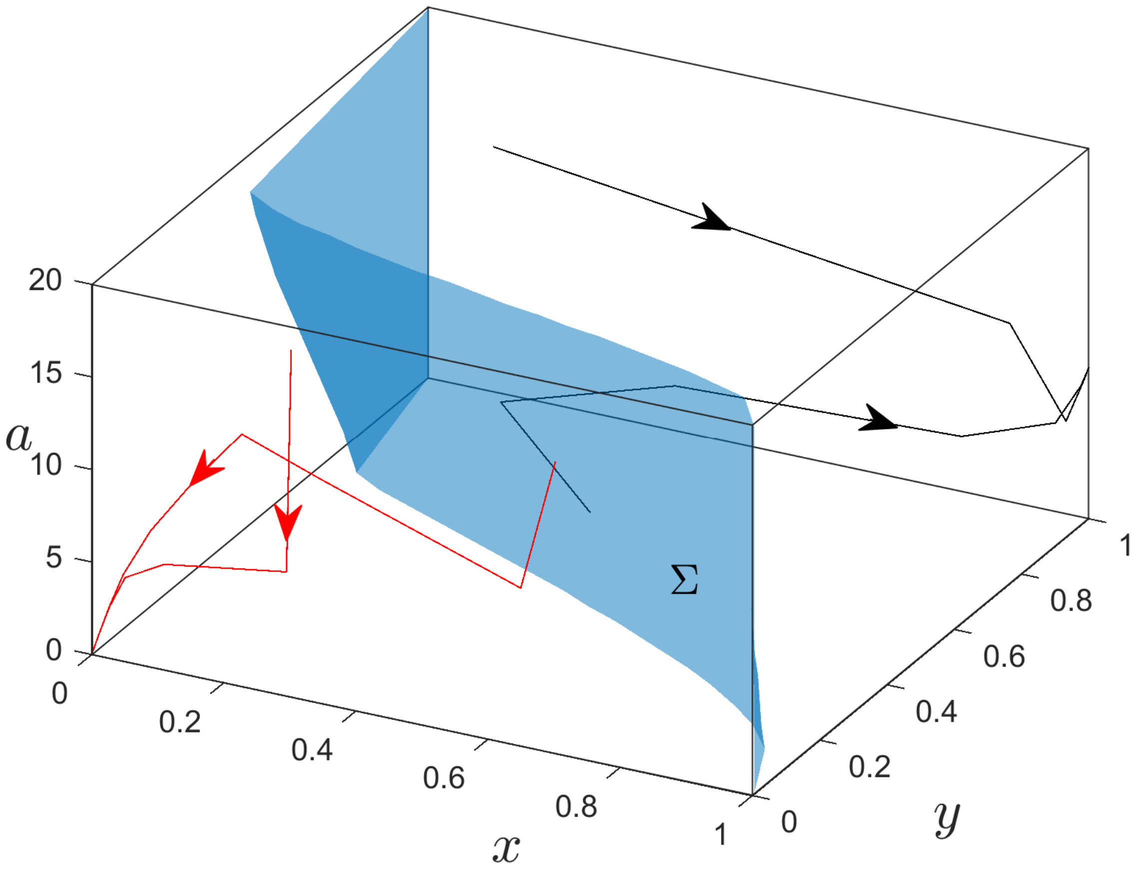


Figure 2: About trajectories tending to the boundary. $q_{ND} = 0.5$, $q_D = 0.2$, $\epsilon = 1$, $p_D = 0.41$, $p_{ND} = 0.3983$, $k = 7.1687$, $E = 100$, $B_{ND}^{PH} = 1.3443$, $B_D^{PH} = 1$, $\tilde{B}_D = 1$, $\tilde{B}_{ND} = 1.05$, $R = 0$, $C_L = 40$, $\bar{a} = 0$

3.6. Invertibility of the map

Now we drop the assumption of the existence of an interior equilibrium. Renaming $(x(t), y(t), a(t))$ as (x, y, a) and $(x(t+1), y(t+1), a(t+1))$ as (x', y', a'), we consider the map $F : (x, y, a) \to (x', y', a')$, generating the dynamic system, under the original conditions on the parameters, in particular $q_{ND} > q_D$.

Hence, F is defined by:

$$x' = x\frac{\exp kM_1(y,a)}{x\exp kM_1(y,a)+(1-x)\exp kM_2(y,a)}$$
$$y' = y\frac{\exp kP_1(x)}{y\exp kP_1(x)+(1-y)\exp j}$$
$$a' = [q_D p_D Ex + q_{ND} p_{ND} E(1-x)]\, y + \bar{a}$$

We distinguish two cases:

1. $q_{ND} p_{ND} > q_D p_D$
2. $q_{ND} p_{ND} < q_D p_D$.

In case 1, consider the Jacobian matrix of the map F. It follows from straightforward computations that, by choosing $kE \leq 1$, its determinant is positive. This way, the local invertibility of F is proved. However, since parallelepipeds are obviously simply connected, that leads to the global invertibility of the map F (Theorem of Hadamard-Caccioppoli, see Krantz and Parks, 2013).

In case 2, being $B_{ND}^{PH} - B_D^{PH} > 0$, no interior equilibrium exists and in fact it will be seen that $x(t) \to 0$. Hence, the possible non-invertibility of F does not affect the asymptotic outcomes of the trajectories.

Finally, as it concerns the invariant sides $x = 0, x = 1, y = 0, y = 1$, on the former three the map induces a one-dimensional dynamics, while the map is easily seen to be invertible on $y = 1$.

3.7. About trajectories tending to the boundary

Let $p_{ND} q_{ND} > p_D q_D$. Assume there exists an interior equilibrium, i.e. a stationary point $Q_0 = (x_0, y_0, a_0) \in \Pi$. Then the following theorem holds.

Theorem 1. *Under the above assumptions, suppose $p_{ND} > p_D$. Then no trajectory starting in Π can tend to the boundary of $\overline{\Pi}$, in particular to the planes $y = 0$ and $y = 1$. Vice-versa, suppose $p_D > p_{ND}$. Then there exist in Π two open regions whose trajectories converge, respectively, to $y = 0$ and $y = 1$. In the former case, x converges to zero; whereas, in the latter, there may exist a further equilibrium $(\widetilde{x}, 1, \widetilde{a})$ which is attracting or surrounded by an attracting curve.*

Proof. See the Appendix. ■

3.8. No interior equilibrium

We investigate the dynamics in the case there is no interior equilibrium, i.e. no equilibrium lying in the open parallelepiped Π. Then the following theorem holds.

