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# Maladaptation to environmental degradation and the interplay between negative and positive externalities

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## ABSTRACT

This paper investigates the possible dynamics that may emerge in an economy in which agents adapt to environmental degradation by increasing the produced output to repair the damages of environmental degradation. The analyzed economy is characterized by both positive and negative externalities. On the one hand, an increase in production-related environmental degradation lowers the net income left at disposal for consumption and investment; on the other hand, it induces an increase in labor and capital to repair environmental damages from production, which enhances the positive externalities occurring in the production process. From the analysis of the model we show that there can be two steady states but only the one with lower capital level can be locally attractive. Both local and global indeterminacy may arise in the model, even with decreasing returns to scale. It follows that one cannot predict a priori which path the economy will follow when converging to an equilibrium, nor the equilibrium the dynamics will eventually converge to. In particular, the trajectories emerging from the model may eventually lead the economy to be trapped in a Pareto-dominated equilibrium with lower capital and higher environmental degradation levels. Moreover, the interplay between positive and negative externalities generates a rich set of possible trajectories that may lead to opposite extreme outcomes, namely, either infinite growth or the collapse of the economy.

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## 1. Introduction

Environmental degradation phenomena caused by human activity are attracting large attention in the media and in the policy debate for their potentially catastrophic consequences that pose mankind and life on the earth at risk. Scientific evidence provides many examples of severe environmental problems with large-scale and possibly unknown consequences. The glaciers melting rate has increased almost everywhere and the ice mass loss rate has tripled in the Antarctic in the last decade (Shepherd et al., 2018); the rise in the sea level has doubled its rate in the last twenty years (Nerem et al., 2018 ); extreme weather events have increased in frequency and intensity registering record numbers in terms of heavy rains and heat waves (USGCRP, 2017); oceans acidification has increased by 30 per cent due to the absorption of carbon dioxide (NASA, 2018). The degradation of the ecosystems has large effects on mankind's health and economic activities. It is estimated that climate change causes about 250.000 deaths every year (WHO, 2018). Moreover, the observed increase in floods and droughts causes crop failures that encourage mass migrations, especially from agricultural countries (Cai et al., 2016; IPCC, 2018).

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In facing these possibly catastrophic scenarios, mitigation and adaptation choices require an unprecedented level of coordination. Mitigation actions by a single agent generate positive (environmental) externalities on other agents and, therefore, agents tend to provide them at a level lower than the social optimum (see, e,g., Shogren and Crocker, 1991). The opposite holds for adaptation choices. They allow the single agent to protect herself from the damages generated by environmental degradation (they generate "positive private effects") but they may contribute to a further increase of environmental degradation ("negative public effects"). In such a case, according to Shogren and Crocker (1991), they tend to be adopted at a level higher than the social optimum, and their adoption process exhibits a self-reinforcing nature: an increase in adaptation effort determines an increase in environmental degradation, which in turn determines a further increase in adaptation effort, and so on.[1]

The aim of this paper is to propose an economic growth model in which adaptation choices may generate indeterminacy of equilibrium selection, even in an economy in which economic agents are endowed with perfect foresight. The economy we analyze is decentralized, that is, the choices of economic agents are not coordinated by a policy maker. In our economy, agents adapt to the negative effects of environmental degradation by sustaining adaptation costs that allow to self-protect from it. To bear such costs, agents need to increase their labor input in order to produce more output. The increase in output contributes to worsening the damages deriving from environmental degradation. In this context, the economy may end up in a self-reinforcing vicious circle that leads to an increase in environmental degradation and in the costs incurred by agents to defend themselves against it, which may eventually reduce the agents' welfare.

Following the terminology that has been recently proposed in the literature (Barnett and O'Neill, 2010; IPCC, 2018; Antoci et al., 2019), the adaptive choices considered in our model can be classified as "maladaptive". The term "maladaptation" (originally introduced by IPCC (2001)), in fact, refers to the large set of situations in which adaptation ends up further increasing environmental degradation. More precisely, maladaptation denotes self-protective strategies (Shogren and Crocker, 1991; Antoci and Bartolini, 2004; Antoci and Borghesi, 2012) which exacerbate environmental problems or shift negative impacts, risks, and exposure to other individuals, population groups or countries (Antoci and Borghesi, 2010). This phenomenon is increasingly observed in modern economies and it is regarded as one of the global emerging environmental challenges (UNEP, 2019).

The empirical literature and many case studies report numerous instances of maladaptive choices. Air conditioning is a paradigmatic example of maladaptation: global warming augments the demand and use of air conditioning systems that is rapidly growing all over the world, particularly in middle-income countries (cf. Davis and Gertler (2015)).[2] This brings about a dramatic increase in electricity consumption[3] which produces additional emissions thus worsening global warming.

Similarly, the rise in average temperatures increases the agricultural demand for irrigation, the use of water pumps in more arid zones (Borghesi, 2014; Beilin et al., 2012), of water transfer schemes across basins (Barnett and O'Neill, 2010), of desalination projects and of snow-making machines (Abegg et al., 2007) which are all very energy-intensive activities contributing to further enhance global warming. The increasing use of fertilizers and pesticides is another example of maladaptation (Klein et al., 2014). To counterbalance the observed land productivity loss, many agents use more and more chemical products. The consumption of these products, however, ends up polluting land and water (thus possibly further reducing the land productivity in the long run), while their production produces additional greenhouse gases in the air which eventually contribute to the desertification and crop yield loss in many areas, as well as to expected changes in land use in the farming sector (Fezzi et al., 2015).

The pharmaceutical production of medicaments to cure environment-related diseases accounts for another large area of maladaptive choices. As extensively shown in the literature (IPCC, 2018), environmental problems are responsible for a large number of health problems.[4] The need to address environment-related health issues pushes an increase in pharmaceutical production which is itself polluting and, therefore, health-damaging.

Also the reconstruction of buildings and infrastructures can be seen as a maladaptive choice if it simply re-establishes the status quo. Global warming causes extreme weather events that damage or destroy houses, roads, bridges etc. The need to repair these physical damages supports production in the building sector which causes additional emissions (UNEP, 2020; Röck et al., 2020).

As these few examples show, maladaptation choices represent a large category and a pervasive phenomenon. Indeed, the empirical literature has documented instances in many other areas and sectors beyond those described above, such as the tourism sector, water management, geoengineering, infrastructural development, disaster relief and resettlement, agriculture practices, land use changes, migration choices, insurance schemes, and urban planning (Hamin and Gurran, 2009; Barnett and O'Neill, 2010; Pouliotte et al., 2009; McEvoy and Wilder, 2012; Klein et al., 2014; Fezzi et al., 2017; Wagner and Weitzman, 2015; Weitzman, 2015; Magnan et al., 2016; UNEP, 2019).

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[1] In their seminal paper Shogren and Crocker (1991) distinguish two kinds of self-protection choices: those that filter (dilute) externalities versus those that transfer them to the others. The adaptation choices described in this paper enhance the environmental degradation suffered by all individuals, including the person who performs the adaptive choice. Think, for instance, of the numerous cases of self-protection from increasing temperatures that eventually contribute to global warming (see below). As such, the adaptation activity does not fully "transfer" the damage to the others (as in the terminology adopted by Shogren and Crocker (1991)), as it ends up being also a self-damaging activity.

[2] In China, for instance, sales of air conditioners have nearly doubled over the last few years, becoming more than eight times as many as those sold in the United States (cf. Euromonitor International (2014)).

[3] See Deschênes and Greenstone (2011) and Auffhammer and Aroonruengsawat (2011) for an analysis of the relationship between electricity consumption and temperature shocks in the US residential sector.

