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# Living in an uncertain world: Environment substitution, local and global indeterminacy

Angelo Antoci (University of Sassari)  
Simone Borghesi (European University Institute and University of Siena)  
Marcello Galeotti (University of Florence)  
Mauro Sodini (University of Pisa)

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## Abstract

Environmental problems are increasingly frequent, intensive and unpredictable. To protect from the observed environmental depletion, economic agents increasingly react by substituting previously free public environmental goods with costly private goods. This substitution mechanism, however, can contribute to enhance the indeterminacy of the possible consequences of mankind activity, further increasing the uncertainty on the future environmental trajectories. To investigate this issue, the paper proposes an intertemporal optimization problem in which agents derive utility from three goods: leisure, a public environmental good and/or private consumption that can be used as a substitute for the environment. The analysis shows that the economy may end up being trapped in the Pareto-dominated stationary state and that both local and global indeterminacy may arise in the model. No indeterminacy, however, emerges if green technologies are used so that production has no negative effects on the environment.

**Keywords:** environmental depletion, substitutability, local and global indeterminacy, uncertainty.

## 1 Introduction

Environmental problems rank progressively higher in the international political agenda attracting increasing attention from the public opinion. A growing number of people, scholars and international institutions worldwide (Climate Strike, 2019; Sachs et al., 2019; IPCC 2014, 2018, 2019) call for immediate action to stop environmental degradation and its large negative effects. Mitigation activities are particularly needed since environmental problems are increasingly frequent, intensive and unpredictable. The combination of these three worrisome features increases the uncertainty on the environmental consequences of

economic growth and of related anthropogenic activities, causing the indeterminacy of future environmental trajectories.<sup>1</sup> As originally pointed out in the early literature on this issue by Pearce et al. (1989, 1991), this might eventually lead the economy into a “grey zone” in which environmental consequences of human action are uncertain and possibly irreversible.

Along with the rise in environmental degradation and uncertainty, another (strictly related) phenomenon has attracted the attention of many scholars: the tendency to replace previously free public environmental goods with costly private consumption goods. In modern industrial economies one can identify a plethora of private goods and services that agents use to self-protect from environmental degradation. Some of the most typical and often-quoted textbook examples include air filters and water treatment plants, mineral water, double-glazing to reduce the acoustic damage from urban traffic, medicines against pollution-related diseases (e.g. asthma and skin diseases). In all these cases, individuals are now forced to pay for goods that were once freely available (i.e. clean air, clean water, silent cities etc.); by consuming these goods they try to restore the utility they used to enjoy from a pristine environment in the status quo (i.e. before environmental degradation took place). This substitution phenomenon - that was originally described in a seminal contribution by Hirsch (1976) who introduced the concept of “defensive consumption” - goes beyond these few textbooks examples and has become so pervasive in modern societies that it may account for up to several points of GDP.<sup>2</sup>

In some cases this substitution mechanism may further increase environmental degradation. Thus, for instance, as reported by Sun et al. (2017) for China, the massive use of air filters to self-protect from outdoor air pollution may contribute to further worsen the air quality problem. The same occurs with the large increase in the use of air conditioning in response to heat waves and global warming. In these cases, therefore, the individual self-adaptation process tends to enhance and transfer the negative externalities to the other agents rather than filter them (Shogren and Crocker, 1991), what has been described with the term maladaptation in the literature on this topic (Adejuwon et al., 2001; Barnett and O’Neill, 2010; Antoci et al., 2019; UNEP, 2019).

This paper tries to relate the two phenomena mentioned above: on the one hand, the progressive substitution of public environmental goods with private consumption goods, and on the other hand, the increasing indeterminacy of environmental consequences of our activity. Our aim is to build a bridge between the two correspondent research lines. More precisely, the present paper shows that the substitution mechanism described above may cause indeterminacy, fur-

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<sup>1</sup>See Caravaggio and Sodini (2018) for a review of the literature on indeterminacy in growth models with environmental goods.

<sup>2</sup>See Antoci and Borghesi (2012) and the literature cited therein, for additional examples of the substitution mechanism. See United Nations (1993, 2003) for alternative classifications of the environmental defensive expenditures generated by these self-protection instruments. See also the more recent and rapidly growing literature on the economic dimension of the individual defensive behaviours against air pollution, with numerous empirical studies on China where this problem is perceived as particularly serious (cf. Williams, 2019; Zhang and Mu, 2018; Yang and Zhang, 2018; Liu et al., 2018).

ther increasing the uncertainty about future environmental trajectories.

Differently from previous studies in the literature on indeterminacy (see, for a review, Bella et al. 2017, Mino 2017), in which indeterminacy is generated by positive externalities in the production process or in the accumulation process of human capital, in our model (local and global) indeterminacy results from negative externalities only, that are generated by the substitution process between the private good and the environmental good. More precisely, economic agents react to the depletion of the environmental resource by an increase in their labour input, which allows them to produce a higher quantity of private good which is consumed as substitute for the environmental resource. The consequent increase in production and consumption of the private good generates a further reduction in the stock of the environmental resource, and so on.

To investigate the issue described above, in this paper we analyse an economy with optimizing agents (the context is that proposed by Wirl, 1997) in which agents' well-being depends on three goods: a produced (private) good, leisure, and a stock of a (free access) renewable environmental resource. The production activity of the private good deteriorates the natural resource, and individuals may defend themselves from environmental degradation by increasing consumption of the private good, which may be perceived as a "substitute" for the environmental resource.

In the proposed model, economic agents have to solve an intertemporal optimization problem in which the state variables are the stock of physical capital  $K$  accumulated by each agent and the stock  $E$  of a free access renewable environmental resource. The control variables are agents' labour input  $L$  and consumption  $C$  of the produced good.

The analysis of the model shows that there exist at most two stationary states,  $P_1$  and  $P_2$ , the former being a poverty trap that is Pareto-dominated by the latter and has a lower level of the environmental resource. As it will be shown below, the poverty trap  $P_1$  can be an attractor only if the environmental good and the private consumption good are substitutes, namely, if the marginal utility of consumption increases as the environmental good decreases. Indeed, if that is the case, individuals have an incentive to work more and more as the environment depletes to afford higher consumption levels, but this leads the economy on a welfare-reducing trajectory.

When  $P_1$  is an attractor then the dynamics are *locally indeterminate*, namely, given the initial conditions there exists a continuum of possible trajectories leading to  $P_1$  so that one cannot predict a priori how the economy will converge to  $P_1$ . Furthermore, numerical simulations suggest that scenarios of *global indeterminacy* may occur, that is, given the initial conditions both stationary states can be reached so that one cannot predict a priori where the economy will eventually converge to.<sup>3</sup> This indeterminacy scenario may be observed because economic agents are unable to coordinate their choices, in that each of them takes the stock  $E$  of the environmental resource as exogenously given.

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<sup>3</sup>See Krugman (1991) and Matsuyama (1991) for seminal contributions on the notion of global indeterminacy in the literature.

Finally, our findings show that indeterminacy may occur in the model if production has a negative impact on the environment. If green technologies are used, the model admits only one saddle point with two-dimensional stable manifold, therefore no indeterminacy occurs in that case.