Theorem 2. *If no interior equilibrium exists, all the trajectories starting inside the parallelepiped Π converge either to the side $y = 0$ or to the side $y = 1$.*

Proof. See the Appendix. ■

3.9. Period doubling

Suppose an interior equilibrium $Q_0 = (x_0, y_0, a_0)$ exists. Consider, as above, the Jacobian matrix J and the matrix $H = J - I$. If α is an eigenvalue of H, $1 + \alpha$ is an eigenvalue of J. Starting from $\det(H) < 0$, with $p_D E < C_L < p_{ND} E$, assuming the existence of two complex conjugate eigenvalues, we can see how bifurcations occur when the real negative eigenvalue α of H varies. In fact, posed $q = -\det(H)$, α must satisfy the equation:

$$\alpha^3 + \alpha^2 + p\alpha + q = 0$$

Explicitly

$$q = k^2 x_0 y_0 (1 - x_0)(1 - y_0)\left[q_{ND}(p_{ND}E - C_L) + q_D (C_L - p_D E)\right](q_{ND} p_{ND} - q_D p_D)(R + 2\varepsilon a_0 - \varepsilon \overline{a})$$

$$\begin{aligned} p = \quad & k^2 x_0 y_0 (1 - x_0)(1 - y_0)\left[q_{ND}(p_{ND}E - C_L) + q_D (C_L - p_D E)\right](q_{ND} p_{ND} - q_D p_D)(R + \varepsilon a_0) + \\ & + k\varepsilon x_0 (1 - x_0) y_0^2 E (q_{ND} p_{ND} - q_D p_D)^2 \end{aligned}$$

The period doubling (flip) bifurcation requires $\alpha = -2$: therefore it cannot occur in our model, since it is easily checked that $q < 2p$.

We observe that such a conclusion holds also in case the insurance price includes a loading, proportional to the expected loss. Hence, we can state that the existence itself of a medical malpractice insurance may prevent the occurrence of period doubling bifurcations.

3.10. No chaos?

We have seen that, when $p_{ND} q_{ND} > p_D q_D$ and thus an interior attractor may exist, a suitable choice of the positive parameter k (such that $kE \leq 1$) leads to invertibility of the map $F : \overline{\Pi} \rightarrow \overline{\Pi}$, while the assumption that the policy price is proportional to the expected loss rules out the possibility of a period doubling bifurcation. Actually, neither result allows to conclude that chaotic behaviors cannot occur, although, in particular, the route to chaos through a cascade of period doubling bifurcations is not admitted. On the other hand, numerical experiments, even when the map is possibly not invertible ($kE > 1$), show that an interior attractor, whenever it exists, consists of a single point or a closed curve. Finally, a further empirical argument can be added. Namely, we can hypothesize that the introduction of insurance policies (mandatory for physicians) plays a stabilizing role in the dynamics generated by defensive medicine and patients' litigation, and we expect that our model captures this empirical intuition.

For example, the Italian statistics relative to medical malpractice insurance and litigations in the arc of time 2010-2020 (IVASS,2021) appear to be quite well explained by our model. In fact, skipping details, the data report an increased length of the trials, a constant decrease of litigation cases and an oscillation in the insurance price, more consistent than that of the compensation amount. Now, the increased length of trials can be explained by a lower value of k (adjustment speed), so that kE remains small (say, ≤ 1), guaranteeing the invertibility of the map. On the other hand, longer trials imply higher legal costs, so that the litigation rate, y, decreases (even if compensation increases). But, in turn, this implies, because of the first equation of the system, that the rate of defensive physicians, x, decreases as well. However (see the third equation of the system), as a consequence, the insurance price increases (even if the compensation does not vary or varies by a small amount), so that, coming back to the first equation, x now increases and so on. In conclusion, we can