[4] Climate change is estimated to cause about 250'000 additional deaths each year and health losses that amount to 2–4 billions USD/year from 2030 (WHO, 2018). According to (Takakura et al., 2017), the economic costs of preventing heat-related illnesses in the workplace may cause GDP losses ranging between 0.5 and 4 per cent of world GDP in 2100 depending on the emissions scenario. Ng and Zhao (2011) estimate that an increase in 1 °C is associated with a fall by 3 per cent in global GDP.

The present paper intends to contribute to the literature on this issue focusing attention on the dynamics that might emerge in the economy in the presence of maladaptive choices. To provide a more exhaustive analytical framework, we will account not only for the negative externalities deriving from environmental degradation but also for the positive ones that may characterize the production process.

As pointed out above, we assume no government action and no mitigation activities by the agents. We are fully aware that mitigation policies are gaining increasing importance and that governments can play a key role in spurring/implementing such policies. However, here we want to focus on the dynamic effects of maladaptation and show that multiple equilibria may exists in a decentralized system in which agents do not care for environmental degradation (at least, not directly for the sake of it but only for its production consequences) and take no mitigation action but try only to defend from it, possibly provoking further environmental damages. In other words, we want to look at the consequences of two (possibly simultaneous) undesirable behaviors: on the one hand, inaction in terms of mitigation, on the other hand, wrong action in terms of adaptation.[5]

The analysis of our model shows that, in the scenario described above, the interplay between positive and negative externalities may generate a rich set of possible dynamic regimes, and very complex global indeterminacy scenarios may emerge (see the seminal paper by Matsuyama (1991)). More specifically, starting from the same initial values of the state variables (“history”, in the terminology of Krugman (1961)), different initial values of the jumping variable (whose choice is conditioned by agents' “expectations”) may collocate the economy along equilibrium trajectories approaching very different outcomes.[6] Along such trajectories, the economy may eventually converge to one or more steady states, to an infinite growth path or to the opposite extreme case in which the amount of capital in the economy goes to zero. Our results suggest, therefore, that the degree of uncertainty surrounding the costs generated by environmental degradation may be extremely high and that opposite outcomes can happen once the domino effects generated by adaptive choices are at work.

Global indeterminacy is the object of many contributions in economic growth theory (see Mino, 2017, for a review of the literature). Other growth models with environmental assets exhibit global indeterminacy scenarios (see, among the others, Antoci et al., 2011; Yanase, 2011; Fernández et al., 2012; Carboni and Russu, 2013, 2014; Bretschger and Schaefer, 2017; Bella and Mattana, 2018; Caravaggio and Sodini, 2018; Russu, 2021). However, this literature neglects the possible role of maladaptive choices in generating global indeterminacy. This paper aims to contribute to that literature in three main respects: (i) it extends the scope of application of global indeterminacy to maladaptation problems, (ii) it focuses on the case in which agents try to adapt to environmental degradation rather than coordinating their activities to mitigate environmental problems, (iii) differently from previous contributions in this research line (e.g. Antoci et al. (2021)), we deliberately assume that people do not care for environmental degradation per se, but only for its production and consumption consequences. While environmental awareness is certainly increasing all over the world, most people seem to care or even realize about environmental degradation only because this impacts their consumption and life habits. To provide an example, people probably do not care for (or hardly perceive) an increase in temperature by 1 °C, but they care about the impact this may have on agricultural production, on the probability of suffering adverse consequences from extreme events, on the insurance costs to protect against the environmental risks and so on. Assuming a “non-environmentalist” utility function allows to enrich and extend the indeterminacy results obtained in previous studies, showing that indeterminacy can occur also if agents have just an “instrumental” view of the environment and do not care for the environment per se.

Finally, differently from previous contributions in the global indeterminacy literature based on bifurcation techniques (e.g., Mattana et al., 2009; Bella et al., 2017), we derive results through an analytical characterization of the invariant surfaces separating different regimes of the trajectories (e.g. Antoci et al., 2011, 2014).

The present paper is structured as follows. Sections 2 and 3 define the set-up of the model and the associated dynamic system. Section 4 deals with the existence and local stability of steady states. Section 5 is devoted to the global analysis of dynamics, while Section 6 summarizes and discusses the main results emerging from the paper.

## 2. Set-up of the model

The economy we analyze is constituted by a continuum of identical economic agents; the size of the population of agents is normalized to unity. At each instant of time $t \in [0, \infty)$, the representative agent produces an output $Y$ by the following Cobb–Douglas technology

$$Y = AK^{\alpha}L^{\beta}, \text{ with } \alpha + \beta < 1 \tag{1}$$

where $K$ is the stock of physical capital accumulated by the representative agent, $L$ is the agent's labor input, and $A$ represents the positive externality

$$A = \overline{K}^{a}\overline{L}^{b}, \text{ with } a, b > 0 \tag{2}$$

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[5] While these assumptions may look too pessimistic as compared to the increasing commitments and call for environmental actions by many individuals and governments worldwide, they may capture the relative inaction in the fight against many environmental problems that has prevailed in past years and the lack of coordination among agents in their adaptation choices. See, for instance, Bird (2014) for a discussion of the relative unbalance between adaptation and mitigation actions against global warming.

[6] On the relationship between history and expectations see the interesting contribution by Bretschger and Schaefer (2017) who study the impact of energy policy on the relevance of expectations compared to history in driving the economy towards different steady states.

$\overline{K}$ and $\overline{L}$ denoting the economy-wide average values of $K$ and $L$, respectively.[7]

We assume that production activities cause environmental degradation and the latter determines a reduction in output via a damage function (see, among the others, Hackett and Moxnes, 2015; Golub and Toman, 2016; Bretschger and Pattakou, 2019). We denote with $\Omega \in (0,1]$ the share of the output $Y$ that can be used either for consumption or investment in physical capital, which is determined by the following damage function

$$\Omega(P) = \frac{1}{1+P^{\gamma}}, \text{ with } \gamma > 0 \tag{3}$$

where the variable $P$ represents an index of environmental degradation caused by the production activity. So, $\Omega(P)\cdot Y$ represents the net output, while $[1-\Omega(P)]\cdot Y$ represents the output required to repair the damages generated by $P$ (i.e. the cost of adaptation to environmental degradation).

We assume that the representative agent's instantaneous utility function depends on leisure $1-L$ and consumption $C$ of the net output $\Omega(P)\cdot Y$. More precisely, we consider a *constant intertemporal elasticity of substitution* (CIES) utility function (a function of this type is used, among the others, by Mino, 1999; Bennet and Farmer, 2000; Itaya, 2008; Antoci et al., 2011)

$$U(C,L) = \frac{\left[C(1-L)^{\theta}\right]^{1-\eta} - 1}{1-\eta} \tag{4}$$

where $\theta$, $\eta > 0$ and $\eta \neq 1$. This function is concave in $C$ and in $1-L$ if $\eta > \frac{\theta}{1+\theta}$. The parameter $\eta$ denotes the inverse of the intertemporal elasticity of substitution in consumption and leisure. Our function possesses the property that income and substitution effects exactly balance each other in the labor supply equation.

Notice that environmental quality does not enter the CIES utility function adopted here. It follows that environmental degradation affects the agents' utility only indirectly, through its impact on consumption and leisure. This is equivalent to assuming (somehow provocatively) that agents do not care for environmental quality per se, but for the consequences of environmental degradation which – in the present context – induce them to work harder to repair the environmental damages.[8]

The time evolution of $K$ (assuming, for simplicity, that the depreciation rate of $K$ is equal to zero) is represented by the differential equation

$$\dot{K} = \Omega A K^{\alpha} L^{\beta} - C \tag{5}$$

where $\dot{K}$ is the time derivative of $K$.