The present work builds upon and extends previous studies (cf. Antoci et al., 2005 and 2007) in the research strand on the substitution of environmental goods with private consumption goods. However, it differs from such studies in two main respects. First, those works assumed an additively separable utility function so that the disutility of labor is not influenced by environmental quality and consumption, whereas here we assume that environmental quality affects the utility deriving from leisure and consumption (the utility function being multiplicative in consumption, leisure and the environment). Second, in the studies mentioned above global indeterminacy was absent (Antoci et al., 2007) or could be observed only assuming high positive externalities (Antoci et al., 2005), while here we show that it can occur also without any positive externality and assuming negative externalities only.

The paper will be structured as follows. Sections 2 and 3 define the setup of the model and the associated dynamic system. Section 4 deals with the existence and local stability of stationary states. Section 5 is devoted to numerical simulations of dynamics. Section 6 extends the model introducing output taxation to charge for the negative externalities generated by the production activity. Section 7 concludes the paper.

## 2 Set up of the model

The economy we analyze is constituted by a continuum of identical economic agents; the size of the population of agents is normalized to unity. At each instant of time  $t \in [0, \infty)$ , the representative agent produces an output  $Y(t)$  by the following constant returns Cobb-Douglas technology:

$$Y = K^\alpha L^{1-\alpha}, \text{ with } 1 > \alpha > 0 \quad (1)$$

where  $K(t)$  is the stock of physical capital accumulated by the representative agent and  $L(t)$  is the agent's labour input.

We assume that the representative agent's preferences are described by a *constant intertemporal elasticity of substitution* (CIES) utility function, augmented by introducing the stock of the environmental good  $E(t)$ :

$$U(C, L, E) = \frac{[CE^\beta(1-L)^\gamma]^{1-\delta} - 1}{1-\delta}$$

where  $C$  and  $1-L(t)$  represent the consumption of the output  $Y(t)$  and leisure, respectively, and parameters satisfy the conditions:  $\beta, \gamma, \delta > 0$  and  $\delta \neq 1$ . A function of this type is used, among others, by Ladrón-De-Guevara et al.

(1999), Bennet and Farmer (2000), Gomez Suarez (2008), and Itaya (2008);<sup>4</sup> it is jointly concave in  $C$  and  $1 - L$  if  $\delta > \frac{\gamma}{1+\gamma}$ . The parameter  $\delta$  denotes the inverse of the intertemporal elasticity of substitution in consumption. Our function displays a constant intertemporal elasticity of substitution and possesses the property that income and substitution effects exactly balance each other in the labour supply equation. Furthermore, it holds:

$$\frac{\partial U(C, L, E)}{\partial C \partial E} = (1 - \delta) \beta E^{\beta-1} \frac{(1 - L)^\gamma}{[CE^\beta (1 - L)^\gamma]^\delta} < 0 \quad \text{if } \delta > 1$$

So, if  $\delta > 1$ , then  $C$  and  $E$  are “Edgeworth substitutes”, namely, the marginal utility of  $C$  increases as the stock  $E$  of the environmental good decreases. The opposite holds if  $\delta \in (0, 1)$ ; in such a case,  $C$  and  $E$  are “Edgeworth complements”.<sup>5</sup>

The evolution of  $K(t)$  (assuming, for simplicity, the depreciation of  $K$  to be zero) is represented by the differential equation:

$$\dot{K} = K^\alpha L^{1-\alpha} - C \tag{2}$$

where  $\dot{K}$  is the time derivative of  $K$ . In order to model the dynamics of  $E$  we start from the well-known logistic equation:

$$\dot{E} = E(\bar{E} - E) \tag{3}$$

where the parameter  $\bar{E} > 0$  represents the carrying capacity of the natural resource, that is, the value of  $E$  to which the stock of the environmental resource converges starting from an initial value  $E(0) > 0$ , and  $\bar{E}/2$  represents the maximum sustainable yield, namely, the value of  $E$  at which the speed of regeneration  $\dot{E}$  of the natural resource reaches its maximum.

We augment equation (3) by adding the negative impact due to the output production process:

$$\dot{E} = E(\bar{E} - E) - \varepsilon \bar{Y} \tag{4}$$

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<sup>4</sup>In particular, Itaya (2008) adopts a similar non separable utility function in consumption, leisure and pollution, in which the latter is a function of physical capital stock, and introduces positive externalities in the production process. He finds that at most a unique balanced growth path can exist, so that local indeterminacy may be observed but not global indeterminacy.

<sup>5</sup>To fix ideas, consider doing sport in a park. If the park is clean and/or the number and extension of green areas increases (i.e.  $E$  increases) then the marginal utility of private consumption (e.g. buying a bike to go riding in the park) increases. In this case,  $C$  and  $E$  go hand in hand (are Edgeworth complements). If the park is dirty and/or green areas shrink (i.e.  $E$  decreases), then agents may prefer to buy a costly season ticket to a gym (where they can use a cyclette or walk on a treadmill) rather than doing sport en plein air. In this case, costly private consumption  $C$  "replaces" the use of the free public good  $E$  ( $C$  and  $E$  are Edgeworth substitutes). Similar examples apply to many other private consumption goods (cf. Antoci and Borghesi, 2012).

where  $\bar{Y}$  is the economy-wide average output and the parameter  $\varepsilon > 0$  measures the negative impact of  $\bar{Y}$  on  $E$ .

We assume that the representative agent chooses the control variables  $C$  and  $L$  so as to solve the following problem:

$$V(K(0), E(0)) := \underset{C, L}{MAX} \int_0^\infty \frac{[CE^\beta(1-L)^\gamma]^{1-\delta} - 1}{1-\delta} e^{-\rho t} dt \quad (5)$$

subject to:

$$\begin{aligned} \dot{K} &= K^\alpha L^{1-\alpha} - C \\ \dot{E} &= E(\bar{E} - E) - \varepsilon \bar{Y} \end{aligned}$$

with  $K(0)$  and  $E(0)$  given,  $K(t)$ ,  $E(t)$ ,  $C(t) \geq 0$  and  $1 \geq L(t) \geq 0$  for every  $t \in [0, +\infty)$ ; the parameter  $\rho > 0$  is the subjective discount rate.

Furthermore, we assume that capital  $K$  is reversible, that is, we allow for disinvestment ( $\dot{K} < 0$ ) at any instant of time. Furthermore, we assume that in solving problem (5), the representative agent considers  $\bar{Y}$  as exogenously determined, since, being economic agents a continuum, the influence on  $\bar{Y}$  by each of them is null. However, since agents are identical, ex post  $\bar{Y} = Y$  holds. This implies that the trajectories resulting from our model are not optimal (i.e. they do not describe the social optimum). However, they represent Nash equilibria in the sense that, along them, no agent has an incentive to modify her choices if the others don't modify theirs.