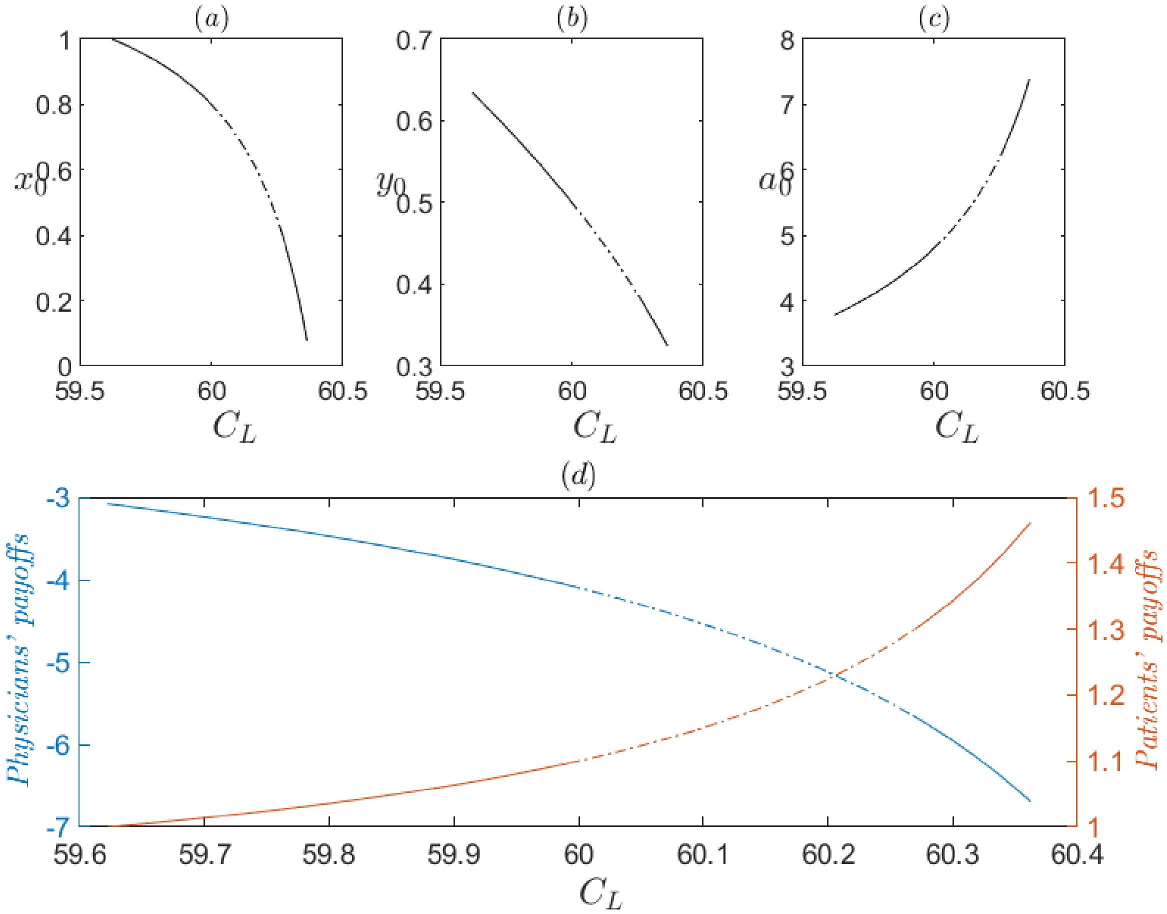


Figure 3: Comparative statics (solid line attractive equilibrium, dot line no attractive equilibrium) varying the parameter values C_L. The other parameter values are: $q_{ND} = 0.4$, $q_D = 0.1$, $\epsilon = 2.1$, $p_D = 0.5962$, $p_{ND} = 0.6038$, $k = 2.8197$, $E = 100$, $B_{ND}^{PH} = 1.9167$, $B_D^{PH} = 1$, $\tilde{B}_D = 1$, $\tilde{B}_{ND} = 1.05$, $R = 0$, $\bar{a} = 0$

hypothesize an oscillation of the system around an equilibrium with a rather low level of the litigation rate, which confirms the stabilizing role played by insurances.

Hence, on the basis of all the above considerations, we are led, anyway, to formulate the following

Conjecture 1. *Assume the system parameters satisfy the conditions of Section 2 and $kE \leq 1$. Then under no parameter configuration the system can exhibit a chaotic behavior.*

4. Interpretation of results

Let us start by remembering that, according to our assumptions $B_{ND}^{PH} > B_D^{PH}$ and $\tilde{B}_{ND} > \tilde{B}_D$, the equilibrium $Q_1 = (0, 0, a_1)$ (where all physicians play *ND* and all patients play *NL*) represents the social optimum. Despite the higher clinical risk, *ND* treatment can be considered as the optimal treatment, both for physicians and patients.

The analysis of the dynamic system

(6) has showed that at most one interior equilibrium, $Q_0 = (x_0, y_0, a_0)$, exists. In Q_0, all the strategies *D*, *ND*, *L*, *NL* coexist (i.e., they are played by strictly positive shares of physicians and patients, $0 < x_0 < 1$ and $0 < y_0 < 1$). In case Q_0 does not exist, all the trajectories starting from inside the parallelepiped:

$$\Pi = \{0 \leq x \leq 1, 0 \leq y \leq 1, \bar{a} \leq a \leq \hat{a}\} \tag{7}$$

tend to either the side $y = 0$ (where no patient chooses strategy *L*) or to the side $y = 1$ (where all patients choose strategy *NL*), depending on the legal cost C_L faced (ex-ante) by each patient choosing strategy *L*.

Necessary conditions for the existence of Q_0 are:

$$(p_D E - C_L)(p_{ND} E - C_L) < 0 \tag{8}$$

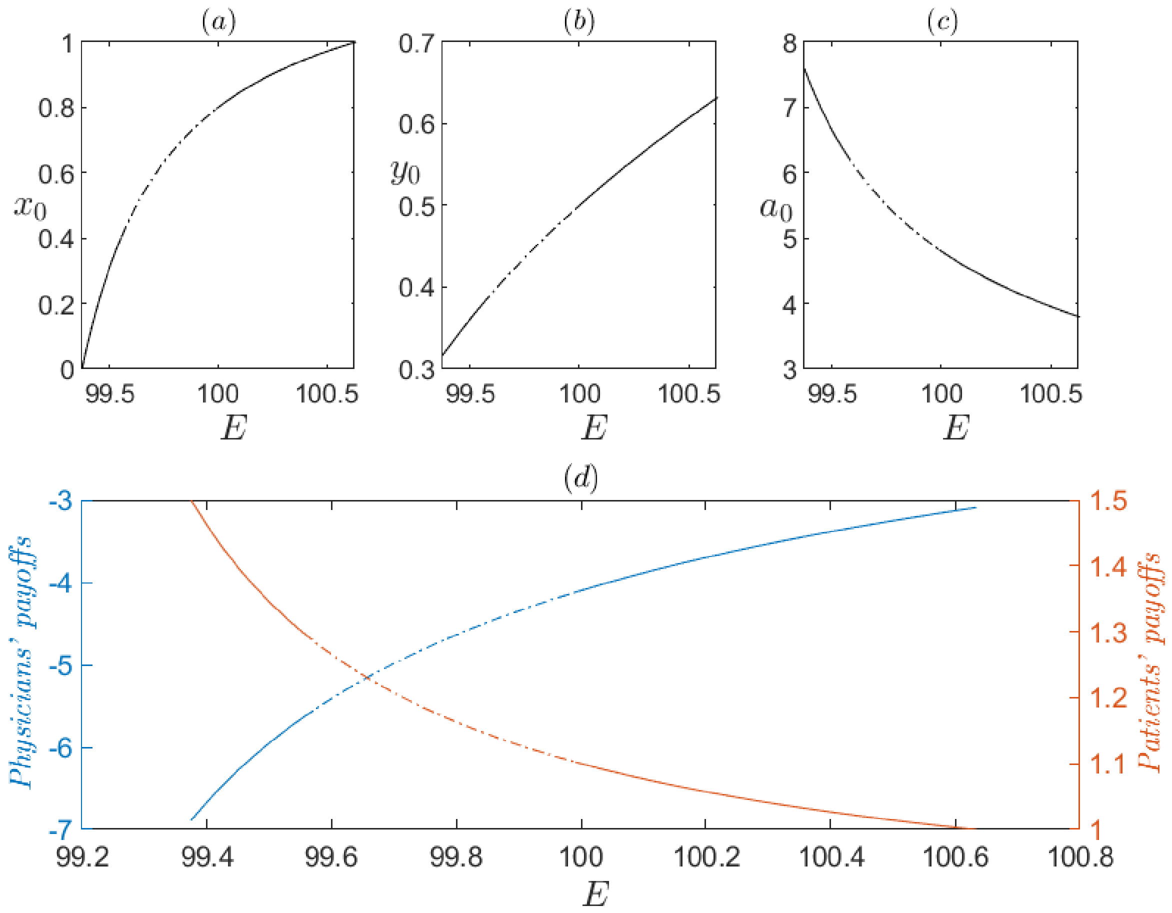


Figure 4: Comparative statics varying the parameter value E. The other parameter values: $q_{ND} = 0.4$, $q_D = 0.1$, $\epsilon = 2.1$, $p_D = 0.5962$, $p_{ND} = 0.6038$, $k = 2.8197$, $E = 100$, $B^{PH}_{ND} = 1.9167$, $B^{PH}_D = 1$, $\tilde{B}_D = 1$, $\tilde{B}_{ND} = 1.05$, $R = 0$, $C_L = 60$, $\bar{a} = 0q_{ND} = 0.4$, $q_D = 0.1$, $\epsilon = 2.1$, $p_D = 0.5962$, $p_{ND} = 0.6038$, $k = 2.8197$, $B^{PH}_{ND} = 1.9167$, $B^{PH}_D = 1$, $\tilde{B}_D = 1$, $\tilde{B}_{ND} = 1.05$, $R = 0$, $C_L = 60$, $\bar{a} = 0$