The time evolution of $P$ is determined by

$$\dot{P} = \delta\overline{Y} - \varepsilon P e^{-\zeta P} \tag{6}$$

where $\dot{P}$ is the time derivative of $P$, $\overline{Y}$ represents the economy-wide average output, and the parameters satisfy the conditions $\varepsilon, \delta > 0$, $\zeta \geq 0$.

The parameter $\delta$ measures the positive impact of $\overline{Y}$ on $P$. According to Eq. (6), environmental degradation $P$ depletes at the rate $-\varepsilon e^{-\zeta P}$, which is a decreasing function of $P$: when environmental degradation increases, the environment's self-regeneration capacity deteriorates and may eventually become zero (see Xepapadeas, 2005).[9]

Eqs. (3), (5), and (6) represent a context in which the increase in adaptation cost $[1-\Omega(P)]\cdot Y$ due to environmental degradation is financed via an increase in gross output $Y$, which in its turn determines an increase in $P$. So, the adaptation choices we consider can be classified as maladaptation (IPCC, 2018; Barnett and O'Neill, 2010).

The representative agent solves the optimization problem

$$\underset{C,L}{MAX} \int_0^{\infty} \frac{\left[C(1-L)^{\theta}\right]^{1-\eta} - 1}{1-\eta} e^{-rt} dt \tag{7}$$

subject to (5) and (6), with $K(0)$ and $P(0)$ given, $K(t)$, $P(t)$, $C(t) \geq 0$ and $1 \geq L(t) \geq 0$ for every $t \in [0,+\infty)$; $r > 0$ is the discount rate.

We assume that, in solving problem (7), the representative agent considers as exogenously determined the total productivity factor $A$ (positive externalities) and the impact on $P$ generated by $\overline{Y}$ (negative externalities), since, being economic agents a continuum, the impact on $A$ and $\overline{Y}$ of each individual is null. However, since agents are identical, ex post $\overline{Y} = Y$, $\overline{K} = K$ and

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[7] Following the seminal papers by Lucas (1988) and Romer (1994), by positive externalities we mean that a higher level of $L$ and $K$ generates improved/increased knowledge that becomes common knowledge (i.e. it is transferred to the rest of society) through a learning-by-doing mechanism. To provide just a few examples in the context of the adaptive choices discussed here, producing medicaments against the numerous health problems provoked by pollution may increase knowledge on how to deal also with other diseases. Analogously, producing air conditioners to defend from increasingly frequent heat waves may improve knowledge on cooling systems used for other purposes (e.g. to refrigerate industrial engines and computer servers). A similar reasoning applies to the reconstruction of buildings or infrastructures that have been damaged/destroyed by extreme weather events, which may bring about knowledge spillovers (e.g. in terms of construction materials and techniques) increasing the productivity in the building sector as a whole.

[8] Although the CIES utility function is sufficiently generic and widely used, the results of the model obviously hinge upon the specific utility function adopted here. Under alternative functional specifications, labor supply might shrink rather than grow in response to a negative productivity shock, thus preventing the self-reinforcing mechanism described here. We thank an anonymous reviewer for pointing this out.

[9] Notice that posing $\zeta = 0$, we get the equation $\dot{P} = \delta\overline{Y} - \varepsilon P$, which exhibits a constant decay rate of $P$, which implies an exponential decay function. If $\zeta > 0$, instead, the decay function is not exponential but takes an inverted-U shape.

$\overline{L} = L$ hold. Therefore, from (1) and (2), we get $Y = AK^{\alpha}L^{\beta} = K^{\alpha+a}L^{\beta+b}$. This implies that the private marginal productivity of $L$ and $K$ (obtained taking $A$ as exogenously given) is lower than the social marginal productivity of these factors (obtained by replacing $A$ with its correspondent value) and that trajectories resulting from our model are not optimal (i.e. they do not describe the social optimum). However, they represent Nash equilibria in the sense that, along them, no agent has an incentive to modify her choices if the others do not modify theirs.

## 3. Dynamics

Following Wirl (1997), the current value Hamiltonian function associated to problem (7) is

$$H = \frac{\left[C(1-L)^{\theta}\right]^{1-\eta}-1}{1-\eta} + \lambda\left(\Omega AK^{\alpha}L^{\beta}-C\right)$$

where $\lambda$ is the co-state variable associated to $K$. By applying the Maximum Principle, the dynamics of the economy are described by the equations

$$\dot{K} = \frac{\partial H}{\partial \lambda} = \Omega AK^{\alpha}L^{\beta} - C$$

$$\dot{\lambda} = \theta\lambda - \frac{\partial H}{\partial K} = \lambda\left(r - \alpha\Omega AK^{\alpha-1}L^{\beta}\right)$$

with the constraint

$$\dot{P} = \delta\overline{Y} - \varepsilon Pe^{-\zeta P} \tag{8}$$

where $C$ and $L$ satisfy the following conditions[10]

$$\frac{\partial H}{\partial C} = C^{-\eta}(1-L)^{\theta(1-\eta)} - \lambda = 0 \tag{9}$$

$$\frac{\partial H}{\partial L} = 0, \text{ i.e. } \theta C^{1-\eta}(1-L)^{\theta(1-\eta)-1}\left(1+P^{\gamma}\right) - \beta AK^{\alpha}L^{\beta-1}\lambda = 0 \tag{10}$$

Since our system meets the Mangasarian hypotheses, the above conditions plus the transversality condition

$$\lim_{t\to+\infty} \lambda(t)K(t)e^{-rt} = 0 \tag{11}$$

are sufficient for solving problem (7). This is the case also if $\alpha+\beta+a+b > 1$ (remember we assumed $\alpha+\beta < 1$), because $\overline{Y}$ and $A = \overline{K}^{a}\overline{L}^{b}$ are considered as exogenously given in the decision problem of the representative agent.

By replacing $\overline{Y} = Y$ and $AK^{\alpha}L^{\beta} = K^{\alpha+a}L^{\beta+b}$, the Maximum Principle conditions yield a dynamic system with two state variables, $K$ and $P$, and one jumping variable, $\lambda$.

Eqs. (9) and (10) allow us, after straightforward computations, to get the following system, defined in $R = \{K > 0, P > 0, 0 < L < 1\}$, equivalent to (12)

$$\begin{aligned}
\dot{K} &= \frac{1}{\vartheta\left(1+P^{\gamma}\right)}K^{a+\alpha}L^{b+\beta-1}\left[(\vartheta+\beta)L-\beta\right]\\
\dot{P} &= \delta K^{a+\alpha}L^{b+\beta} - \varepsilon Pe^{-\zeta P}\\
\dot{L} &= \frac{L(1-L)}{(1-b-\beta)(1-L)+\left(1-\frac{\vartheta(1-\eta)}{\eta}\right)L}\left[(a+\alpha)\frac{\dot{K}}{K} - \gamma\frac{P^{\gamma-1}}{1+P^{\gamma}}\dot{P} + \frac{1}{\eta}\left(r - \alpha\frac{1}{1+P^{\gamma}}K^{a+\alpha-1}L^{b+\beta}\right)\right]
\end{aligned} \tag{12}$$

where $1-\vartheta(1-\eta)/\eta > 0$, according to the concavity of the utility function (4), which requires $\eta > \theta/(1+\theta)$.

In such a context, the jumping variable is $L$, instead of $\lambda$. As a consequence, given the initial values of the state variables, $K_0$ and $P_0$, the representative agent has to choose the initial value $L_0$ of $L$ so as to solve the maximization problem (7).

## 4. Steady states and local indeterminacy

This section deals with the steady states of system (12).