### 3 Dynamics

The current value Hamiltonian function associated to problem (5) is:

$$H = \frac{[CE^\beta(1-L)^\gamma]^{1-\delta} - 1}{1-\delta} + \lambda (K^\alpha L^{1-\alpha} - C) + \mu [E(\bar{E} - E) - \varepsilon \bar{Y}]$$

where  $\lambda$  and  $\mu$  are the co-state variables associated to  $K$  and  $E$ , respectively. Since the representative agent considers the economy-wide average output  $\bar{Y}$  as exogenously determined, the co-state variable  $\mu$  does not enter the first order conditions determining the values of the control variables  $C$  and  $L$ , and consequently equations (2) and (4) do not depend on it. So, by applying the Maximum Principle, we get:

$$\dot{K} = \frac{\partial H}{\partial \lambda} = K^\alpha L^{1-\alpha} - C \quad (6)$$

$$\dot{\lambda} = \rho \lambda - \frac{\partial H}{\partial K} = \lambda (\rho - \alpha K^{\alpha-1} L^{1-\alpha}) \quad (7)$$

where  $C$  and  $L$  satisfy the following conditions<sup>6</sup>:

$$\frac{\partial H}{\partial C} = C^{-\delta} E^{\beta(1-\delta)} (1-L)^{\gamma(1-\delta)} - \lambda = 0 \quad (8)$$

$$\frac{\partial H}{\partial L} = 0 \quad \text{i.e.} \quad -\gamma C^{1-\delta} E^{\beta(1-\delta)} (1-L)^{\gamma(1-\delta)} + (1-\alpha)\lambda(1-L)K^\alpha L^{-\alpha} = 0 \quad (9)$$

The representative economic agent considers the time evolution of  $E$ , given by equation (4), as exogenously determined, in that  $\dot{E}$  is negligibly affected by the representative agent's choice of the output level  $Y$  (see Wirl, 1997).

By solving equations (8) and (9) with respect to the variable  $\lambda$ , and equating the right hand sides of both expressions, we obtain:

$$C = \frac{K^\alpha(1-L)(1-\alpha)}{\gamma L^\alpha} \quad (10)$$

The substitution of (10) in (9) gives the equation which determines the choice of  $L$  by the representative agent:

$$\frac{L^{\alpha\delta}}{(1-L)^{(\gamma+1)\delta-\gamma}} = \frac{\lambda K^{\alpha\delta}}{E^{\beta(1-\delta)}} \left( \frac{1-\alpha}{\gamma} \right)^\delta \quad (11)$$

By taking logarithms of both sides of equation (11) we obtain:

$$[\gamma - (\gamma + 1)\delta] \ln(1-L) + \alpha\delta \ln L = \ln \lambda + \alpha\delta \ln K + \delta \ln(1-\alpha) + \beta(\delta-1) \ln E - \delta \ln(\gamma)$$

By differentiating with respect to time we get:

$$[(\gamma + 1)\delta - \gamma] \frac{\dot{L}}{1-L} + \alpha\delta \frac{\dot{L}}{L} - \frac{\dot{\lambda}}{\lambda} - \alpha\delta \frac{\dot{K}}{K} + \beta(1-\delta) \frac{\dot{E}}{E} = 0$$

from which:

$$\dot{L} = \frac{L(1-L) \left[ \frac{\dot{\lambda}}{\lambda} + \alpha\delta \frac{\dot{K}}{K} - \beta(1-\delta) \frac{\dot{E}}{E} \right]}{[(\gamma + 1)\delta - \gamma - \alpha\delta] L + \alpha\delta} \quad (12)$$

Finally, taking into account that (ex post)  $\bar{Y} = K^\alpha L^{1-\alpha}$  holds and that the growth rate of  $\lambda$  is:

$$\frac{\dot{\lambda}}{\lambda} = \left( \rho - \alpha \frac{L^{1-\alpha}}{K^{1-\alpha}} \right)$$

we get the equilibrium dynamics:

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<sup>6</sup>Notice that the utility function we adopted implies that the representative agent always chooses  $C > 0$  and  $0 < L < 1$ .

$$\dot{K} = \frac{1}{\gamma} \frac{K^\alpha}{L^\alpha} [L(1 - \alpha + \gamma) - (1 - \alpha)] \quad (13)$$

$$\dot{E} = E(\bar{E} - E) - \varepsilon K^\alpha L^{1-\alpha} \quad (14)$$

$$\dot{L} = f(L) \left[ \rho - \alpha \frac{L^{1-\alpha}}{K^{1-\alpha}} + \frac{\alpha \delta}{K} \dot{K} - \frac{\beta(1-\delta)}{E} \dot{E} \right] \quad (15)$$

where:

$$f(L) = \frac{L(1-L)}{[(\gamma+1)\delta - \gamma - \alpha\delta]L + \alpha\delta}$$

and  $f(L) > 0$ , recalling  $\delta > \gamma/(\gamma+1)$ .

## 4 Local stability analysis

The following two propositions deal with the numerosity of the stationary states of system (13)-(15), and their local stability properties.

**Proposition 1** *There exist two stationary states of system (13)-(15),  $P_1 = (K^*, E_1, L^*)$  and  $P_2 = (K^*, E_2, L^*)$ , where  $0 < E_1 \leq E_2$  are the real solutions of the equation  $E(\bar{E} - E) = \varepsilon(K^*)^\alpha(L^*)^{1-\alpha}$ ,  $L^* = \frac{1-\alpha}{1-\alpha+\gamma}$ , and  $K^* = \left(\frac{\rho}{\alpha}\right)^{\frac{1}{1-\alpha}} L^*$ , if:*

$$\varepsilon \leq \frac{(1-\alpha+\gamma)\bar{E}^2}{4(1-\alpha)} \left(\frac{\rho}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}. \quad (16)$$

*No stationary state exists if the opposite of (16) holds.*

**Proof.** The stationary states of system (13)-(15) must solve equations  $\dot{K} = 0$ ,  $\dot{E} = 0$ ,  $\dot{L} = 0$ . From equation (13) we get directly the stationary value of  $L$ ,  $L^*$ . From equation (15), it follows that  $\dot{L} = 0 \iff \dot{\lambda} = 0$ ; therefore by substituting (10) and the expression of  $L^*$  in (7), we obtain the stationary value of  $K$ ,  $K^*$ . The stationary values of  $E$  are obtained by solving the equation  $E(\bar{E} - E) = \varepsilon(K^*)^\alpha(L^*)^{1-\alpha}$ . Direct calculations show that solutions exist if and only if  $\varepsilon \leq \frac{\bar{E}^2}{4(K^*)^\alpha(L^*)^{1-\alpha}} = \frac{(1-\alpha+\gamma)\bar{E}^2}{4(1-\alpha)} \left(\frac{\rho}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$ . If such an inequality holds strictly, then the solutions are distinct, otherwise two coincident solutions exist. ■

**Proposition 2** *The stationary state  $P_1$  is, generically, either a local attractor or a saddle with a one-dimensional stable manifold; it is always a saddle when  $\delta \in (0, 1)$ . The stationary state  $P_2$  is, generically, either a repeller or a saddle with a two-dimensional stable manifold.*

**Proof.** Consider the Jacobian matrix  $J(P_i)$ :

$$J(P_i) = \begin{pmatrix} 0 & 0 & a \\ b & c & d \\ e & g & h \end{pmatrix}$$

where, in particular,  $a > 0$ ,  $b = -\alpha\varepsilon (K^*)^{\alpha-1} (L^*)^{1-\alpha}$ ,  $c = \bar{E} - 2E_i$ ,  $e = e' + e''$ ,

$$e' = \alpha(1 - \alpha)f(L^*) (K^*)^{\alpha-2} (L^*)^{1-\alpha}$$

$$e'' = \alpha\varepsilon f(L^*) \frac{\beta(1 - \delta)}{E_i} (K^*)^{\alpha-1} (L^*)^{1-\alpha}$$

and:

$$g = -f(L^*) \frac{\beta(1 - \delta)}{E_i} (\bar{E} - 2E_i)$$

Hence, in order to calculate  $\det J(P_i)$ , by adding to the third row the second one multiplied by  $f(L^*) \frac{\beta(1 - \delta)}{E_i}$ , it easily follows that:

$$\det J(P_i) = -ace'$$

and therefore:

$$\text{sign} \det J(P_i) = -\text{sign} (\bar{E} - 2E_i)$$

Consequently  $P_1$  is, generically, either a local attractor or a saddle with one-dimensional stable manifold, while  $P_2$  is, generically, either a repeller or a saddle with two-dimensional stable manifold. In particular, given the above conditions on the system parameters, straightforward computations show that, when  $\delta \in (0, 1)$ ,  $P_1$  is a saddle. In fact, consider the characteristic polynomial:

$$p(x) = \det(xI - J(P_1)) = x^3 + qx^2 + rx + s$$

where in particular  $s = -\det(P_1) > 0$ . Then it is easily computed that, when  $\delta \in (0, 1)$ , if  $q, r > 0$ , it follows  $qr < s$ , so that, by Routh-Hurwitz conditions, there exist two eigenvalues with positive real part. ■