$$\left(B^{PH}_{ND} - B^{PH}_D\right)\left(q_{ND}p_{ND} - q_D p_D\right) > 0 \tag{9}$$

while a necessary condition for its local attractiveness is:

$$p_{ND} > p_D \tag{10}$$

Since $B^{PH}_{ND} > B^{PH}_D$ (i.e., the choice ND provides the physician a higher expected benefit, when the uncertain harm $H > 0$ does not occur) by assumption, condition (9) requires:

$$q_{ND}p_{ND} > q_D p_D \tag{11}$$

If (11) holds, then the probability that the uncertain harm occurs and the judge finds the physician guilty is higher for physicians adopting the non defensive strategy ND. Condition (11) may be interpreted as representing a context in which the judicial system is not efficient. The higher clinical risk q_{ND} (remember that, by assumption, $q_{ND} > q_D$) of the socially optimal strategy ND is not compensated by a lower enough probability p_{ND} of winning the litigation against a physician playing ND. Notice that, if the stability condition (10) holds, then condition (11) holds.

To interpret condition (8), remember that $p_{ND}E$ and $p_D E$ represent the expected compensations (when the uncertain harm occurs) for litigious patients when matched with physicians adopting ND and D, respectively, while C_L represents the (ex ante) legal cost of strategy L. So, condition (8) requires that the relative performance of strategies L and NL depends on the strategy played by the physician. More specifically, in the context in which the stability condition (10) holds, condition (8) requires that the (expected) gain of strategy L is higher than that of strategy NL if the physician adopts ND (i.e. $p_{ND}E - C_L > 0$) and lower if the physician adopts D (i.e. $p_D E - C_L < 0$). Vice-versa it occurs when the opposite of condition (10) holds.

When the interior equilibrium exists, then Neimart-Sacker bifurcations –where stability shifts from an equilibrium point to a curve– occur. There is a striking difference when $p_{ND} > p_D$ (i.e. the stability condition (10) holds) or vice-versa. If $p_{ND} > p_D$, then most trajectories in the parallelepiped tend to an interior attractor, either the equilibrium point Q_0 or a cycle. If, instead, $p_{ND} < p_D$, then there exists a surface, containing the interior equilibrium point Q_0, "separating" trajectories tending to $y = 0$ (where all patients choose *NL*) from trajectories tending to $y = 1$ (where all patients choose *L*). In the former case, x converges to zero; whereas, in the latter, there may exist a further equilibrium $(\widetilde{x}, 1, \widetilde{a})$ which is attracting or surrounded by an attracting curve (if it does not exist, then, along trajectories where $y \to 1$, $x \to 1$). Furthermore, we were led by analytical, numerical and empirical arguments to formulate a conjecture. In fact, in a system like ours (three-dimensional and non-linear) chaotic behaviors frequently appear. Vice-versa, the arguments discussed in 3.10 motivated us to conjecture that in our system (when $0 < kE \leq 1$) chaos never occurs, which in a sense amounts to hypothesize a stabilizing role, in the dynamics, played by the introduction of (mandatory) malpractice insurance policies.

Finally, if the interior equilibrium Q_0 does not exist, then we have the following cases (see the proof of Theorem 2, in Appendix):

1. If:

$$C_L > \max(p_{ND}E, p_D E) \tag{12}$$

then $y \to 0$ and consequently $x \to 0$.

2. If:

$$C_L < \min(p_{ND}E, p_D E) \tag{13}$$

then $y \to 1$. In that case, if $p_D q_D > p_{ND} q_{ND}$, $x \to 0$. Conversely, an equilibrium may exist in the plane $y = 1$, $\widetilde{Q} = (\widetilde{x}, 1, \widetilde{a})$, $0 < \widetilde{x} < 1$, which is an attractor or surrounded by an attractive closed curve. If such an equilibrium does not exist, then $x \to 0$.

3. If:

$$p_{ND}E < C_L < p_D E \text{ and } p_D q_D > p_{ND} q_{ND} \text{ or } p_D q_D < p_{ND} q_{ND} \tag{14}$$

then $y \to 0$ and $x \to 0$.

4. If:

$$p_D E < C_L < p_{ND}E \text{ (and therefore } p_D q_D < p_{ND} q_{ND}) \tag{15}$$

then $y \to 1$. If there exists an equilibrium in the plane $y = 1$, $\widetilde{Q} = (\widetilde{x},1,\widetilde{a})$, $0 < \widetilde{x} < 1$, such equilibrium is an attractor or surrounded by an attractive closed curve, to which all the trajectories starting in the open parallelepiped converge. If that equilibrium does not exist, then $x \to 0$.

If condition (12) holds, then the legal cost C_L is higher than the expected compensations $p_{ND}E$ and $p_D E$, and the system converges to the socially optimal equilibrium $Q_1 = (0, 0, a_1)$, where all physicians play *ND* and all patients play *NL*, with $a_1 = \overline{a} \geq 0$.

If condition (13) holds, the legal cost C_L is lower than the expected compensations $p_{ND}E$ and $p_D E$. In such a case, the share y of litigious patients always tends to 1. However, if the judicial system is efficient (i.e. $p_D q_D > p_{ND} q_{ND}$), the share x of defensive physicians tends to 0 and the system converges to the equilibrium $Q_2 = (0, 1, a_2)$, where all physicians play *ND* and all patients play *L*. If this is not the case (i.e. $p_D q_D < p_{ND} q_{ND}$), then an equilibrium may exist $\widetilde{Q} = (\widetilde{x}, 1, \widetilde{a})$, with $0 < \widetilde{x} < 1$ (where both strategies *D* and *ND* coexist), which is an attractor or surrounded by an attractive closed curve. If such an equilibrium does not exist, then the system converges to $Q_2 = (0, 1, a_2)$.