Previous studies (for a review of the literature on this issue see Benhabib and Farmer, 1999; Mino, 2017) found indeterminacy assuming increasing or constant social returns to scale ($a+b+\alpha+\beta \geq 1$). In the present context, instead, we will not constraint the value of the sum of the exponents $a+b+\alpha+\beta$ to allow for any possible social returns to scale. For this purpose, we will exclude the case in which $b+\beta \geq 1$ as this would automatically imply constant or increasing returns to scale, and will limit our analysis to the case $b+\beta < 1$ which allows the overall sum of the exponents $a+b+\alpha+\beta$ to be greater, equal or less than 1. We will describe and summarize results based on the possible values of $a+\alpha$.

The following theorem holds.

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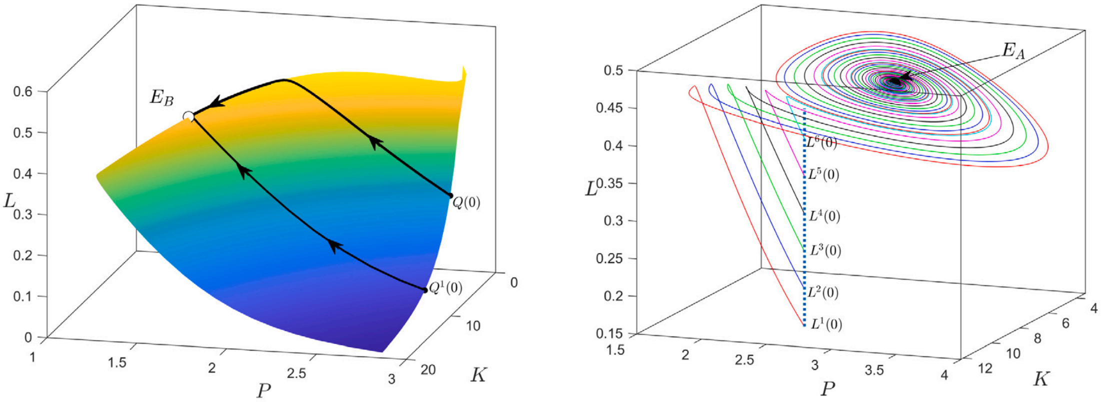
[10] Notice that the utility function we adopted implies $C > 0$ and $0 < L < 1$.



(a) (b)

**Fig. 1.** Parameter values: $\alpha = 0.08$, $\beta = 0.8$, $\gamma = 0.9$, $\delta = 1$, $\epsilon = 0.85$, $\eta = 0.6$, $\theta = 1$, $\zeta = 0.5$, $a = 0.02$, $b = 0.1$, $r = 0.002$.

**Theorem 1.** *Assume $\zeta > 0$. Then*

1. *If $a + \alpha = 1$, there exists at most one steady state $E^*$ in the region $R = \{K, P > 0, 0 < L < 1\}$. It exists iff $\alpha\left(\frac{\beta}{\vartheta+\beta}\right)^{b+\beta} > r$, and it is either a repeller or a saddle point with a two-dimensional stable manifold (the Jacobian determinant in $E^*$ is positive).*
2. *If $a + \alpha < 1$, there exist, generically, zero or two steady states in $R$. In the latter case, call them $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$, with $K_B > K_A$, $P_B < P_A$, and $L^* = \frac{\beta}{\vartheta+\beta}$. The steady state $E_A$ is either an attractor or a saddle point with a one-dimensional stable manifold, while the steady state $E_B$ is either a repeller or a saddle point with a two-dimensional stable manifold (the Jacobian determinant is negative in $E_A$, and positive in $E_B$).*
3. *If $a + \alpha > 1$, there exist, generically, zero or two steady states in $R$, $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$, with $K_B > K_A$, $P_B > P_A$. The stability properties of $E_A$ and $E_B$ are as in case $a + \alpha < 1$.*

**Proof.** Let $E^* = (K^*, P^*, L^*)$ be a steady state of system (12). Then the Jacobian matrix is easily checked to have the form

$$J\left(E^*\right) = \begin{pmatrix} 0 & 0 & p \\ q & s & t \\ u & v & w \end{pmatrix} \tag{13}$$

while $\det J\left(E^*\right)$ is easily computed to have the sign of $\left(\frac{\partial \dot{P}}{\partial K}\frac{\partial \dot{\lambda}}{\partial P} - \frac{\partial \dot{P}}{\partial P}\frac{\partial \dot{\lambda}}{\partial K}\right)\left(E^*\right)$. When the Jacobian determinant is negative, the matrix (13) has either one or three eigenvalues with negative real part; when it is positive, the matrix (13) has either zero or two eigenvalues with negative real part.

In case $a + \alpha = 1$, being $\frac{\partial \dot{\lambda}}{\partial K} = 0$, such a determinant turns out to be positive, which proves claim 1. Suppose now $a + \alpha \neq 1$. Then the two possible steady states, $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$, with $K_B > K_A$ and $L^* = \frac{\beta}{\vartheta+\beta}$, are the intersections of two curves in the positive quadrant of the $(K, P)$ plane: namely, the graphics of the functions $K = f(P) = \left[\frac{\varepsilon}{\delta(L^*)^{b+\beta}} Pe^{-\zeta P}\right]^{\frac{1}{a+\alpha}}$ and $K = g(P) = \left[\frac{\vartheta}{\alpha(L^*)^{b+\beta}}\left(1 + P^{\gamma}\right)\right]^{\frac{1}{a+\alpha-1}}$. It is easily checked that the former graphic is *bell-shaped* (with a maximum at $P = \zeta^{-1}$) if $\zeta > 0$, while the latter one is the graphic of a function decreasing or increasing if, respectively, $a + \alpha < 1$ or $a + \alpha > 1$. Now, let $E^* = (K^*, P^*, L^*)$ be, as above, a steady state of system (12). Hence, it follows from straightforward computations that, when $a + \alpha < 1$, $sign(\det J\left(E^*\right)) = sign\left(f'\left(P^*\right) - g'\left(P^*\right)\right)$, whereas, when $a + \alpha > 1$, $sign(\det J\left(E^*\right)) = sign\left(g'\left(P^*\right) - f'\left(P^*\right)\right)$: which easily implies claims 2 and 3 of the theorem.[11] ■

According to Theorem 1, the value of $a+\alpha$ affects the number and properties of the steady states. Remember that the output $Y$ is produced according to the production function $Y = AK^{\alpha}L^{\beta}$ (with $\alpha+\beta < 1$), where $A = \overline{K}^{a}\overline{L}^{b}$ (with $a, b > 0$) is a positive externality. So we can interpret the condition $\alpha + a > 1$ (respectively, $\alpha + a < 1$ ) as representing a scenario in which the positive externalities generated by the economy-wide average value $\overline{K}$ of $K$ are "high" ("low"). According to Theorem 1, if positive externalities are high $(\alpha + a > 1)$, then the values of $K$ and $P$ at the steady states $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$ are positively correlated, so that the steady state $E_A$ has both lower capital and lower pollution (i.e. $K_B > K_A$, $P_B > P_A$). Vice-versa, if positive externalities

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[11] In the limit case $\zeta = 0$ there exists exactly one steady state if $a + \alpha < 1$ or $a + \alpha - 1 > \gamma(a + \alpha)$ (the Jacobian determinant being positive in the first case and negative in the second one). Vice-versa, if $0 < a + \alpha - 1 < \gamma(a + \alpha)$ there exist, generically, zero or two steady states and, in the latter case, the Jacobian determinant is negative in $E_A$, and positive in $E_B$.

are low ($\alpha + a < 1$), the values of $K$ and $P$ at the steady states are negatively correlated. In this case, the steady state $E_A$ has lower capital but higher pollution than the steady state $E_B$ (i.e. $K_B > K_A$, $P_B < P_A$).