According to Proposition 1, there exist at most two stationary states,  $P_1 = (K^*, E_1, L^*)$  and  $P_2 = (K^*, E_2, L^*)$ , and their coordinates only differ with respect to the values of  $E$ , which satisfy the condition  $0 < E_1 \leq E_2$ . This occurs since, given our utility function, the marginal rate of substitution is homothetic in consumption and leisure, and hence the environmental externality (i.e. the value of  $E$ ) does not affect the allocation choices (i.e. the value of  $C$  and  $L$ ) and capital accumulation in the stationary states (see, e.g., Azariadis et al., 2013; Xepapadeas, 2005).

If  $\varepsilon < \frac{(1-\alpha+\gamma)\bar{E}^2}{4(1-\alpha)} \left(\frac{\rho}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$ , being  $E_1 < E_2$ , the stationary state  $P_1$  is Pareto-dominated by  $P_2$ ; that is,  $P_1$  is a poverty trap, when it is attractive. It is easy to check (see the proof of Proposition 2) that  $E_1 \leq \bar{E}/2 \leq E_2$ , where

$\bar{E}/2$  represents the maximum sustainable yield, namely, the value of  $E$  which maximizes  $\dot{E}$  in the logistic equation (3).

According to Proposition 2, the stationary state  $P_2$  can be (generically) reached by the economy only when it is a saddle with a two-dimensional stable manifold. In such a case, it possesses saddle-point stability: given initial conditions  $K(0)$  and  $E(0)$  close enough to the values of  $K$  and  $E$  in  $P_2$  ( $K^*$  and  $E_2$ , respectively), then there generically exists a unique initial value  $L(0)$  of the jumping variable  $L$  such that the trajectory starting from  $(K(0), E(0), L(0))$  converges to  $P_2$ . Furthermore, Proposition 2 shows that the economy can converge to the Pareto-dominated stationary state  $P_1$  only when it is attractive. In this case, the dynamics are locally indeterminate (see Benhabib and Farmer, 1999): given initial conditions  $K(0)$  and  $E(0)$  close enough to the values of  $K$  and  $E$  in  $P_1$  (that is,  $K^*$  and  $E_1$ ) there exists a continuum of initial values  $L(0)$  of the jumping variable  $L$  such that the trajectory starting from  $(K(0), E(0), L(0))$  converges to  $P_1$ . On the contrary, when  $P_1$  is a saddle with a one-dimensional manifold, then it cannot be generically reached.

It is worth to stress that the stationary state  $P_1$  can be attractive only if  $\delta > 1$ , that is, if  $C$  and  $E$  are Edgeworth substitutes (the marginal utility of  $C$  increases as the stock  $E$  of the environmental good decreases). In other words, when replacing the environmental good with private consumption provides higher marginal utility to the agents, the latter will be induced to work increasingly more to afford higher consumption levels, but this ends up leading the economy on the “wrong” path so to speak, namely, on a welfare-reducing trajectory converging to  $P_1$ . Furthermore, as shown in the numerical simulations illustrated in the following section, the substitutability between  $E$  and  $C$  may give rise to global indeterminacy scenarios: given the initial conditions  $K(0)$  and  $E(0)$ , both the stationary states  $P_1$  (or a limit cycle surrounding it) and  $P_2$  can be reached, varying the initial choice  $L(0)$  of the jumping variable  $L$ . Such global indeterminacy scenarios derive from a lack of coordination among economic agents as each one takes the stock  $E$  of the environmental resource as exogenously given.

If, on the contrary,  $\delta \in (0, 1)$  (i.e.  $C$  and  $E$  are Edgeworth complements), then the economy is unlikely to “fall into a poverty trap” (i.e.  $P_1$  cannot generically be reached), and the unique stationary state that can be (generically) reached by the economy is the Pareto dominant one,  $P_2$ . Consequently, the global indeterminacy scenarios illustrated in the next section cannot occur.

## 5 Numerical simulations

To dig deeper into the model and its implications, in this section we perform some numerical simulations that allow to visualize the possible dynamics emerging from the analysis. The following set of parameter values have been used in the simulations:  $\bar{E} = 1.3$ ;  $\alpha = 0.3$ ;  $\beta = 24.84$ ;  $\gamma = 1.3$ ;  $\delta = 1.03$ ;  $\rho = 0.04$ . The parameter values have been selected so as to illustrate the most interesting and representative dynamics deriving from the analysis. The value of the

parameter  $\varepsilon$  is free to vary to show how results change at different levels of the environmental impact of production.

The first graph (cf. Figure 1) shows a bifurcation diagram obtained by letting  $\varepsilon$  range between 0.25 and 0.5059. Consider first the higher parabola in Fig.1 and let us proceed from higher to lower values of  $E$  along the vertical axis. The upper branch of that parabola (indicated in black in the figure, for  $E$  approximately above 0.6) shows the values of  $E$  corresponding to the Pareto-dominant stationary state  $P_2$ . The latter can be reached, being a saddle point with two negative eigenvalues, only if the agents coordinate themselves on its two-dimensional stable manifold. The lower branch of the parabola (for  $E$  approximately below 0.6 where the vertex of the parabola is located) shows the values of  $E$  corresponding to the poverty trap  $P_1$ . The latter can be divided in two portions:  $P_1$  is locally attractive along the red portion, while along the blue portion it is a saddle with one-dimensional stable manifold (hence generically it cannot be reached) surrounded by an attractive limit cycle generated by a Hopf-bifurcation.<sup>7</sup> The red parabola-like curve in Figure 1 shows, instead, the maximum and minimum values of  $E$  along the limit cycle. The diagram shows, therefore, that both local and global indeterminacy can occur in the model. Indeed, for given initial values of  $K$  and  $E$ , by choosing different values of  $L$  we can converge to the poverty trap  $P_1$  (or to a limit cycle around it) following different transition paths, which denotes local indeterminacy.

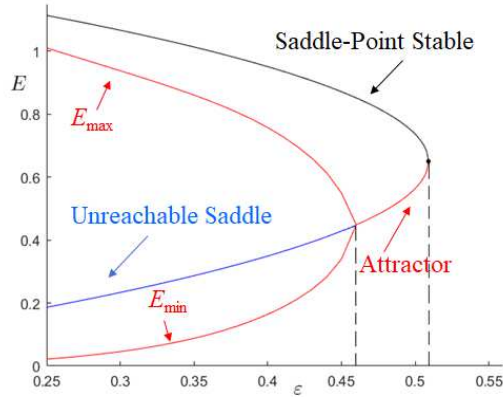


Figure 1. Bifurcation diagram with respect to  $\varepsilon$ .