Condition (14) represents a context in which the judicial system is efficient (i.e. $p_D > p_{ND}$), and the legal cost C_L of strategy *L* is higher than $p_{ND}E$ but lower than $p_D E$. In such a case, the system converges to the socially optimal equilibrium $Q_1 = (0, 0, a_1)$, no matter what is the sign of $p_D q_D - p_{ND} q_{ND}$.

Finally, condition (15) represents a context in which the conditions $p_{ND}E - C_L > 0 > p_D E - C_L > 0$ and consequently $p_D q_D < p_{ND} q_{ND}$ hold. In such a context, if an equilibrium exists in the plane $y = 1$, $\widetilde{Q} = (\widetilde{x}, 1, \widetilde{a})$, $0 < \widetilde{x} < 1$, then that equilibrium is an attractor or surrounded by an attractive closed curve. If such an equilibrium does not exist, then the system converges to $Q_2 = (0, 1, a_2)$.

5. Numerical Simulations

Numerical simulations have the goal of illustrating

i) the analytical results obtained in the previous sections (see Example 1);

ii) how the values of x (the share of defensive physicians), y (the share of litigious patients), and a (the price of the insurance policy) –evaluated at the interior equilibrium $Q_0 = (x_0, y_0, a_0)$– vary as the values of parameters C_L, E, and q_{ND} increase (see Example 2).

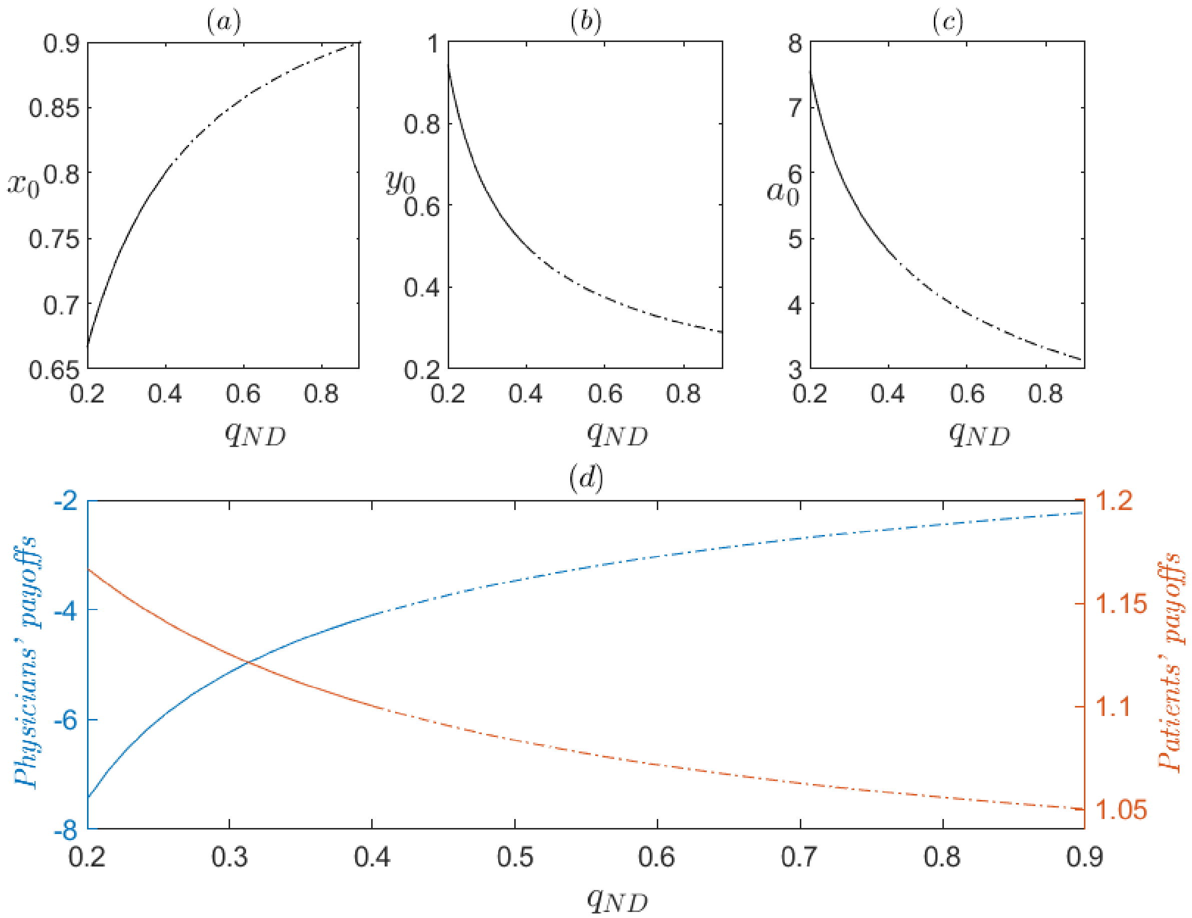


Figure 5: Comparative statics varying the parameter value q_{ND}. The other parameter values are: $q_D = 0.1$, $\epsilon = 2.1$, $p_D = 0.5962$, $p_{ND} = 0.6038$, $k = 2.8197$, $E = 100$, $B_{ND}^{PH} = 1.9167$, $B_D^{PH} = 1$, $\tilde{B}_D = 1$, $\tilde{B}_{ND} = 1.05$, $R = 0$, $C_L = 60$, $\bar{a} = 0$

Example 1. Given a large number of parameters in the system (6), an "algorithm" for their choice is described in the Appendix.

Table 1 summarizes the parameter values used for the numerical simulation shown in Figure 1, which illustrate the behaviors in case an interior equilibrium exists and $p_{ND} > p_D$: the attractor is either a closed curve (panels (a) and (c)) or a point (panel (b)).