As to the dynamic properties of the two steady states $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$, the theorem proves that only the steady state $E_A$ (characterized by a lower accumulation of physical capital, in both the cases $\alpha + a \gtrless 1$) can be locally attractive, while only the steady state $E_B$ can have a two-dimensional stable manifold.

When the steady state $E_A$ is attractive, then local indeterminacy occurs: if the economy starts from initial values $K(0)$ and $P(0)$ sufficiently close to $K_A$ and $P_A$, respectively, then there exist a continuum of initial values $L(0)$ such that the trajectory from $(K(0), P(0), L(0))$ approaches $E_A$. If $E_A$ is a saddle with a one-dimensional stable manifold, it cannot (generically) be reached by the economy. When the steady state $E_B$ is a saddle point with a two-dimensional stable manifold, then it possesses saddle-point stability: if the economy starts from initial values $K(0)$ and $P(0)$ sufficiently close to $K_B$ and $P_B$, respectively, then there (generically) exists a unique initial value $L(0)$ of the jumping variable $L$ such that the trajectory starting from $(K(0), P(0), L(0))$ approaches $E_B$. If $E_B$ is a repeller, it cannot (generically) be reached by the economy.

Fig. 1(a) illustrates the dynamics around the steady state $E_B$, when it possesses saddle-point stability. The black trajectories belong to the two-dimensional stable manifold (the colored surface), and so they approach $E_B$ starting from different initial values of the state variables, $K$ and $P$. Fig. 1(b) illustrates the dynamics around the steady state $E_A$, when it is locally attractive. All the trajectories approaching $E_A$ start from the same initial values of the state variables, $K$ and $P$.

What is the welfare level at the two steady states?[12] It is easy to check that, in both the cases $\alpha + a \gtrless 1$, the value in $E_A$ of the objective function of problem (7) is lower than in $E_B$. So $E_A$ is always a poverty trap, when it is attractive. This result is obvious under the assumption $\alpha + a < 1$ since, in such a context, the economy is poorer and more polluted at $E_A$ than at $E_B$ ($K_B > K_A$ and $P_B < P_A$ hold). In the context $\alpha + a > 1$, instead, the economy is poorer but less polluted at $E_A$ than at $E_B$ ($K_B > K_A$ and $P_B > P_A$). However, even in this case the economy is better-off at $E_B$ than at $E_A$. Indeed, the higher positive externalities in $E_B$ – generated by a higher equilibrium value of $K$ – overcome the higher adaptation costs generated by a higher equilibrium value of $P$. It follows that the net welfare effect of the interplay between positive and negative externalities is always higher at $E_B$ than at $E_A$.

Fig. 2 illustrates numerical examples concerning the values of the state variables $K$ and $P$ at the steady states $E_A = \left(K_A, P_A, L^*\right)$ and $E_B = \left(K_B, P_B, L^*\right)$, corresponding to different values of parameters $\gamma$ and $\delta$ (panel (a)), and of parameters $a$ and $b$ (panel (b)). The color scale is set in such a way that the steady state values are increasing from blue to yellow. The red curve in the diagram denotes the Hopf bifurcation curve $H$. The latter separates the parameter space in two regions such that a limit cycle arises when crossing the curve $H$. The black curve $LP$ in the diagram represents the limit point curve. The system admits one steady state along this bifurcation curve, two steady states to its left and no steady state to its right. In the white region, therefore, no steady state exists, being to the right of the curve $LP$. In these examples, the steady state $E_B$ is always saddle-point stable. In panel (a), the steady state $E_A$ is locally attractive on the right of the Hopf bifurcation curve $H$, while it has a one-dimensional stable manifold on the left of $H$. The opposite holds in panel (b).

Notice that local indeterminacy can be observed even if positive externalities are very low (i.e., $a$ and $b$ are very low, see panel (b)), and in the context in which the sum $\alpha + \beta + a + b$ is lower than 1. In fact, consider panel (b). In this case, as stated above, $E_A$ is locally attractive (therefore, there is local indeterminacy) on the left of the Hopf bifurcation curve $H$. As the figure shows, this area exists even at extremely low values of both $a$ and $b$ (i.e. very low positive externalities) and low values of $\alpha$ and $\beta$ ($\alpha = 0.08$ and $\beta = 0.5$ as indicated in the caption), so that the sum $\alpha + \beta + a + b$ is much lower than 1.

This result enriches the literature by showing that indeterminacy may occur even when social returns to scale are very low (much below 1). Indeed, early studies (see, among the others, Benhabib and Farmer, 1994; Boldrin and Rustichini, 1994) found indeterminacy assuming high social returns to scale (much larger than 1). Subsequent studies proved that indeterminacy may emerge also with constant social returns to scale.[13]

Finally, Fig. 3 digs deeper into the results shown in Fig. 2 providing further insights on the possible outcomes of the model from a different perspective. The figure helps the reader visualize the existence of a bifurcation which separates the steady states $E_A$ and $E_B$ at different values of environmental degradation. Panel 3a (3b) reports on the vertical axis the value of pollution (capital) at the steady state, and on the horizontal axis the production-related environmental degradation level. The shape and color of the curves illustrate the properties of the steady states that can be attractive (along the red portion of the curves), or saddle-point stable (with 1 or 2-dimensional stable manifolds, along the dotted and dashed portions of the curves, respectively). To get a better understanding of the diagram, it is convenient to interpret the diagram moving leftwards along the horizontal axis (i.e. from high to low values of $\delta$). As the figure shows, at very high values of $\delta$ (to the right of LP in the diagram) no steady state exists as there is no portion of the curve corresponding to such values. In simple words, this suggests that if environmental degradation is too high (i.e. if production is very polluting) the system does not converge to any steady state. If $\delta$ decreases to $\delta_{LP} = 1.052078439$, then there exists a unique steady state (indicated as LP in the figure). If $\delta$ keeps falling and gets lower than $\delta_{LP}$ (moving further to the left of LP along the horizontal axis), then the steady state splits into two alternative equilibria, corresponding to the two branches of the curve (one for $E_A$ and the other for $E_B$). As the figure shows, only the poverty trap $E_A$ can be attractive, as the red portion occurs only along the branch corresponding to $E_A$. Notice that the latter is first attractive (red portion) and then becomes a saddle point with a one-dimensional stable manifold (dotted portion). Point $H$ ($\delta_H = 0.8300851851$) indicates the Hopf bifurcation showing where this transformation occurs, and thus also where a limit cycle may possibly arise around $E_A$.