Furthermore, by selecting different values of  $L$  we can converge either to the poverty trap  $P_1$  (or to a limit cycle around it) or to the Pareto-dominant point  $P_2$ , which denotes global indeterminacy.

<sup>7</sup>The value of the environmental impact of production at the Hopf bifurcation is  $\varepsilon = 0.461$ , whereas it is  $\varepsilon = 0.509$  at the birth of the stationary states (i.e. at the vertex of the upper parabola).

To better visualize the local indeterminacy scenario described above, Figure 2 illustrates two trajectories converging to the poverty trap  $P_1$ . As emerges from Figure 2, the economy can eventually converge to the same stationary state starting from the same initial values of  $K$  and  $E$ , but different initial values of  $L$ .<sup>8</sup>

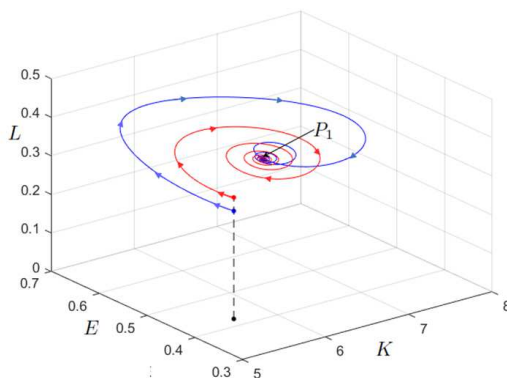


Figure 2. Local indeterminacy

By computing the values of the maximized objective function  $V(K(0), E(0))$  (see (5)) associated to the trajectories in Figure 2, we find that agents get a higher utility by choosing the lower initial level of  $L$  (the starting point of the blue trajectory) instead of the higher one (the starting point of the red trajectory):  $V(K(0), E(0))|_{L=L_{low}} = -536.32 > V(K(0), E(0))|_{L=L_{high}} = -624.70$ .

Figure 3 shows a trajectory converging to a locally attractive limit cycle  $\Gamma$  around  $P_1$  in the three-dimensional space  $(K, E, L)$ ;<sup>9</sup> such a trajectory fluctuates around  $P_1$  (which looks as an “hurricane eye”) for ever.

Figure 4 illustrates, instead, a scenario of global indeterminacy: starting from initial conditions  $K(0)$  and  $E(0)$  close enough to the values of  $K$  and  $E$  in  $P_2$ , both stationary states  $P_1$  (the Pareto-dominated one) and  $P_2$  can be reached starting from different initial choices  $L(0)$  of the jumping variable  $L$ .

<sup>8</sup>The initial values of the state variables are  $K = 5.6027$  and  $E = 0.423$  for both trajectories, whereas labour  $L$  is initially equal to 0.28 for the lower (blue) trajectory and 0.315 along the upper (red) trajectory. The coordinates of the stationary state  $P_1$  to which the system eventually converges are (6.2252, 0.47, 0.35). The figure has been obtained assuming  $\varepsilon = 0.47$ .

<sup>9</sup>The trajectory drawn in Figure 3 starts from the initial values:  $K = 4.3577$ ,  $E = 0.4244$ ,  $L = 0.245$ . The coordinates of the stationary point  $P_1$  are (6.2252, 0.5635, 0.35). The figure has been obtained assuming  $\varepsilon = 0.45$ .

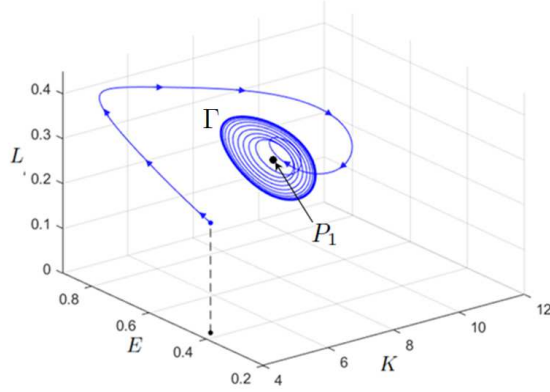


Figure 3. A locally attractive limit cycle

Notice that, in the three-dimensional space  $(K, E, L)$ , the initial value of labour is higher along the blue trajectory leading to the poverty trap  $P_1$  than along the red one leading to  $P_2$ .<sup>10</sup>

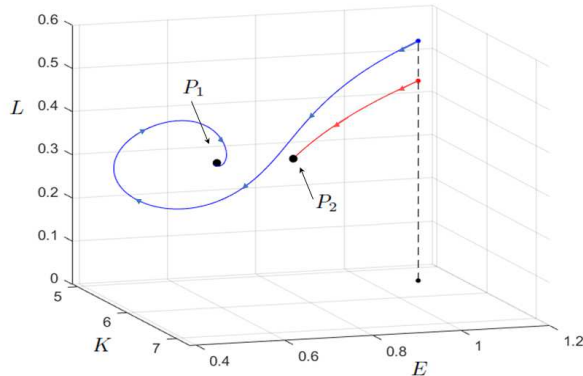


Figure 4. Global indeterminacy scenario where  $P_1$  is locally attractive while  $P_2$  is a saddle with a two-dimensional stable manifold.

By computing the values of  $V(K(0), E(0))$  associated to the trajectories in Figure 4, we find that the utility  $(V(K(0), E(0)))|_{L=L_{low}} = -215.32$  evaluated

<sup>10</sup>The initial value of labour is  $L(0) = 0.554959705397776$  for the blue trajectory leading to  $P_1$  and  $L(0) = 0.462466421164813$  for the red transition path leading to  $P_2$ . The initial values of capital and environment along the trajectories drawn in Figure 4 are  $K(0) = 5.44851477221256$  and  $E(0) = 1.10673341382514$ . The coordinates of the stationary points are  $P_1 = (6.225232318, 0.5634869258, 0.35)$  and  $P_2 = (6.225232318, 0.7365130742, 0.35)$ . Notice that these points do not lie on the vertical wall of the cube but are located in front of it. The figure has been obtained assuming  $\varepsilon = 0.5$ .

along the red trajectory, starting from a lower initial level of  $L$ , is higher than the utility  $(V(K(0), E(0)))|_{L=L_{high}} = -511.36$  corresponding to the blue trajectory, converging to  $P_1$ . This suggests a sort of “overshooting” process in the individuals’ labour choice: if people “work and consume too much”, this leads them to the Pareto-dominated outcome  $P_1$ , in which the stock of the environmental resource ends up being lower than in the first-best outcome  $P_2$ . Stated differently (and somehow provocatively), one can conclude that people would all be better-off by working less. This conclusion may look surprising at first sight, as agents are assumed to be rational in the present context, but it can be easily explained as the outcome of a coordination failure. Indeed, economic agents are unable to coordinate their choices in this context, since each of them takes the stock  $E$  of the environmental resource as exogenously given. Global indeterminacy, in our model, is generated by the substitution process between the private good and the environmental good. Economic agents react to the depletion of the environmental resource by increasing their labour input, which allows them to produce a higher quantity of private good which is consumed as substitute for the environmental resource. The consequent increase in production and consumption of the private good generates a further reduction in the stock of the environmental resource, and so on.