α	q_{ND}	q_D	ϵ	p_D	p_{ND}	k	E	B_{ND}^{PH}	B_D^{PH}	$\tilde{B}_D$	$\tilde{B}_{ND}$	R	C_L	$\bar{a}$
-0.2	0.5	0.2	1	0.39979	0.40020	13.62940	100	1.34326	1	1	1.05	0	40	0
-0.5	0.5	0.2	1	0.39925	0.40074	7.16873	100	1.34434	1	1	1.05	0	40	0
-0.85	0.5	0.2	1	0.39661	0.40338	3.32896	100	1.34963	1	1	1.05	0	40	0

Table 1: Parameter values of Figure 1. The equilibrium point is $(x_0, y_0, a_0) = \left(\frac{5}{7}, \frac{1}{2}, 5.7143\right)$

Remark 3. *Our parameter values lead q to be $\frac{3}{16}$, which implies the two Neimark-Sacher bifurcations to occur at $\alpha_{NS}^1 = -0.2$ and $\alpha_{NS}^2 = -0.75$. These values divide the interval $(-1, 0)$ in three parts: $I_1 = (\alpha_{NS}^1, 0)$, $I_2 = (\alpha_{NS}^2, \alpha_{NS}^1)$, $I_3 = (-1, \alpha_{NS}^2)$. We choose the different values of α as $\alpha_1 = -0.2 \in I_1$, $\alpha_2 = -0.5 \in I_2$, $\alpha_3 = -0.85 \in I_3$.*

Figure 2, instead, illustrates the dynamics when $p_D > p_{ND}$: a surface through Q_0 separates trajectories tending to $y = 0$ from trajectories tending to $y = 1$. Parameter values for this simulation are provided in the figure caption.

Example 2. Panels (a)–(c) of Figures 3–5 show how the values of x (the share of defensive physicians), y (the share of litigious patients), and a (the price of the insurance policy) –evaluated at the interior equilibrium $Q_0 = (x_0, y_0, a_0)$– vary as the values of parameters C_L, E, and q_{ND} increase. Panel (d) of Figures 3–5 shows how the payoffs of physicians (blue lines) and patients (orange lines) –evaluated at Q_0– change in response to variations in C_L, E, and q_{ND} . Continuous lines indicate that Q_0 is locally attractive, while dotted lines indicate that Q_0 is not attractive and "surrounded" by an attractive closed trajectory.

Figure 3 shows that if the (ex ante) legal cost C_L increases, then the number of physicians adopting the virtuous strategy *ND* and the number of non-litigious patients increase; however, the price of the insurance policy a increases. Such a result can be explained taking into account that the *ND* strategy is characterized by a higher clinical risk ($q_{ND} > q_D$). So, a reduction in the share of defensive physicians generates an (expected) increase in the number of medical services that have an adverse outcome. In the numerical example illustrated in Figure 3, such an increase is not offset by the increase in the number of non-litigious patients.

Note that the growth in C_L generates an increase in patient payoffs (panel (d) of Figure 3) but a decrease in physician payoffs. The former result is due to an increase in the number of physicians adopting the strategy *ND*, characterized by higher expected benefits for patients and physicians ($B^{PH}_{ND} > B^{PH}_{D}$, $\tilde{B}_{ND} > \tilde{B}_D$). The latter result is due to the growth **o**f the insurance policy price a, which negatively affects physicians' payoffs.

A symmetric result is generated by an increase in the compensation E for patients winning the litigation (see Figure 4): the number of physicians adopting the *non-virtuous* strategy D and the number of litigious patients increase, the price of the insurance policy a decreases, patients' payoffs decrease, and physicians' payoffs increase.

Figure 5 illustrates the effects due to an increase in clinical risk q_{ND} of the virtuous strategy *ND*, in a context in which the judicial system is not efficient (i.e. $q_{ND}p_{ND} > q_D p_D$), and therefore defensive medicine is an effective self-protection choice against litigious patients. Observe that, as expected, the number of physicians adopting the defensive strategy D increases and the number of litigious patients decreases. The increase in the share of defensive physicians generates an (expected) reduction in the number of medical services that have an adverse outcome, and consequently a reduction in the policy price a. The inverse relationship between clinical risk q_{ND} of strategy ND and the share y of litigious patients may seem a paradoxical result. However, it can be explained by a predator-prey relationship between patients and physicians. Accordingly, a decrease in clinical risk q_{ND} favors not-defensive physicians and pushes, ceteris paribus, the share of defensive physicians below; therefore, the fitness of litigious patients improves (with respect to not-litigious ones) and their equilibrium share y increases. Increasing safety in clinical practice, then, may increase malpractice litigation against physicians when adverse events occur.

Parameters values per the numerical simulation in this example are given in the figure captions.

6. Conclusions

The main results of the analysis can be summarized as follows:

1. If one interior equilibrium exists, then $q_{ND}p_{ND} - q_D p_D > 0$. The occurrence of Neimark- Sacker bifurcations is investigated.
2. In the above case, the global dynamics strongly depends on the sign of $p_{ND} - p_D$. Namely, if $p_{ND} - p_D > 0$, the trajectories starting in the open parallelepiped Π converge, generically, to an interior attractor, either the equilibrium point or a closed curve surrounding it. Vice-versa, if $p_{ND} - p_D < 0$, there exists a surface containing the equilibrium point, which separates trajectories tending to the side $y = 0$ of Π from trajectories tending to the side $y = 1$ of Π.
3. Choosing the parameter k (say, a speed adjustment) sufficiently small (e.g., $k \leq E^{-1}$), in case the dynamics is not trivial (i.e., one interior equilibrium exists), the map $F : (x, y, a) \to (x', y', a')$, defining the discrete dynamics, is invertible.
4. Having chosen the insurance price proportional to the expected loss plus a fixed tariff, a period doubling bifurcation can never occur.

According to 1, an interior equilibrium (where strategies *D*, *ND*, *L*, and *NL* coexist) can exist if the probability that the uncertain harm occurs and the judge finds the physician guilty is higher for physicians adopting the non defensive strategy *ND*. That is, the higher clinical risk q_{ND} (remember that, by assumption, $q_{ND} > q_D$) of the socially optimal strategy *ND* is not compensated by a lower enough probability p_{ND} of winning the litigation against a physician playing *ND*. So, such a condition may be interpreted as representing a context in which the judicial system is not efficient. In the opposite case $q_{ND}p_{ND} - q_D p_D < 0$ (the judicial system is efficient, and therefore the interior equilibrium Q_0 does not exist), if the legal cost C_L is high enough w.r.t. the expected compensations $p_{ND}E$ and $p_D E$, then the dynamic system converges to the socially optimal equilibrium $Q_1 = (0, 0, a_1)$, where all physicians play *ND* and all patients play *NL* There remains the open question whether chaotic behaviors could occur under the assumed configurations of the parameters: if they could not, as we are led to conjecture, that could be interpreted as a stabilizing role played by the introduction of malpractice insurance policies.