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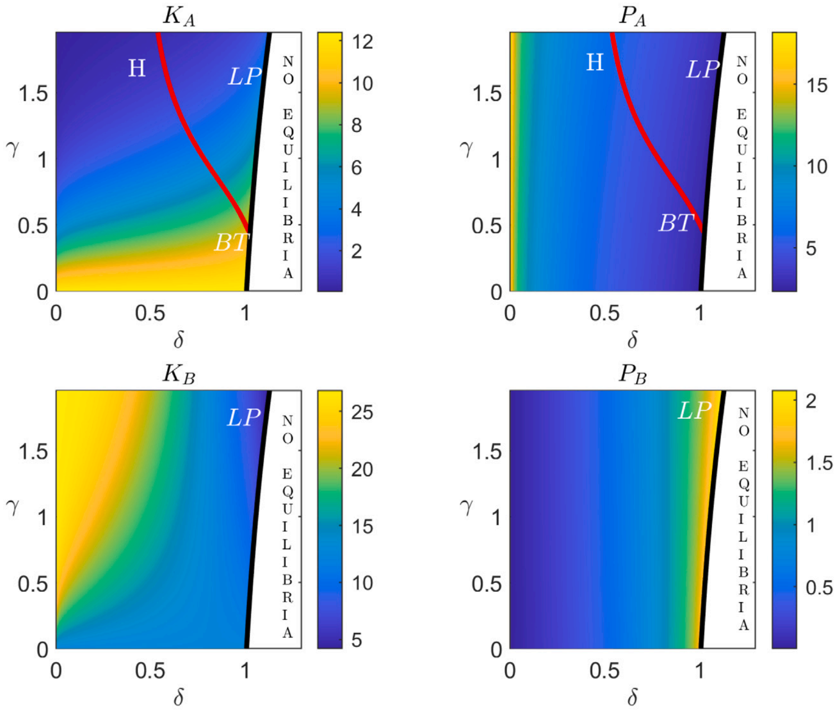
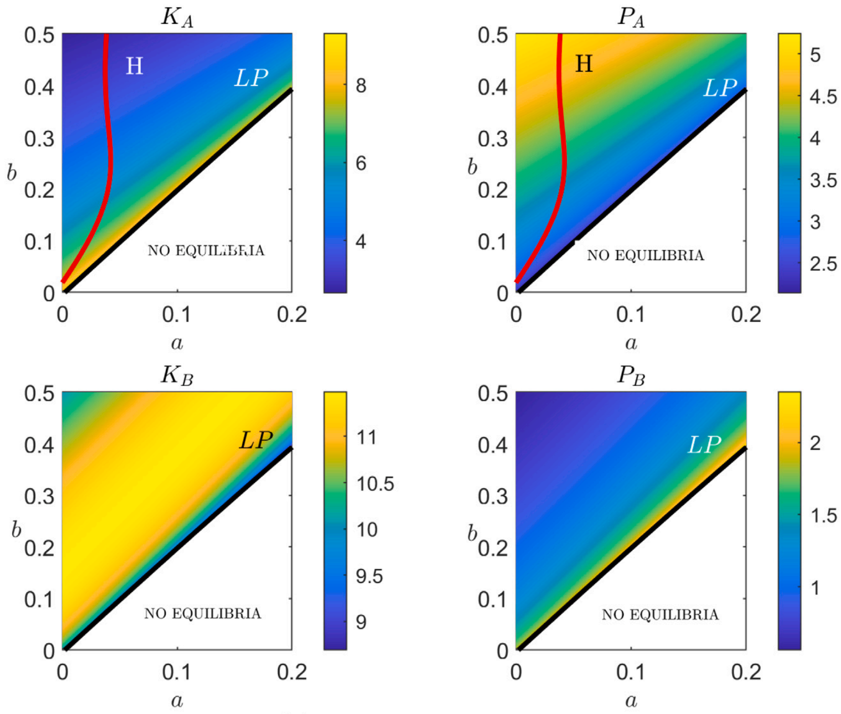
[12] Notice that, since utility depends on consumption and leisure and labor is constant at the steady state ($L = L^*$, see Theorem 1), differences in welfare reflect differences in consumption levels at the steady states.

[13] See, among the others, Benhabib and Nishimura (1998), Bennet and Farmer (2000), Brito and Venditti (2010), Antoci et al. (2014). For a review of the literature on this issue see Benhabib and Farmer (1999), Mino (2017).

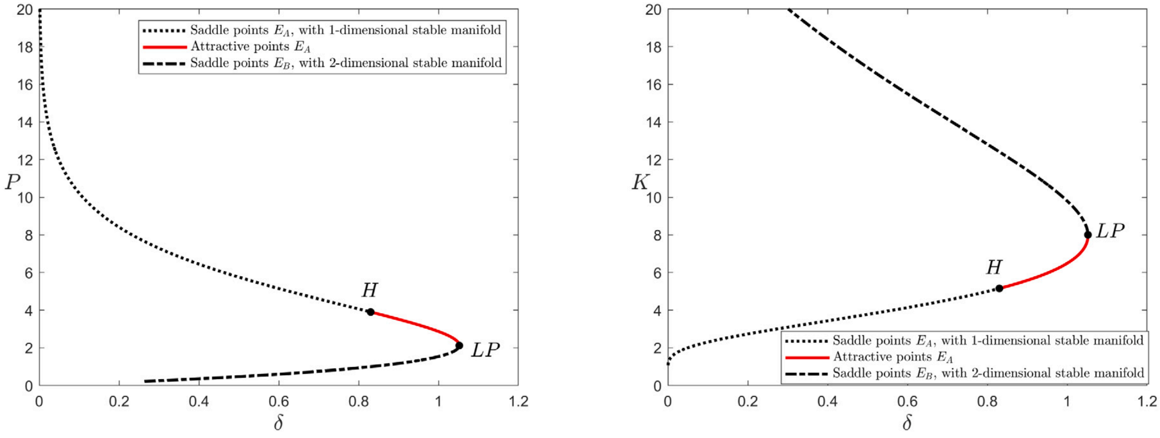


(a) $a = 0.02,\ b = 0.1,\ \beta = 0.8$



(b) $\delta = 0.9,\ \gamma = 0.9,\ \beta = 0.5$

**Fig. 2.** The steady state values of the state variables ($K$ and $P$) as function of the parameters: $\delta$ and $\gamma$ (panel (a)); $a$ and $b$ (panel (b)). The color scale is set such that the steady state values are increasing from blue to yellow. H= Hopf curve, LP= limit Point curve, BT=Bogdanov Takens point. Parameter values: $\alpha = 0.08,\ \eta = 0.6,\ \epsilon = 0.85,\ \theta = 1,\ \zeta = 0.5,\ r = 0.002$.



(a) (b)

**Fig. 3.** Parameter values: $a = 0.02$, $b = 0.1$, $\alpha = 0.08$, $\beta = 0.8$, $\eta = 0.6$, $\epsilon = 0.85$, $\theta = 1$, $\gamma = 0.9$, $\zeta = 0.5$, $r = 0.002$.

## 5. Taxonomy of dynamic regimes and global indeterminacy

This section deals with global analysis of the dynamic system (12), in order to highlight the dynamic regimes that can be observed, and the role played by agents' expectations in determining the future evolution of the economy. As we will show, in our model, the economy may face global indeterminacy scenarios: given the initial values of the state variables, $K$ and $P$, different initial values of $L$ may collocate the economy along equilibrium trajectories approaching different $\omega$-limit sets (for example, different steady states). In such a context, performing local stability analysis alone may be misleading, since it refers only to a neighborhood of a steady state, whereas the initial values of the jumping variable $L$ may not belong to such a neighborhood.

The mathematical results about global dynamics are in the Supplementary material. According to such results, there always exist trajectories along which the variables $K$, $P$ and $L$ all go to zero, thus leading to a clean environment but at the cost of having no production at all. Moreover, there always exist other trajectories along which both $K$ and $P$ tend to $+\infty$, as $t \to +\infty$, and $L$ tends to 1, if and only if the following conditions are satisfied:

1. $\gamma \leq 1$: that is, the negative impact of $P$ on net output $\Omega Y$ is low enough;
2. $(a + \alpha)(1 - \gamma) < 1$: that is, the positive externality due to the economy-wide average value of $K$ (measured by the parameter $a$) is low enough, given the value of $\gamma$. Notice that this condition is always satisfied if $a + \alpha \leq 1$.

In other words, conditions 1 and 2 suggest that if environmental degradation has relatively low negative effects on net income, and capital has sufficiently low positive externalities then $K$ and $P$ will keep growing for ever while economic agents keep working more and more (eventually all the time) to repair the damages produced from environmental degradation. If condition 1 does not hold, then $K$ does not tend to $+\infty$ when $(P, L)$ tend to $(+\infty, 1)$. In this case, in fact, environmental degradation has high negative effects on net income which prevent capital from keep growing even if agents work all the time. If condition 1 holds, but condition 2 is violated (i.e. $(a + \alpha)(1 - \gamma) > 1$), then $K$ tends to $+\infty$ in finite time, and consequently the transversality condition (11) cannot be satisfied.