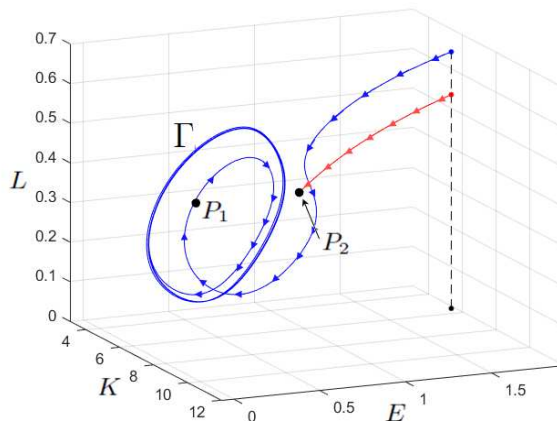


Figure 5. Global indeterminacy scenario where  $P_1$  is a saddle with a one-dimensional stable manifold, surrounded by a locally attractive cycle, while  $P_2$  is a saddle with a two-dimensional stable manifold.

Finally, Figure 5 illustrates a global indeterminacy scenario in which a locally attractive limit cycle (arisen via a Hopf bifurcation) coexists with  $P_2$ , which is saddle-point stable.<sup>11</sup> Notice that the trajectory approaching  $P_2$  starts

<sup>11</sup>The initial value of labour is  $L(0) = 0.6472817136$  for the blue trajectory leading to  $P_1$  and  $L(0) = 0.539401428034914$  for the red transition path leading to  $P_2$ . The initial values of capital and environment for the trajectories drawn in Figure 5 are  $K(0) =$

from an initial value  $L(0)$  lower than the initial value of the trajectory converging to the cycle. Furthermore, the value of  $V(K(0), E(0))$  along the former is higher than along the latter, that is  $V(K(0), E(0))|_{L=L_{low}} = -35.73 > V(K(0), E(0))|_{L=L_{high}} = -641.38$ .

## 6 Taxing negative externalities

In this section we introduce a tax on output  $Y$ , charging for negative externalities. For simplicity, we assume that  $Y$  is taxed at a constant rate  $\tau \in (0, 1)$ , and that the revenues  $\tau Y$  are used for environmental protection (environmental defensive expenditures). In such a context, equations (2) and (4) become:

$$\begin{aligned}\dot{K} &= (1 - \tau)K^\alpha L^{1-\alpha} - C \\ \dot{E} &= E(\bar{E} - E) - \varepsilon \bar{Y} + \sigma \tau \bar{Y}\end{aligned}$$

where the parameter  $\sigma > 0$  measures the impact of defensive expenditures on the dynamics of  $E$ .

By applying the Maximum Principle, and following the same steps illustrated in Section 4, we get the dynamic system:

$$\dot{K} = \frac{1 - \tau}{\gamma} \frac{K^\alpha}{L^\alpha} [L(1 - \alpha + \gamma) - (1 - \alpha)] \quad (17)$$

$$\dot{E} = E(\bar{E} - E) + (\sigma\tau - \varepsilon)K^\alpha L^{1-\alpha} \quad (18)$$

$$\dot{L} = f(L) \left[ \rho - \alpha(1 - \tau) \frac{L^{1-\alpha}}{K^{1-\alpha}} + \frac{\alpha\delta}{K} \dot{K} - \frac{\beta(1 - \delta)}{E} \dot{E} \right] \quad (19)$$

The following proposition is analogous to Proposition 1, and deals with the existence conditions of stationary states under the taxation mechanism described above.

**Proposition 3** *Let  $\tilde{L}^* = \frac{1-\alpha}{1-\alpha+\gamma}$ ,  $\tilde{K}^* = \left[ (1-\tau) \frac{\alpha}{\rho} \right]^{\frac{1}{1-\alpha}} \tilde{L}^*$  and consider the equation*

$$E(\bar{E} - E) = (\varepsilon - \sigma\tau) \left( \tilde{K}^* \right)^\alpha \left( \tilde{L}^* \right)^{1-\alpha}. \quad (20)$$

*i) There exist two stationary states of system (17)-(19),  $Q_1 = (\tilde{K}^*, \tilde{E}_1, \tilde{L}^*)$  and  $Q_2 = (\tilde{K}^*, \tilde{E}_2, \tilde{L}^*)$ , where  $0 < \tilde{E}_1 \leq \tilde{E}_2$  are the real solutions of the equation of (20) if:*

$$\sigma\tau < \varepsilon \leq \sigma\tau + \frac{\bar{E}^2(1 - \alpha + \gamma) \left( \frac{\rho}{(1-\tau)\alpha} \right)^\alpha}{4(1 - \alpha)}. \quad (21)$$

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5.44851477221256 and  $E(0) = 1.10673341382514$ . The coordinates of the stationary points are  $P_1 = (6.225232318, 0.3491884152, 0.35)$  and  $P_2 = (6.225232318, 0.9508115848, 0.35)$ . The figure has been obtained assuming  $\varepsilon = 0.4$ .

ii) There exists a unique stationary state,  $Q_2 = (\tilde{K}^*, \tilde{E}_2, \tilde{L}^*)$  (where  $\tilde{E}_2 > \tilde{E}_1$  is the unique positive solution of (20)), if:

$$\varepsilon \leq \sigma\tau. \quad (22)$$

iii) No stationary state exists if neither condition (21) nor condition (22) hold.

The proof of Proposition 3 follows the same steps of the proof of Proposition 1. From comparison of Propositions 1 and 3, one can notice that the value of  $K$  at the stationary state turns out to be lower if output taxation is introduced in the economy, as expected. The stationary state value of labor, instead, is not affected by taxation (i.e. is the same as in Proposition 1), which reflects the property of the utility function assumed in the model (labor being constant as the output changes).

Proposition 3 suggests that the economy can converge to a unique stationary state  $Q_2$  with high environmental quality (i.e. high level of the environmental resource). However, this requires taxation ( $\tau$ ) and the efficacy of defensive expenditures ( $\sigma$ ) to be sufficiently high with respect to the negative impact on  $E$  of the production activity ( $\varepsilon$ ), so that condition (22) is satisfied. As  $\varepsilon$  increases above  $\sigma\tau$  (within the range of values indicated in (21)) an additional (less desirable) stationary state  $Q_1$  with lower environmental quality arises. Finally, further increases in  $\varepsilon$  (above the upper bound of (21)) lead to a situation without stationary states.

The economy can thus shift across different situations depending on the relative values of  $\varepsilon$  and  $\tau$ . This is clearly illustrated in Figure 6 below. The figure shows, in the parameter plane  $(\varepsilon, \tau)$ , the regions in which: a) only the stationary state  $Q_2$  exists (the region in white); b) no stationary state exists (the region in yellow); c) two stationary states exist, with  $Q_1$  locally attractive and  $Q_2$  possessing saddle-point stability (the region in light grey); d) two stationary states exist, with  $Q_1$  non reachable but surrounded by an attractive limit cycle, and  $Q_2$  possessing saddle-point stability (the region in dark grey). Crossing the line which separates the dark gray region from the light gray one gives rise to a Hopf bifurcation (which generates the attractive cycle).

Figure 6 thus confirms the results pointed out above, suggesting that both local and global indeterminacy hold in the model even when allowing for output taxation. However, it also enriches the possible insights deriving from the model, showing that the Pareto-dominated stationary state disappears if taxation is sufficiently high and the environmental impact of production is sufficiently low.