7. Appendix

7.1. Proof of Theorem 1

At a given time $t > 1$, $a(t) = a(x, y) = \overline{a} + (p_{ND}q_{ND} - x(p_{ND}q_{ND} - p_D q_D))Ey$, where $x = x(t-1)$, $y = y(t-1)$. Assume an interior equilibrium $Q_0 = (x_0, y_0, a_0)$, exists. Hence, the curve $M_1 - M_2 = 0$, in the square $(x, y) \in [0, 1]^2$, has the shape of a hyperbolic arc joining two points $(0, \widetilde{y}_0)$ and $(\widetilde{x}_1, \widetilde{y}_1)$, $0 < \widetilde{y}_0 < y_0$, $x_0 < \widetilde{x}_{1,} \leq 1$, $y_0 < \widetilde{y}_{1,} \leq 1$. Denote by $f(x, y) = 0$ the equation of the curve. Suppose, first, $p_{ND} > p_D$. Then, y increases when $x < x_0$, while x increases when $f(x, y) > 0$, Consider a trajectory starting from $Q(0) = (x(0), y(0), a(0))$ such that $x(1) < x(0) < x_0$, $y(0) > y_0$ is sufficiently close to 1. Hence $f(x(0), y(0)) > 0$, so that $x(2) > x(1)$ and so on. If along the trajectory $x(t)$ tended to x_0 and $y(t)$ to some $\overline{y} > y_0$, posing $x(0) = x_0$, $y(0) = \overline{y}$, $a(0) = a(x_0, \overline{y})$, a trajectory would move from that point satisfying $y(1) = y(0)$ and $x(1) > x(0) = x_0$. Hence the original trajectory must continue until, eventually, $x(t) > x_0$, so that $y(t)$ starts decreasing. Vice-versa, suppose to start from $Q(0)$ with $y(0)$ sufficiently close to zero, in particular $0 < y(0) < \widetilde{y}_0 < y_0$. Then, for y to decrease x must be larger than x_0. So, assume $x(0) > x_0$.. By arguments as above, one can see that, at some time t, $x(t)$ will become smaller than x_0 and y will start increasing.

Now, consider instead $p_D > p_{ND}$. Then y increases when $x > x_0$ and decreases when $x < x_0$. Let us start from $Q(0) = (x(0), y(0), a(0))$ with $y(0)$ close to 1. In particular, if the *right* end-point of $f(x, y) = 0$ is $(1, \widetilde{y}_1)$ with $\widetilde{y}_1 < 1$, we can choose $y(0) > \widetilde{y}_1$. Then, if $x(0) > x_0$, y keeps increasing and so does x, until both tend to the value 1. Suppose, instead, the *right* end-point of $f(x, y) = 0$ is $(\widetilde{x}_1, 1)$ with $x_0 < \widetilde{x}_1 \leq 1$. Let $a(0)$ be such that x is increasing, with $y(0)$ sufficiently high and $x_0 < x(0) < \widetilde{x}_1$ satisfying $f(x(0), y(0)) > 0$. Then x and y increase. However, it may happen that at some instant $t - 1 > 0$ $f(x(t-1), y(t-1)) < 0$, so that $x(t+1) < x(t)$. However, if $y(0)$ has been chosen very close to 1, the increase of y, in particular $y(t-1) - y(t-2)$ is very small, so that, being $f(x(t-2), y(t-2)) > 0$, $f(x(t-1), y(t-1)) = -\delta$, $\delta > 0$ being small. Specifically, $x(t+1) > x_0$. Hence y keeps increasing, so that $y(t+1) > y(t)$ and, after a certain number k of steps, $f(x(t+k), y(t+k)) > 0$. This proves that eventually $y(t)$ tends to 1, while $x(t)$ tends to $\widetilde{x}_1$ if $(\widetilde{x}_1, 1, a(\widetilde{x}_1, 1))$ is a sink. Let us start, instead, from a point $Q(0) = (x(0), y(0), a(0))$ with $y(0) < \widetilde{y}_0$, $x(0) < x_0$, and $a(0)$ such that $x(1) < x(0)$. Then $f(x(0), y(0)) < 0$ and both x and y keep decreasing, tending to $(0, 0)$.

7.2. Proof of Theorem 2

In the following we will assume $p_{ND} > p_D$. If, instead, $p_D > p_{ND}$, analogous arguments apply.

Assuming there is no interior equilibrium in Π, we distinguish the following cases:

1. $C_L > p_{ND}E$
2. $C_L < p_D E$
3. $p_D E < C_L < p_{ND}E$.

Clearly, in case 1 (litigating is never convenient), $y \to 0$. Then, being $B_D^{PH} < B_{ND}^{PH}$, $x \to 0$ as well, while $a \to \overline{a}$.

Vice-versa, in case 2, $x \to 0$ if there is no equilibrium interior to the side $y = 1$; otherwise, in the dynamics restricted to $y = 1$, $(0, \overline{a})$ becomes a saddle, while the interior equilibrium, say (x_1, a_1), can be either attracting or surrounded by an attracting curve, through a Neimart-Sacker bifurcation.

Consider, finally, case 3. Posed

$$x_0 = \frac{q_{ND}(p_{ND}E - C_L)}{q_{ND}(p_{ND}E - C_L) + q_D(C_L - p_D E)}$$

it is easily checked that $y' > y$ if $x < x_0$, while $y' < y$ if $x > x_0$.

We distinguish two sub-cases:

a) no interior equilibrium exists inside $y = 1$;

b) there is an interior equilibrium (x_1, a_1) inside $y = 1$.

In case a) $M_1(y, a) < M_2(y, a)$ whatever is y, so that x keeps decreasing and y increases when $x < x_0$. Therefore, the trajectories tend, generically, to $(x, y, a) = (0, 1, \widetilde{a})$, where $\widetilde{a} = p_{ND}q_{ND} + \overline{a}$.

Consider, now, case b). Since $a(x, y)$ decreases with x, we can assume, generically, $0 < x_1 < x_0$. Hence, after some steps:

$$(p_{ND}q_{ND} - p_D q_D)\left[\varepsilon E(p_{ND}q_{ND} - x_0(p_{ND}q_{ND} - p_D q_D)) + \varepsilon\overline{a} + R\right] < B_{ND}^{PH} - B_D^{PH} \tag{16}$$

whereas:

$$(p_{ND}q_{ND} - p_D q_D)\left[\varepsilon E p_{ND}q_{ND} + \varepsilon\overline{a} + R\right] > B_{ND}^{PH} - B_D^{PH}$$

In order to simplify the notations, we will set in the following $R = \overline{a} = 0, B_{ND}^{PH} - B_{D}^{PH} = 1$ (nothing changes by different choices satisfying the above inequalities).