The above results can be summarized by saying that, for all sets of admissible parameters, there exist trajectories (filling open regions) showing extreme opposite behaviors: that is, tending either to the boundary plane $P = 0$ or to the boundary plane $P = +\infty$.

### 5.1. *The context with "low" positive externalities ($a + \alpha < 1$)*

The analysis in Supplementary material focuses on the context in which two steady states exist, a saddle $E_B = \left(K_B, P_B, L^*\right)$ and an attractor $E_A = \left(K_A, P_A, L^*\right)$. In such a context, beyond the trajectories described above, there exist the trajectories converging to the steady states $E_A$ and $E_B$. Furthermore, either the sink $E_A$ (the poverty trap) or the saddle $E_B$ can be reached starting from the same initial values $\left(K_0, P_0\right)$ of the state variables $(K, P)$. Precisely, starting from a sufficiently small neighborhood $U$ of $E_B$, the economy can follow three different development paths:

(a) It will converge to the saddle $E_B$ if economic agents choose an initial value $L_0$ of the jumping variable $L$ equal to a suitable $\widetilde{L}$.
(b) If $L_0 > \widetilde{L}$ it will converge to the poverty trap $E_A$.
(c) If $L_0 < \widetilde{L}$ it will converge to $(K, P, L) = (0, 0, 0)$.

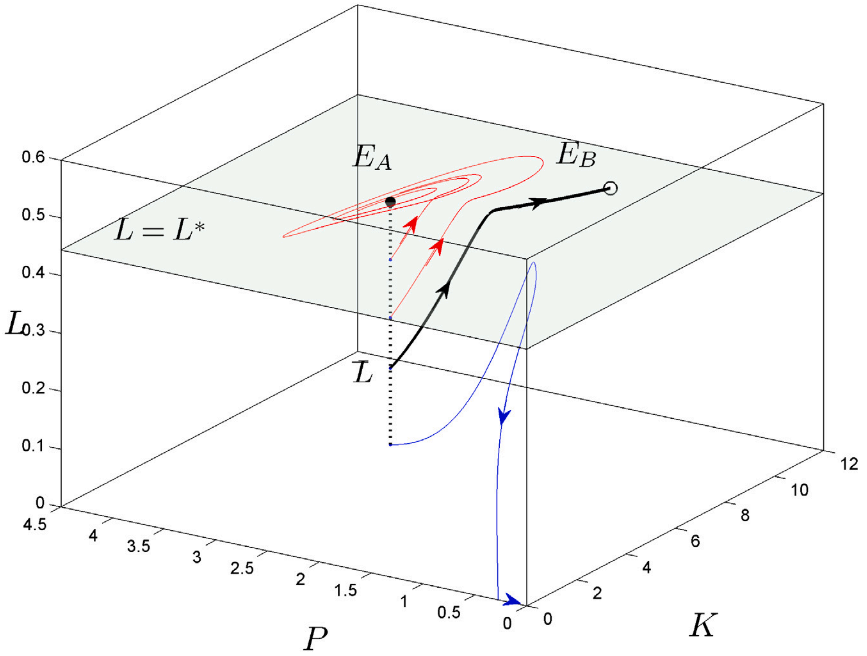


**Fig. 4.** Global indeterminacy in the space $(K, P, L)$: case with $a + \alpha < 1$. Parameter values: $\alpha = 0.08$, $\beta = 0.8$, $\delta = 1$, $\epsilon = 0.85$, $\eta = 0.6$, $\theta = 1$, $\zeta = 0.5$, $a = 0.02$, $b = 0.15$, $r = 0.002$.

To provide an heuristic explanation of our results, consider the initial value $L_0 = \widetilde{L}$ as a benchmark value: starting from $\left(K_0, P_0, \widetilde{L}\right)$, the economy will approach the steady state $E_B$, which always Pareto dominates the locally attractive steady state $E_A$. If economic agents choose an initial value $L_0$ higher than $\widetilde{L}$ (i.e. they work "hard") then they produce a higher gross output $Y$. This generates an increase in environmental degradation $P$ and, consequently, an increase in the adaptation cost $[1 - \Omega(P)] \cdot Y$ to environmental degradation. Such a process of adaptation is self-reinforcing and drives the economy towards the poverty trap $E_A$, where the accumulation of physical capital is lower and environmental degradation is higher, with respect to the steady state $E_B$.

What does it happen in the opposite case, that is, if the initial choice $L_0$ is lower than $\widetilde{L}$ (i.e. agents work "little")? In such a case, the trajectory starting from $\left(K_0, P_0, L_0\right)$ will converge to $(K, P, L) = (0, 0, 0)$. In other words, if agents work little and have low positive externalities at the beginning, the economy eventually leads to an equilibrium without environmental degradation but also without capital.

Fig. 4 illustrates the above global indeterminacy results, to help the reader visualize them. As the figure shows, starting from given values of $P$ and $K$ but different levels of $L$, the economy can converge to totally different final outcomes. Consider, for instance, the vertical dashed line in the 3D-space $(K, P, L)$. All points along the vertical line correspond to equal values of $P$ and $K$ but different levels of $L$. If $L_0 < \widetilde{L}$, as the arrows show, the economy moves along the trajectory converging to the lower right vertex of the cube in which $P = K = L = 0$. In this case, at the end of the day people will enjoy a clean environment without pollution ($P = 0$), but the economic system eventually collapses ($K = L = 0$). If $L_0 = \widetilde{L}$ the economy moves along the solid bold line and converges to the steady state $E_B$. If $L_0 > \widetilde{L}$ the system converges to the Pareto-dominated steady state $E_A$ in which pollution is higher and capital lower than in $E_B$. Stated differently, agents basically work too much as a result of a coordination failure, leading the economy to end up in a poverty trap.

### *5.2. The context with "high" positive externalities ($a + \alpha > 1$)*

When positive externalities are "high", then starting from the same initial values $\left(K_0, P_0\right)$ belonging to the set $U$, the economy can follow again three different development paths:

(a) It will converge to the saddle $E_B$ if economic agents choose an initial value $L_0$ of the jumping variable $L$ equal to a suitable $\widetilde{L}$.

(b) If $L_0 < \widetilde{L}$ it will converge to the poverty trap $E_A$.

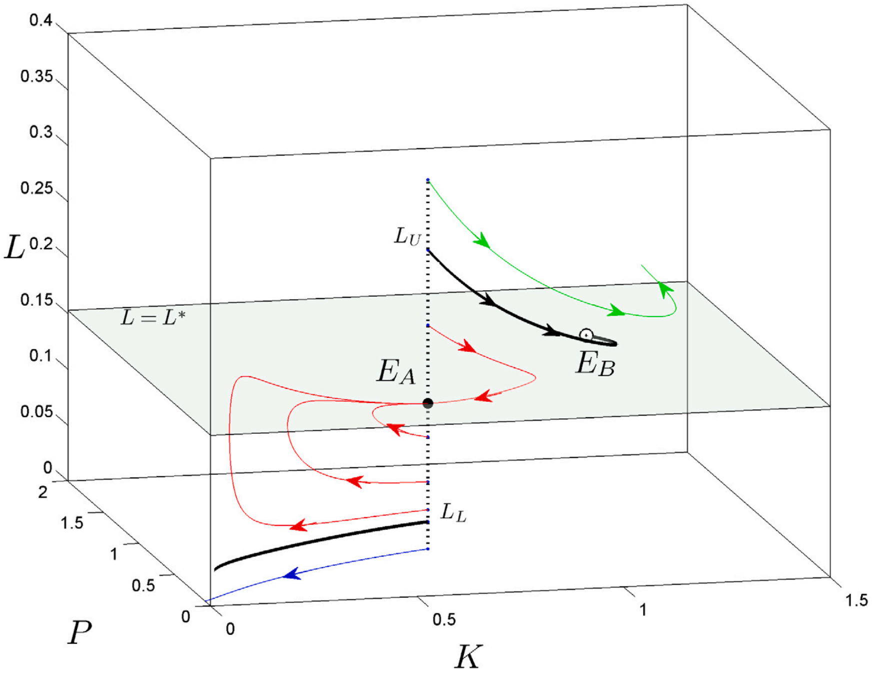


**Fig. 5.** Global indeterminacy in the space $(K, P, L)$: case with $a + \alpha > 1$. Parameter values: $\alpha = 0.94$, $\beta = 0.018$, $\gamma = 0.04$, $\delta = 1$, $\epsilon = 1.15$, $\eta = 0.185$, $\theta = 0.1$, $\zeta = 0.7$, $a = 0.1$, $b = 0.32$, $r = 0.249$.