Consider, for instance, the case in which the environmental impact of production is constant (say,  $\varepsilon = 0.6$  in the figure), and suppose to progressively increase the taxation level, thus moving horizontally and rightward in the diagram. The economy will go through all four regions as  $\tau$  increases, passing from a situation without stationary states (yellow area) up to the case in which only the Pareto-dominant stationary state  $Q_2$  exists (white area). As one might expect, the latter area (representing the preferable outcome) will be reached much earlier (at lower  $\tau$  levels) and without crossing the four different regions if  $\varepsilon$  is relatively low (say,  $\varepsilon = 0.2$  in the figure). On the contrary, the white area

will not be reached if  $\varepsilon$  is sufficiently high (above 0.7 in the diagram). In this case, in fact, even a high taxation level may be unable to counterbalance the environmental effects provoked by production.<sup>12</sup>

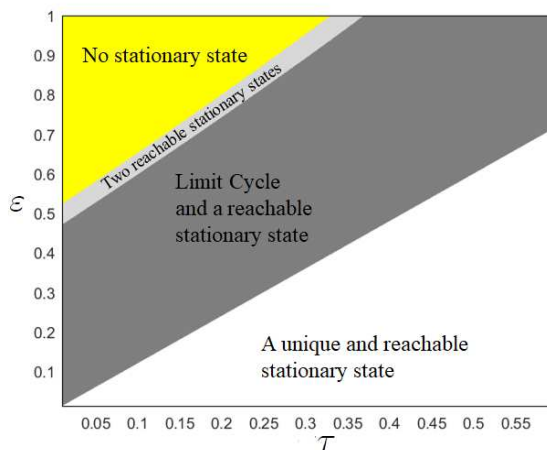


Figure 6. Bifurcation diagram in the parameter space  $(\tau, \varepsilon)$ . Parameter set:  $\bar{E} = 1.3$ ,  $\alpha = 0.3$ ,  $\beta = 24.84$ ,  $\gamma = 1.3$ ,  $\delta = 1.03$ ,  $\rho = 0.04$ ,  $\sigma = 1.2$ .

## 7 Conclusions

A frequently observed phenomenon in modern societies is the progressive substitution of environmental public goods with private consumption goods. What was once freely available as a gift from nature (e.g. clean air and water, green areas, fertile lands etc.) it is nowadays often hardly enjoyable either because it has become highly polluted or because it has gone extinct. The increasing depletion of the environment has progressively induced economic agents to look for costly alternatives in terms of private consumption goods that can satisfy the same needs. In the current debate on how to react to environmental problems (mitigation versus adaptation), this process can be seen as a sort of spontaneous private adaptation policy adopted by people in response to growing environmental degradation. From a theoretical viewpoint, this substitution process is in line with the idea of substitutability underlying the well-known notion of weak sustainability (Solow, 1974, 1986, 1993; Hartwick, 1977, 1978). Moreover, this mechanism of replacing the environment seems to reveal an anthropogenically-focused vision of nature, which gives mankind central stage in the universe.

<sup>12</sup>Notice that results depend also on the value of  $\sigma$  (assumed constant in Figure 6), which measures how effective tax revenues are in preserving the environment. The larger is  $\sigma$  the higher is the efficacy of output taxation in counterbalancing the negative effects of production and so the larger will also be the set of values leading to the desirable white area in the diagram.

Although the philosophical discussion on these aspects is certainly fascinating, here we want to stress a much more simple and pragmatic aspect: no matter whether replacing the environment with private consumption goods is feasible and ethically correct, it may lead to a highly uncertain and welfare-reducing outcome. To show that this is the case, we have investigated an intertemporal optimization problem characterized by a continuum of identical, perfectly rational agents who derive their utility from leisure, consumption and the stock of the environment (which is degraded by the overall production level) and decide how much to work and consume so as to maximize their own utility. The analysis of the model shows that there may exist at most two stationary states,  $P_1$  and  $P_2$ , the former being Pareto-dominated by the latter. Moreover, as proved above:

1. If consumption  $C$  and environment  $E$  are Edgeworth substitutes, then the poverty trap  $P_1$  is either an attractor or a saddle with a one-dimensional stable manifold (i.e. it is not generically reachable)
2. If  $C$  and  $E$  are Edgeworth complements, then the poverty trap  $P_1$  is always a saddle with a one-dimensional stable manifold.
3. The stationary state  $P_2$  is either a repeller or a saddle with a two-dimensional stable manifold.

Four main findings emerge from the analysis. First, the poverty trap  $P_1$  can be attractive only if the private consumption good  $C$  is an Edgeworth substitute for the public environment good  $E$ , namely, only if a lower stock of the environmental good increases the marginal utility of consumption. If, on the contrary, a lower environmental stock decreases the marginal utility of consumption (i.e.  $C$  and  $E$  are Edgeworth complements), then  $P_1$  cannot be attractive. This applies to all forms of consumption which cannot be detached from the presence (and quality) of the environmental good to be enjoyed by consumers. Think, for instance, of the costly purchase of the equipment for fishing or scuba diving which gives decreasing marginal utility as the sea gets more and more polluted (since pollution reduces the stock of fishes that can be captured or seen in the sea). Similarly, the marginal utility of an open-air picnic is likely to decrease as the surrounding environment gets more and more depleted (a picnic at the beach, the lake or the mountain becomes much less enjoyable if nature around is filled with litter than if it is well preserved). In the real world, consumption goods obviously differ with respect to the feature described above: some are Edgeworth substitutes, others Edgeworth complements. In a simplified, aggregate model like the one proposed in this paper the prevailing effect between these two opposite forces is what determines at the end of the day whether the poverty trap will be reached or not.

In the second place, our findings suggest that both local and global indeterminacy may arise in the model. This result holds even when extending the model to introduce output taxation and environmental defensive expenditures proportional to tax entries. Local indeterminacy occurs whenever, for given

initial values of  $K$  and  $E$ , there exists a continuum of initial values of  $L$  such that the trajectory converges to an attractive stationary state or limit cycle. In this case, therefore, the transition dynamics (i.e. the trajectory leading to the stationary state or to the limit cycle) is indeterminate: We know the stationary state (limit cycle) that will eventually be reached by the trajectory, but not how to get there. In the case of global indeterminacy, instead, we do not even know where the trajectories will eventually lead to. This occurs whenever, for given initial values of  $K$  and  $E$ , both the stationary states  $P_1$  (or a limit cycle around it) and  $P_2$  can be reached starting from different initial values of the jumping variable  $L$ . It is remarkable that local and global indeterminacy (and the accompanying strong uncertainty on the final outcome of human activities) arise in the model although economic agents are perfectly rational but they are unable to coordinate their activities. The agents' rationality, therefore, cannot prevent the unpredictability of the aggregate environmental effects, which is the effect of a coordination failure.

In the third place, both uncertainty and irreversibility may occur in the model. Indeed, as shown in the paper, the trajectories can converge to the Pareto-dominated attractor  $P_1$ , so that the economy is eventually and irreversibly trapped in a stationary state characterized by a high level of environmental degradation. To avoid this undesirable situation, taxation should be sufficiently high and the environmental impact of production sufficiently low. Under these simultaneous conditions, in fact, the Pareto-dominated stationary state  $P_1$  (and thus also indeterminacy) may disappear.

Finally, it is worth stressing that the indeterminacy can be excluded even if the production activity has no negative impact on the environment ( $\varepsilon$  being zero). Indeed, if a totally green technology were developed with no detrimental impact on the environment, then a unique saddle point would emerge in the model. The latter would have a two-dimensional stable manifold: given the initial values of  $K$  and  $E$ , there exists a unique initial value of  $L$  from which converge to the stationary point, so that no uncertainty would arise with green technologies.