In any case, $a = a(x, y)$ with $x \geq x_0$ implies $x' < x$, so that eventually x becomes smaller than x_0 and y starts increasing.

For reversing such dynamics, x should become, again, greater than x_0. Therefore, since x' may increase only for values $x < x_1$, it should happen, at some time, $x' - x > x_0 - x_1$.

Now:

$$x' - x = \frac{xe^{M_1}}{xe^{M_1} + (1-x)\, e^{M_2}} - x = \frac{x(1-x)\left(e^{M_1 - M_2} - 1\right)}{xe^{M_1 - M_2} + 1 - x}$$

Then, by straightforward computations, recalling $R = \overline{a} = 0$:

$$M_1 - M_2 < \varepsilon E \left(p_{ND}q_{ND} - p_D q_D\right)\left(p_{ND}q_{ND} - x_1\left(p_{ND}q_{ND} - p_D q_D\right)\right) - 1$$

Since, from (16):

$$\varepsilon E < \frac{1}{\left(p_{ND}q_{ND} - p_D q_D\right)\left(p_{ND}q_{ND} - x_0\left(p_{ND}q_{ND} - p_D q_D\right)\right)}$$

we obtain:

$$M_1 - M_2 < \frac{\left(x_0 - x_1\right)\left(p_{ND}q_{ND} - p_D q_D\right)}{p_{ND}q_{ND} - x_0\left(p_{ND}q_{ND} - p_D q_D\right)} = \mu \tag{17}$$

As the possibility that $x' - x > x_0 - x_1$ is clearly as much higher as closer are x_0 and x_1, we can approximate e^{μ} by $1 + \mu$, so that:

$$x' - x < \frac{x(1-x)\,\mu}{1 + x\mu}$$

Recalling $x < x_1$, straightforward computations lead to the conclusion that $x' - x < x_0 - x_1$ if:

$$\frac{x_1\left(1 - x_0\right)\left(p_{ND}q_{ND} - p_D q_D\right)}{p_{ND}q_{ND} - x_0\left(p_{ND}q_{ND} - p_D q_D\right)} < 1$$

which clearly holds, being $p_{ND}q_{ND} - p_D q_D < p_{ND}q_{ND}$ and $x_0(1 - x_1) + x_1 < 1$.

In conclusion, y keeps increasing and all the trajectories starting in the open parallelepiped tend to $y = 1$.

7.3. Steps for finding parameter values for the numerical simulation in Figure 1.

For the sake of convenience, recall the following equations

$$\alpha^3 + \alpha^2 + p\alpha + q = 0, \quad \text{(characteristic equation of } H). \tag{18}$$

Explicitly

$$q = k^2 x_0 y_0 (1 - x_0)(1 - y_0)\left[q_{ND}(p_{ND}E - C_L) + q_D(C_L - p_D E)\right](q_{ND}p_{ND} - q_D p_D)(R + 2\varepsilon a_0 - \varepsilon\overline{a}) \tag{19}$$

$$\begin{aligned} p = {} & k^2 x_0 y_0 (1 - x_0)(1 - y_0)\left[q_{ND}(p_{ND}E - C_L) + q_D(C_L - p_D E)\right](q_{ND}p_{ND} - q_D p_D)(R + \varepsilon a_0) + \\ & + k\varepsilon x_0(1 - x_0) y_0^2 E\,(q_{ND}p_{ND} - q_D p_D)^2 . \end{aligned} \tag{20}$$

where x_0, y_0, a_0 are the coordinates of the interior equilibrium.

Eigenvalues of Neimark-Sacker bifurcation

$$\alpha_{NS}^{1,2} = \frac{-1 \pm \sqrt{1 - q}}{2}. \tag{21}$$

The calculation steps are described below

STEP 1: Pose $C_L = \dfrac{p_{ND} + p_D}{2} E$, $R = 0$ and $\bar{a} = 0$, then it is easy check that

$$x_0 = \frac{q_{ND}}{q_{ND} + q_D} \qquad y_0 = \frac{B_{ND}^{PH} - B_D^{PH}}{\epsilon\, aP}, \quad a_0 = x_0 y_0 q_D w E. \tag{22}$$

and

$$q = \frac{2\,z^2 y_0^2 (1-y_0) q_{ND}{}^2 q_D^2\ \left(P + 1/2\ (-q_{ND} + q_D)\, w\right) P \epsilon w}{(q_{ND} + q_D)^3} \tag{23}$$

$$p = \frac{q}{2} + \frac{z \epsilon y_0^2 q_D q_{ND}}{(q_{ND} + q_D)^2} P^2 \tag{24}$$

where $P = q_{ND} q_{ND} - q_D p_D$, $z = kE$ and $w = p_{ND} + q_D$.

STEP 2: Arbitrarily choose the values of q_{ND}, $q_D (q_D < q_{ND})$, $y_0 (0 \in (0,1)$, $\epsilon > 0$, and w.

STEP 3: Arbitrarily choose a value of q, say $\bar{q}$ in $(0, \frac{1}{4})$. Calculate the N-S bifurcations, by the (21).

STEP 4: Solve the system (in the variables P and z) above

$$\begin{cases} q(P,z) = q_0 \\ \bar{\alpha}^3 + \bar{\alpha}^2 + p(P,z)\bar{\alpha} + q(P,z) = 0 \end{cases}$$

where $\bar{\alpha}$ is a suitable value in $(-1, 0)$ and $q(P,z)$ and $p(P,z)$ are given by (23) and (24) respectively. Denoting $\bar{P}$ and $\bar{z}$ this solution, if $0 < \bar{P} < 1$ and $\bar{z} > 0$ holds, go to the next step; otherwise, go back to STEP2.

STEP 5 Solve the system of equations $\bar{P} = q_{ND} q_{ND} - q_D p_D$ and $w = p_{ND} + p_D$, to find the values of parameters p_{ND} and p_D. If $q_{ND} > q_D$ holds, go to the next step; otherwise, go back to STEP 2.

STEP 6 By $\bar{z} = kE$, choose a positive value of E, and calculate k or vice-versa. Calculate $\Delta B^{PH} = B_{ND}^{PH} - B_D^{PH}$ and a_0 by (22). Keeping $\Delta B^{BH} > 0$, fix an value of B_{ND}^{PH} and calculate B_D^{PH} or vice-versa. Finally, choose arbitrary values of the parameters $\tilde{B}_D$ and $\tilde{B}_{ND}$, such a that $\tilde{B}_{ND} > \tilde{B}_D$ holds.

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