**Table 1**
Numerical simulation of welfare levels.

| Convergence | K(0) | P(0) | L(0) | $\mathcal{J}$ |
|---|---|---|---|---|
| $(K(t), P(t)) \to (+\infty, +\infty)$ | 0.583 | 0.3315 | 0.3525 | −11.043 |
| $(K(t), P(t)) \to (K_A, P_A)$ | 0.583 | 0.3315 | 0.0575 | −7.5173 |

(c) If $L_0 > \widetilde{L}$ it will converge to $\left(\widehat{K}, +\infty, 1\right)$, where $\widehat{K} = +\infty$ if $\gamma \le 1$; so the economy will follow an unlimited growth path $((K, P) \to (+\infty, +\infty) L_0 > \widetilde{L}$ and $\gamma \le 1)$.

According to (b), when agents work “little” ($L_0 < \widetilde{L}$), the trajectory starting from $\left(K_0, P_0, L_0\right)$ will approach the poverty trap $E_A$. Working less, economic agents are not able to benefit from the positive externalities, and the economy converges to the poverty trap $E_A$ with lower capital accumulation and lower environmental degradation than in $E_B$.

According to (c), when agents work “hard” ($L_0 > \widetilde{L}$), then the economy will follow a growth path characterized by an unbounded growth of environmental degradation and of the stock of physical capital (remember that the latter occurs provided the negative impact of environmental degradation is not too high, that is, $\gamma \le 1$).

Fig. 5 illustrates the global indeterminacy scenario described above. Observe that, if $L_0 < L_L$ along the vertical dashed line, the dynamics of the system lead to the lower left vertex of the cube (in which $P = K = L = 0$) and the economy eventually collapses (see the blue line in the figure). If $L_L < L_0 < L_U$, trajectories lead to the poverty trap $E_A$ along the red lines. If $L_0 = L_U$, the economy moves along the solid bold line converging to the saddle $E_B$ in which both pollution and capital are higher than in $E_A$. Finally, if $L_0 > L_U$ trajectories tend to infinity along the green line. This suggests that pollution and capital can keep growing for ever when positive externalities are high.

Table 1 reports the results of a numerical exercise that were performed to compute welfare levels along the possible trajectories. Starting from the same initial values of $K$ and $P$ (columns 2 and 3), a higher level of labor may lead to a never-ending growth of capital and pollution (cf. row 2 in the table). Along this trajectory, however, welfare will be lower than along the path leading to the sink $E_A$ (see column 5). This confirms that an unlimited growth process can be undesirable since it turns out to be welfare-reducing.

## 6. Conclusions

Environmental degradation requires mitigation and adaptation choices. Adaptation activities, however, may sometimes exacerbate environmental problems or shift negative impacts, risks, and exposure to other individuals, population groups or countries, what is known as “maladaptation“ (IPCC, 2001, 2018; Barnett and O’Neill, 2010).

The present paper tries to contribute to the debate on this issue, which is considered as one of the global emerging environmental challenges (UNEP, 2019), and to enrich the analytical framework by taking both negative and positive externalities into account. To this purpose, we investigated an intertemporal optimization problem characterized by negative environmental externalities that reduce the net output at disposal of the agents and positive externalities in production that increase the productivity of labor and capital. The co-existence of positive and negative externalities generates two counteracting mechanisms which are simultaneously at work. On the one hand, an increase in production-related environmental degradation lowers the net income left at disposal (for consumption and investment) of the individuals; on the other hand, it generates a push effect inducing people to work harder and accumulate more capital to repair the higher environmental damages deriving from production. The consequent increase in labor and capital enhances the positive externalities occurring in the production process. As it emerges from the model, the co-existence of these two mechanisms may lead to a welfare-increasing or welfare-reducing outcome depending on the relative size of the (positive versus negative) externalities and thus on which one of the two opposite forces will eventually prevail.

The analysis of the model shows that even with optimizing agents both local and global indeterminacy arise in the context described above, so that one cannot predict a priori which path the economy will follow when converging to an equilibrium, nor the equilibrium the dynamics will eventually converge to. This suggests that the degree of uncertainty surrounding the effects of maladaptive behaviors is extremely high and that the trajectories may eventually lead the economy to be trapped in a Pareto-dominated equilibrium. This result – which derives from lack of coordination among rational, self-interested, maximizing agents – calls for policy intervention and coordinated mitigation activities which were here deliberately ignored to focus on the dynamic effects of self-adaptation only. As shown in previous contributions (e.g. Bretschger and Schaefer (2017), Antoci et al. (2021)), suitable policy interventions may prevent coordination failures leading the economy towards a Pareto-dominant steady state. For instance, using a two-sector model with clean and dirty capital, Bretschger and Schaefer (2017) show that government intervention may lead agents to select the trajectories converging to the Pareto-dominant steady state in which clean capital prevails even if the economy is originally in the neighborhood of the Pareto-dominated steady state. In the context examined by Antoci et al. (2021), when a sufficiently high output tax is introduced in the model the Pareto-dominated steady state can no longer be reached and the economy converges towards a unique saddle-point stable steady state, so that global indeterminacy eventually disappears. A similar outcome may be obtained also in the present context introducing an output tax and using the correspondent revenues to abate pollution.[14]

Much research remains to be done to further enrich the present analysis in the future. While this paper concentrates on the possible perverse effects of adaptation, the model could be extended to account for mitigation. In particular, it would be interesting to see how results change if the revenues accruing from mitigation policies (e.g. a pollution tax) are used to finance adaptation activities. The present analytical framework, moreover, could be extended to a 2-sector model in which each sector uses a different stock of capital: one for adaptation, the other for mitigation. Indeed, one could argue that adaptation requires a specific capital which differs from the one used for mitigation. The presence of two capital stocks would likely affect the dynamics of the model: based on their expectations, agents might coordinate actions leading the economy to converge to either a poverty trap in which adaptation actions prevail or to a virtuous equilibrium in which mitigation actions prevail.

Finally, it is important to stress that the global indeterminacy and complex dynamics pointed out above were obtained from a very simple environmental dynamics. Complexity is obviously bound to increase even further if one accounts for the complex dynamics characterizing many forms of environmental degradation (such as, for instance, climate change). Future research should, therefore, be devoted to investigate more complex environmental dynamics that can better approximate the complex (and partially still uncertain) relationship between economic activity and environmental degradation

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[14] Numerical simulations confirm our a priori expectations: allowing the output taxation rate to range between 0 and 1, we find that the Pareto-dominated steady state can no longer be reached if the taxation rate gets above a given threshold level. The correspondent results, omitted here for space reasons, are available from the authors upon request.

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