## References

- [1] Adejuwon J., Azar C., Baethgen W., Hope C., Moss R., Leary N., Richels R. and van Ypersele J.P., 2001. Overview of impacts, adaptation, and vulnerability to climate change. In *Climate Change 2001: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Third Assessment Report of the Intergovernmental Panel on Climate Change*, 75–103.
- [2] Antoci A., Bartolini S., 2004. Negative externalities, defensive expenditures and labour supply in an evolutionary context. *Environment and Development Economics* 9(5), 591–612.

- [3] Antoci A., Galeotti M., Russu P., 2005. Consumption of private goods as substitutes for environmental goods in an economic growth model. *Nonlinear Analysis: Modelling and Control* 10, 3–34.
- [4] Antoci A., Galeotti M., Russu P., 2007. Undesirable economic growth via economic agents' self-protection against environmental degradation. *Journal of The Franklin Institute* 344, 377–390.
- [5] Antoci A., Borghesi S., 2012. Preserving or escaping? On the welfare effects of environmental self-protective choices. *Journal of Socio-Economics* 41, 248–254.
- [6] Antoci A., Gori L., Sodini M., Ticci E., 2019. Maladaptation and global indeterminacy. *Environment and Development Economics* 24, 643–659.
- [7] Azariadis C., Chen C.-L., Lu C.-H., Wang Y.-C., 2013. A two-sector model of endogenous growth with leisure externalities. *Journal of Economic Theory* 148, 843–857.
- [8] Barnett J., O'Neill S., 2010. Maladaptation. *Global Environmental Change* 2, 211–213.
- [9] Bella G., Mattana P., Venturi B., 2017. Shilnikov chaos in the Lucas model of endogenous growth. *Journal of Economic Theory* 172, 451–477.
- [10] Benhabib J., Farmer R.E., 1999. Indeterminacy and sunspots in macroeconomics. In: J.B. Taylor, M. Woodford (eds.), *Handbook of Macroeconomics*, North-Holland, Amsterdam, 387–448.
- [11] Caravaggio A., Sodini M., 2018. Nonlinear dynamics in coevolution of economic and environmental systems. *Frontiers in Applied Mathematics and Statistics* 4, 1–17.
- [12] Climate Strike, 2019. [Climatestrike.net](https://climatestrike.net). Retrieved 2019-09-13.
- [13] Gomez Suarez M. A., 2008. Utility and production externalities, equilibrium efficiency and leisure specification. *Journal of Macroeconomics* 30, 1496–1519.
- [14] Hartwick J.M., 1977. Intergenerational equity and the investing of rents from exhaustible resources. *The American Economic Review* 67(5), 972–974.
- [15] Hartwick J.M., 1978. Substitution among exhaustible resources and intergenerational equity. *The Review of Economic Studies* 45(2), 347–354.
- [16] Hirsch F., 1976. *Social Limits to Growth*. Harvard University Press, Cambridge, Ma.

- [17] IPCC, 2014. Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Core Writing Team, Pachauri R.K. and Meyer L.A. (eds.)]. IPCC, Geneva, Switzerland.
- [18] IPCC, 2018. Global Warming of 1.5°C: An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty. IPCC, Geneva, Switzerland.
- [19] IPCC, 2019. Climate Change and Land: An IPCC special report on climate change, desertification, land degradation, sustainable land management, food security, and greenhouse gas fluxes in terrestrial ecosystems. IPCC, Geneva, Switzerland.
- [20] Itaya J.-I., 2008. Can environmental taxation stimulate growth? The role of indeterminacy in endogenous growth models with environmental externalities *Journal of Economic Dynamics and Control* 32, 1156-1180.
- [21] Krugman P., 1991. History versus expectations. *Quarterly Journal of Economics* 106, 651-667.
- [22] Ladrón-De-Guevara A., Ortiguera S., Santos M. S., 1999. A two-sector model of endogenous growth with leisure. *The Review of Economic Studies* 66(3), 609-631.
- [23] Liu T., He G., Lau A., 2018. Avoidance behavior against air pollution: evidence from online search indices for anti-PM2.5 masks and air filters in Chinese cities, *Environmental Economics and Policy Studies* 20, 325-363.
- [24] Matsuyama K., 1991. Increasing returns, industrialization, and indeterminacy of equilibrium. *Quarterly Journal of Economics* 106, 617-650.
- [25] Mino K., 2017. Growth and Business Cycles with Equilibrium Indeterminacy. *Advances in Japanese Business and Economics* 13, Springer, Tokyo Japan.
- [26] Pearce D., Markandya A., Barbier E., 1989. *Blueprint for a Green Economy*. Routledge, London.
- [27] Pearce D., Barrett S., Markandya A., Barbier E., Turner R.K., Swanson T., 1991. *Blueprint 2: Greening the World Economy*. Earthscan Publications Ltd, London.
- [28] Sachs J., Schmidt-Traub G., Kroll C., Lafortune G., Fuller G., 2019. *Sustainable Development Report 2019*. Bertelsmann Stiftung and Sustainable Development Solutions Network (SDSN), New York.

- [29] Shogren J.F., Crocker T.D., 1991. Cooperative and noncooperative protection against transferable and filterable externalities. *Environmental and Resource Economics* 1, 195–214.
- [30] Solow R.M., 1974. Intergenerational equity and exhaustible resources. *Review of Economic Studies: Symposium of the Economics of Exhaustible Resources* 29–46.
- [31] Solow R.M., 1986. On the intergenerational allocation of natural resources. *Scandinavian Journal of Economics* 88(1), 141–9.
- [32] Solow R. M., 1993. An almost practical step towards sustainability. *Resources Policy* 16, 162–72.
- [33] Sun C., Kahn M. E. and Zheng S., 2017. Self-protection investment exacerbates air pollution exposure inequality in urban China. *Ecological Economics* 131, 468–474.
- [34] Surminski S., 2013. Private-sector adaptation to climate risk. *Nature Climate Change* 3, 943–945.
- [35] UNEP, 2019. *Frontiers 2018/2019: Emerging Issues of Environmental Concern*. United Nations Environment Programme, Nairobi.
- [36] United Nations, 1993. *Handbook of National Accounting: Integrated Environmental and Economic Accounting*. Studies in Methods, Series F, No. 61, Sales No. E. 93 XVII.12. United Nations, New York.
- [37] United Nations, 2003. Commission of the European Communities, International Monetary Fund, Organisation for Economic Cooperation and Development, World Bank. *Handbook for Integrated Environmental and Economic Accounting*. United Nations, New York.
- [38] Williams A.M., 2019. Understanding the micro-determinants of defensive behaviors against pollution. *Ecological Economics* 163, 42–51.
- [39] Wirl F., 1997. Stability and limit cycles in one-dimensional dynamic optimizations of competitive agents with a market externality. *Journal of Evolutionary Economics* 7, 73–89.
- [40] Xepapadeas A., 2005. Economic Growth and the Environment, pp. 1219–1271. In Maler K.-G., Vincent J. R. (eds.) *Economywide and International Environmental Issues*, vol. 3 of the *Handbook of Environmental Economics*. North-Holland.
- [41] Yang J., Zhang B., 2018. Air pollution and healthcare expenditure: Implication for the benefit of air pollution control in China. *Environment International* 120, 443–455.

- [42] Zhang J., Mu Q., 2018. Air pollution and defensive expenditures: Evidence from particulate-filtering facemasks. *Journal of Environmental Economics and Management* 92, 517–536.